

Modern Michelson-Morley Experiment using Cryogenic Optical Resonators

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(Received 27 January 2003; published 10 July 2003)

We report on a new test of Lorentz invariance performed by comparing the resonance frequencies of two orthogonal cryogenic optical resonators subject to Earth's rotation over ~ 1 yr. For a possible anisotropy of the speed of light c , we obtain $\Delta_{\theta}c/c_0 = (2.6 \pm 1.7) \times 10^{-15}$. Within the Robertson-Mansouri-Sexl (RMS) test theory, this implies an isotropy violation parameter $\beta - \delta - \frac{1}{2} = (-2.2 \pm 1.5) \times 10^{-9}$, about 3 times lower than the best previous result. Within the general extension of the standard model of particle physics, we extract limits on seven parameters at accuracies down to 10^{-15} , improving the best previous result by about 2 orders of magnitude.

DOI: 10.1103/PhysRevLett.91.020401

PACS numbers: 03.30.+p, 12.60.-i, 06.30.Ft, 11.30.Cp

Special relativity (SR) underlies all accepted theories of nature at the fundamental level. Therefore, it has been and must be tested with ever increasing precision to provide a firm basis for its future application. Such tests are also motivated by the efforts to unify gravity with the other forces of nature, one of the outstanding open challenges in modern science. In fact, many currently discussed models of quantum gravity do violate the principles of SR. In string theory, for example, violation of Lorentz invariance might be caused by spontaneous symmetry breaking [1]; in loop gravity, effective low-energy equations (e.g., modified Maxwell equations [2]) that violate Lorentz invariance have been derived. Since the natural energy scale for quantum gravity is the Planck scale, $E_p \sim 10^{19}$ GeV, a direct experimental investigation of quantum gravity effects is not feasible. It is, however, possible to search for *residual* effects at an attainable energy scale in experiments of outstanding precision. High-precision bounds on Lorentz violation might give valuable hints for or against particular models of quantum gravity.

A sensitive probe for Lorentz violation (in electrodynamics) is the Michelson-Morley (MM) experiment [3], that even predated the formulation of SR. It tests the isotropy of the speed of light, one of SR's foundations. In the classic setup, one compares the speed of light c in two orthogonal interferometer arms by observing the interference fringes. If c depends on the direction of propagation, the fringes move if the setup is rotated (using, e.g., Earth's rotation or a turntable). In 1979, Brillet and Hall [4] introduced the modern technique of measuring a laser frequency stabilized to a resonance of an optical reference cavity. The frequency of such a resonance is given by $\nu_{\text{cav}} = mc/(2L)$, where L denotes the cavity length and $m = 1, 2, \dots$ the mode number. Thus, a violation of the isotropy of c can be detected by rotating the cavity and looking for a resulting variation of ν_{cav} by comparing the frequency of the cavity-stabilized laser against a suitable reference. In our experiment

(Fig. 1), we compare the frequencies ν_x and ν_y of *two* similar cavities oriented in orthogonal directions, in analogy with the classical interferometer tests. Compared to the single-cavity setup, this arrangement doubles the hypothetical signal amplitude and provides some common-mode rejection of systematic effects.

Our results are obtained making use of the high dimensional stability of cryogenic optical resonators (COREs). Constructed from crystalline sapphire, COREs show a low thermal expansion coefficient ($10^{-10}/\text{K}$ at 4.2 K) and a remarkable absence of creep (i.e., intrinsic length changes due to material relaxation); upper limits for the CORE frequency drift are < 2 kHz/6 month and

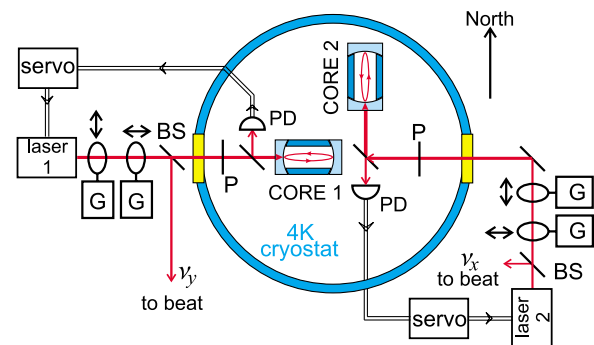


FIG. 1 (color online). Setup: Inside a 4 K cryostat, two COREs are located in a copper block to provide common-mode rejection of thermal effects. LN2 is refilled automatically every ~ 3 h, LHe manually every ~ 2 days. Laser beams are coupled to the COREs via windows, with polarizers P and lock detectors PD inside the cryostat. For active beam positioning, beams pass through galvanometer (G) mounted glass plates. The horizontal and vertical displacements are adjusted to maximize coupling into the cavities, as measured using the $2f_m$ signal from the detector in reflection. Not shown is the setup for measuring the frequency difference $\nu_x - \nu_y$ of the lasers, in which the beams are overlapped on a high-speed photodetector and the beat frequency is measured against a quartz oscillator stabilized to the Global Positioning System.

<0.1 Hz/h [5], which makes COREs particularly well suited for high-precision measurements such as relativity tests [6,7]. For the MM experiment, it allows us to rely solely on Earth's rotation. This avoids the systematic effects associated with active rotation, which previous experiments had to use to overcome the creep of room temperature resonators made from glass ceramics, e.g., ULE (ultralow expansion), on the time scale of a day.

A suitable theoretical framework for analyzing tests of Lorentz invariance is the general standard model extension (SME) [8,9]. A Lagrangian formulation of the standard model is extended by adding all possible observer Lorentz scalars that can be formed from known particles and Lorentz tensors. In the Maxwell sector, the Lagrangian is $\mathcal{L} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu}$ [10]. The tensor $(k_F)_{\kappa\lambda\mu\nu}$ (the greek indices run from 0–3) has 19 independent components; they vanish, if SR is valid. Ten of its components, that can be arranged into traceless symmetric 3×3 matrices $(\tilde{\kappa}_{e+})^{AB}$ and $(\tilde{\kappa}_{o-})^{AB}$, describe polarization-dependent effects. They are restricted to $<2 \times 10^{-32}$ by polarization measurements on light from astronomical sources [9] and are assumed to be zero in the following. The remaining nine components describe boost invariance and isotropy of c and can be arranged into traceless 3×3 matrices $(\tilde{\kappa}_{o+})^{AB}$ (symmetric) and $(\tilde{\kappa}_{e-})^{AB}$ (antisymmetric) plus one additional parameter. They lead to a shift $\delta\nu$ of the resonance frequency of a cavity [9] with a characteristic time signature and can thus be measured in cavity experiments.

For our experiment (Fig. 1), we use two $L = 3$ cm long COREs that feature linewidths of 100 and 50 kHz, respectively (Finesses $\sim 10^5$), located inside a liquid helium (LHe) cryostat with a liquid nitrogen (LN2) shield. Refills and the evaporation of coolants cause mechanical deformations of the cryostat, which change the resonator position. An automatic beam positioning system actively compensates for these movements (Fig. 1). The frequencies ν_{las} of two diode-pumped Nd:YAG lasers at 1064 nm are stabilized (“locked”) to resonances of the COREs using the Pound-Drever-Hall method. The Nd:YAG laser crystal strain (generated by a piezo attached to the crystal) and temperature were used for tuning the frequency. The phase modulation at frequencies f_m around 500 kHz is also generated by crystal strain modulation using mechanical resonances of the piezo. The light reflected from the COREs is detected inside the cryostat (Fig. 1) on Epitaxx 2000 InGaAs detectors. Down-conversion to dc at $3f_m$ rather than $1f_m$ reduces the influence of residual amplitude modulation and gives a higher signal-to-noise (S/N) ratio for the same circulating laser power inside the cavity. For the detector signal, amplifiers consisting of eight paralleled BF1009 dual-gate MOSFETs provide low current noise in spite of the high detector capacitance (500 pF). At ~ 80 nW laser power impinging on the COREs, we achieve an error S/N ratio of $\sim 1.5 \times 10^4 \sqrt{\tau/s}$. Between 80–200 nW have been used, minimizing the change of ν_{cav} due to laser heating

of the COREs (~ 10 Hz/ μ W). The error signal is generated using minicircuits ZHL-32 high-level amplifiers and SAY-1 23 dB m double-balanced mixers operating at ~ 0 dB m rf amplitude and thus well below saturation. This provides highly linear operation and proves very important for a low sensitivity to systematic disturbances. On a time scale of minutes, we reach a minimum relative frequency instability of the lasers locked to the COREs of 7×10^{-16} , referred to a single CORE (Fig. 2). Such a level is reached by the best ULE-cavity stabilized lasers [11] only if a large linear drift ~ 2 Hz/s is subtracted.

A number of systematic disturbances (e.g., residual amplitude modulation, parasitic etalons, or mixer offset voltages) that cause lock-point shifts are compensated for using a new technique: By an additional phase modulation of the laser beams, a part of the laser power is shifted into sidebands, thus reducing the amplitude of the useful part of the error signal. If the parasitic effects do not have sufficient frequency selectivity to discriminate these sidebands against the carrier, modulating the amplitude of the sidebands makes the lock-point shifts time dependent, so they can be identified and compensated for [12]. This proved instrumental in achieving the reliable and repeatable laser system performance required for accumulating enough data.

Except for a 10 d break around January 1, 2002, the COREs were operated continuously at 4.2 K over more than one year. Usable data (discounting data sets shorter than 12 h and data taken during adjustment or LHe refills) starts June 19, 2001, and was acquired over 390 days until July 13, 2002. A total of 146 data sets of 12 h to 109 h in length, totaling 3461 h, are available (Figs. 2 and 3). Forty-nine almost equally distributed data sets are longer than 24 h. For extracting results, simultaneous least-squares fits with a constant offset, a linear drift, and the amplitude of a sinusoidal signal at fixed frequency and phase as suggested by the test theory are performed. Before fitting, the data sets are divided into subsets of 12 h (or 24 h for the 24 h signals). This drops a fraction of the data, but makes the resulting sinewave amplitudes independent of offsets in the data. We obtain 199 fits of 12 h data subsets (Fig. 3).

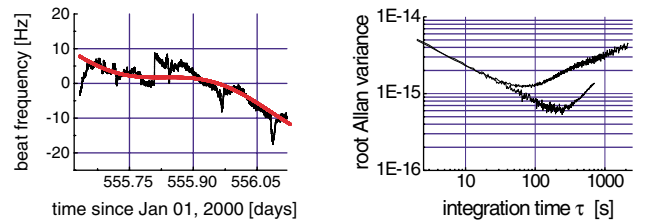


FIG. 2 (color online). Left: Typical data set fitted with a 12 h sinewave amplitude, a linear drift, and a constant offset. Peaks occur every few hours due to automatic LN2 refills. Right: Root Allan variance calculated from this data (upper curve), and from a quiet part between two LN2 refills (118 min starting at 555.87 days; lower curve).

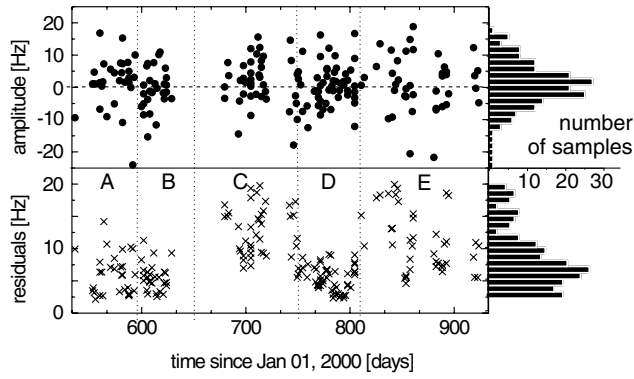


FIG. 3. Fit results (upper part; dots) and fit residuals $\sqrt{\chi^2}$ (lower part; crosses) for the amplitude of the cosine component of an assumed isotropy violation signal at $2\omega_{\oplus}$. Strongly perturbed data sets are omitted when $\sqrt{\chi^2} > 20$ Hz (25 subsets). In eight of these cases, a good fit was achieved after discarding the first 1–3 h (which likely contain residual perturbations from a preceding LHe refill); these were then reintroduced to the analysis. The histograms (bin size = 1 Hz) on the right-hand side give the distribution of the fit results and residuals.

The individual fit results (Fig. 3) are combined for the final result by coherent (vector) averaging. In the SME, the hypothetical Lorentz violation signal $(\nu_x - \nu_y)/\bar{\nu} = \sum_i A_i^S \sin(\omega_i T_{\oplus}) + A_i^C \cos(\omega_i T_{\oplus})$, where $\bar{\nu} = (\nu_x + \nu_y)/2$, has Fourier components at six frequencies ω_i . As defined in [9], $T_{\oplus} = 0$ on March 20, 2001, 11:31 UT. The signal components as given in Table I are derived [13] from Eq. (39) in [9] (the cavity length L is not substantially affected by the hypothetical Lorentz violation [14]). Because our data extends over more than one year, in the vector average we can resolve the six signal frequencies [15] and extract elements of the traceless symmetric matrix

$$\tilde{\kappa}_{e-} = \begin{pmatrix} a & 1.7 \pm 2.6 & -6.3 \pm 12.4 \\ 1.7 \pm 2.6 & b & 3.6 \pm 9.0 \\ -6.3 \pm 12.4 & 3.6 \pm 9.0 & -(a+b) \end{pmatrix} \times 10^{-15},$$

TABLE I. Signal amplitudes A_i^S, A_i^C from the SME, based on [9], and fit results. $\omega_{\oplus} \approx 2\pi/(23 \text{ h } 56 \text{ min})$ and $\Omega_{\oplus} = 2\pi/1 \text{ yr}$ denote the angular frequencies of Earth's sidereal rotation and orbit; $\chi \approx 42.3^\circ$ is the colatitude of Konstanz, $\eta \approx 23.4^\circ$ the angle between the ecliptic and Earth's equatorial plane. The signal includes contributions of order 1, or suppressed by either Earth's orbital velocity $\beta_{\oplus} \sim 10^{-4}$ or by the velocity of the laboratory due to Earth's rotation $\beta_L \sim 10^{-6}$. For each ω_i , we include only the largest term, in effect dropping all terms proportional to β_L . A fit result that has been used to extract a parameter of the SME is set in bold face. The unused fit results lead to additional (but weaker) limits on the elements of $\tilde{\kappa}_{o+}$.

ω_i	A_i^S	Fit (Hz)	A_i^C	Fit (Hz)
$\omega_{\oplus} - \Omega_{\oplus}$	$\frac{\beta_{\oplus}}{2} \sin\chi \cos\chi [(\cos\eta + 1)(\tilde{\kappa}_{o+})^{XY} - \cos\eta(\tilde{\kappa}_{o+})^{XZ}]$	1.82 ± 1.91	$\frac{\beta_{\oplus}}{2} \sin\chi \cos\chi \sin\eta (\tilde{\kappa}_{o+})^{YZ}$	0.67 ± 1.58
ω_{\oplus}	$-\sin\chi \cos\chi (\tilde{\kappa}_{e-})^{YZ}$	-0.51 ± 1.26	$-\sin\chi \cos\chi (\tilde{\kappa}_{e-})^{XZ}$	0.89 ± 1.74
$\omega_{\oplus} + \Omega_{\oplus}$	$\frac{\beta_{\oplus}}{2} \sin\chi \cos\chi [(\cos\eta - 1)(\tilde{\kappa}_{o+})^{XY} - \sin\eta(\tilde{\kappa}_{o+})^{XZ}]$	-0.43 ± 1.68	$\frac{\beta_{\oplus}}{2} \sin\chi \cos\chi \sin\eta (\tilde{\kappa}_{o+})^{YZ}$	1.83 ± 1.83
$2\omega_{\oplus} - \Omega_{\oplus}$	$-\frac{\beta_{\oplus}}{4} (1 + \cos^2\chi)(\cos\eta + 1)(\tilde{\kappa}_{o+})^{YZ}$	-0.01 ± 0.57	$-\frac{\beta_{\oplus}}{4} (1 + \cos^2\chi)(1 + \cos\eta)(\tilde{\kappa}_{o+})^{XZ}$	0.25 ± 0.55
$2\omega_{\oplus}$	$\frac{1}{2} (1 + \cos^2\chi)(\tilde{\kappa}_{e-})^{XY}$	0.37 ± 0.56	$\frac{1}{4} (1 + \cos^2\chi)[(\tilde{\kappa}_{e-})^{XX} - (\tilde{\kappa}_{e-})^{YY}]$	0.97 ± 0.53
$2\omega_{\oplus} + \Omega_{\oplus}$	$\frac{\beta_{\oplus}}{4} (1 + \cos^2\chi)(1 - \cos\eta)(\tilde{\kappa}_{o+})^{YZ}$	0.50 ± 0.55	$\frac{\beta_{\oplus}}{4} (1 + \cos^2\chi)(1 - \cos\eta)(\tilde{\kappa}_{o+})^{XZ}$	-0.75 ± 0.58

with $a - b = 8.9 \pm 4.9$ (in the sun-centered celestial equatorial reference frame of [9]). Likewise,

$$\tilde{\kappa}_{o+} = \begin{pmatrix} 0 & 14 \pm 14 & -1.2 \pm 2.6 \\ -14 \pm 14 & 0 & 0.1 \pm 2.7 \\ 1.2 \pm 2.6 & -0.1 \pm 2.7 & 0 \end{pmatrix} \times 10^{-11}$$

for the antisymmetric matrix. One sigma errors are quoted. The limits on $\tilde{\kappa}_{o+}$ are weaker because its elements enter the experiment suppressed by $\beta_{\oplus} \sim 10^{-4}$, Earth's orbital velocity. All elements of $\tilde{\kappa}_{o+}$ and all but one element of $\tilde{\kappa}_{e-}$ are obtained. Compared to [16], we improve the accuracy by about 2 orders of magnitude. Moreover, while [16] give limits on linear combinations of the elements of $\tilde{\kappa}_{o+}$, the present experiment allows individual determination, because of the > 1 yr span of our data.

For comparison to previous work (e.g., [4,6,17,18]), we also analyze our experiment within the Robertson-Mansouri-Sexl test framework [19]. It assumes generalized Lorentz transformations that contain parameters α, β , and δ . In a preferred frame Σ (usually the cosmic microwave background), the speed of light $c_0 = \text{const}$. In a frame S moving with the velocity \vec{v} with respect to Σ , $c/c_0 = 1 - (A + B\sin^2\theta)v^2/c_0^2$, where θ is the angle between the direction of c and \vec{v} . Both $A = \alpha - \beta + 1$ and $B = \beta - \delta - \frac{1}{2}$ vanish in SR. In this formalism, SR follows from (i) Kennedy-Thorndike (KT), (ii) MM, and (iii) Doppler shift experiments. The latter determine the time dilation coefficient α ($= -\frac{1}{2}$ in SR) to $|\alpha + \frac{1}{2}| < 8 \times 10^{-7}$ [18]. KT experiments test velocity invariance of c , using the periodic modulation of v provided by Earth's orbit [6] or rotation [17]. Measuring the frequency of a cryogenic microwave cavity (that is proportional to c) against an H maser, [17] obtained $A = (-3.1 \pm 6.9) \times 10^{-7}$.

While all three experiments are required for a complete verification of SR within this framework, the MM experiment currently offers the highest resolution. In case of a violation of isotropy, $B \neq 0$, for our experiment we obtain a periodic change of the beat frequency $\nu_x - \nu_y$ at

$2\omega_{\oplus}$ [20]. For such a signal, the experiment yields an amplitude 1.03 ± 0.53 Hz, or $B = (-3.1 \pm 1.6) \times 10^{-9}$. As the quality of the data is not uniform (Fig. 3), taking a weighted vector average is more appropriate here [15]. We divide the data into the intervals A–E (Fig. 3) with approximately uniform data quality within each. The averages over the intervals are then combined to the final result, weighted according to their standard error. This leads to a signal of 0.73 ± 0.48 Hz, or

$$B = \beta - \delta - \frac{1}{2} = (-2.2 \pm 1.5) \times 10^{-9},$$

which we regard as the final result within the RMS framework. It has an inaccuracy about 3 times lower than the best previous result $B = (3.0 \pm 4.9) \times 10^{-9}$ [4,22].

In summary, we performed a modern Michelson-Morley experiment by comparing the frequencies of two crossed cryogenic optical resonators subject to Earth's rotation over a period spanning more than one year. Within the Robertson-Mansouri-Sexl framework, our limit on the isotropy violation parameter is about 3 times lower than that from the classic experiment of Brilliet and Hall [4]. Moreover, we obtain limits on seven parameters from the comprehensive extension of the standard model [8,9], down to 10^{-15} , about 2 orders of magnitude lower than the only previous result [16].

While the high long-term stability of COREs allows one to use solely Earth's rotation, active rotation could still improve the accuracy significantly. Because of the low drift of COREs compared to ULE cavities, the optimum rotation rate would be relatively slow, which is desirable for minimizing the systematics. At a rate of $\sim 0.2/\text{min}$ ($\sim 5/\text{min}$ was used in [4]), one could utilize the optimum $\sim 7 \cdot \times 10^{-16}$ frequency stability (Fig. 2) of the COREs, more than 10 times better than on the 12 h time scale used thus far. Since ~ 500 measurements (2 per turn) could be accumulated per day, one should thus be able to reach the 10^{-17} level of accuracy. Further improvements include fiber coupling and COREs of higher finesse. Finally, space based missions with resonators are currently studied (OPTIS [23] and SUMO [24]).

We thank Claus Lämmerzahl for many valuable discussions and Jürgen Mlynek for making this project possible. This work has been supported by the Deutsche Forschungsgemeinschaft and the Optik-Zentrum Konstanz.

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