Scalable Quantum Processor with Trapped Electrons

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A quantum computer can be implemented by trapping electrons in vacuum within an innovative confining structure. Universal processing is realized by controlling the Coulomb interaction and applying electromagnetic pulses. This system offers scalability, high clock speed, and low decoherence.

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Penning traps are able to confine charged particles by means of a combination of a static quadrupole potential with a homogeneous magnetic field [1]. Experiments with a single electron have been very successful over the past years and have achieved an astonishing precision in determining the value of fundamental constants [2]. Moreover, the experimental control has reached the quantum level for the cyclotron motion of the electron, which has been prepared in the lowest Fock states [3]. Furthermore, the ground state cooling of the axial motion is within the current experimental capabilities [2,4]. In addition to the extremely high accuracy in controlling and detecting the particle dynamics, the system is also well isolated from the environment and almost unaffected by dissipative decoherence. For these reasons, a trapped electron has been proposed as a good candidate to implement an elementary quantum processor [5,6]. By exploiting the electron quantized degrees of freedom, we can encode up to three qubits in each particle [6,7]. However, the real challenge is to devise a scalable quantum computer where the number of qubits is, in principle, unlimited. This can be achieved only by using more than one particle. It is, therefore, fundamental to extend the existing schemes to trap and manipulate an electron to several particles. The present Letter answers to this demand. The solution to the problem is not simply putting more electrons in the same conventional Penning trap. Theory [8] and experiments [9] show that, when dealing with more than one particle, the resulting dynamics is rather complicated, even chaotic, because of the strong Coulomb repulsion. Hence, we need a completely new trap design to hold and manipulate each particle and, at the same time, to guarantee its single addressability. On the other side, we require the possibility to perform two-particle operations. These tasks can be accomplished by realizing a structure capable of creating a linear array of Penning traps. Each trap confines a single electron which oscillates with its own frequency. When two neighboring particles are put into resonance, they may exchange a quantum of excitation [10]. If we are dealing with the lowest Fock states of the axial motion, i.e., $|0\rangle_z$ and $|1\rangle_z$, this operation amounts to a swapping gate. This ability, combined with a universal set of quantum gates on every single electron, allows us to implement conditional dynamics between different particles. In comparison with rf ion traps [11], we have, at least, three major advantages: (i) faster clock frequency of 2–3 orders of magnitude due to the lower mass of the electron, (ii) weaker decoherence effects due to reduced field fluctuations, (iii) dense coding with more qubits per site. Furthermore, with respect to solid state proposals [12], vacuum traps for electrons minimize influence from environment found in semiconductor devices and could create more accurate structures [13].

Our aim is to hold the particles inside the same physical device, creating a periodic potential capable of producing a linear array of Penning traps. The device able to realize such a confining structure consists of a cylindrical trap, whose basis are flat electrodes at zero potential, whereas the lateral surface is made up by a set of ring electrodes, held at different potentials (see Fig. 1). In our design, the distance d between two neighboring particles ranges from 500 to 2 μm. The electrostatic potential of this cavity is analytically calculable [14]. Around each energy minimum, it approximates to a quadrupole potential which axially confines the electron. The corresponding axial oscillation frequency ω_z depends on the voltage of the trap electrodes and is, therefore, under control of the experimenter. This is important because one can tune the axial frequency of two neighboring electrons to put them on and off resonance. Indeed, the Coulomb interaction couples these harmonic oscillators only when their characteristic frequencies are close enough. In the opposite case, when the axial frequencies are far detuned from each other, the electron motion is

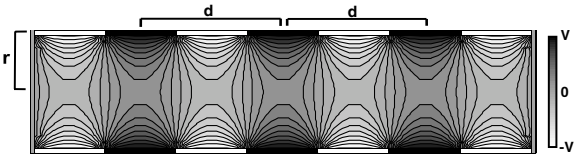

FIG. 1. Schematic section of the trapping device for 3 electrons and corresponding electrostatic potential. We use 7 ring electrodes with radius r held at alternate potentials V and $-V$. The particles are trapped in the potential maxima along the axial direction. By varying the number of ring electrodes we can obtain any number of trapping sites.

negligibly affected by the presence of the other charged particles.

To a good approximation, in the far off resonance regime, the electrons in the cylindrical trap may still be described in terms of *single* particles confined to a traditional Penning trap. The resulting motion in each microtrap consists of three independent harmonic oscillators: the cyclotron, the axial, and the magnetron motion [2]. Here we restrict our analysis to the axial motion and the electron spin. Being a spin-1/2 particle, the electron has two possible orientations, $|\downarrow\rangle$ and $|\uparrow\rangle$, of its spin in the trap magnetic field $\mathbf{B} = B\mathbf{k}$, which represent quite naturally the logical states $|0\rangle$ and $|1\rangle$. Problems arise when we want to encode qubits in multilevel systems, such as the axial harmonic oscillator. However, a solution is provided by small anharmonicities that lift the degeneracy between different transitions. Hence, for the axial motion it is necessary to introduce small anharmonicities in the quadrupole field. Indeed, taking into account the electrostatic corrections, the transition frequency between adjacent axial levels of quantum numbers k and $k+1$ is given by $\omega_z(k) \simeq \omega_z + \delta_e(k+1)$, where the shift amounts to $\delta_e \equiv 3e\hbar C_4/(m^2\omega_z^2)$ [2,15] with e, m being the charge and electron mass. The coefficient C_4 can be increased by varying the potential of suitable "decompensation" electrodes. By appropriately choosing C_4, it can be shown [15] that the level shift becomes larger than the estimated energy linewidth. Thus, in principle, one can control single axial transitions by applying electromagnetic pulses with sufficiently narrow bandwidth.

As described in [6], the manipulation of the electron spin is performed by applying a small transverse oscillating magnetic field resonant with the spin precession frequency $\omega_s \equiv g|e|B/(2mc)$, with g the electron giromagnetic factor. In this case the relevant part of the system Hamiltonian becomes, in interaction picture (IP) and rotating wave approximation (RWA),

$$H_{\mathrm{IP}}^{(\mathrm{spin})} \simeq \hbar\frac{\chi}{2}(\sigma_+ e^{-i\theta} + \sigma_- e^{i\theta}), \tag{1}$$

where $\chi \equiv g|e|b/(2mc)$, b and θ are, respectively, the strength and the phase of the magnetic field, and $\sigma_\pm \equiv (\sigma_x \pm i\sigma_y)/2$. If the small magnetic field is applied for a time t, it produces a spin state rotation

$$|\downarrow\rangle \rightarrow \cos\left(\frac{\chi t}{2}\right)|\downarrow\rangle - ie^{-i\theta}\sin\left(\frac{\chi t}{2}\right)|\uparrow\rangle, \tag{2}$$

$$|\uparrow\rangle \rightarrow \cos\left(\frac{\chi t}{2}\right)|\uparrow\rangle - ie^{i\theta}\sin\left(\frac{\chi t}{2}\right)|\downarrow\rangle. \tag{3}$$

It can be shown that with an appropriate combination of these operations, one can perform any single-qubit gate on the spin qubit. We define the above interaction as a $p_s(\chi t, \theta)$ pulse.

To manipulate the axial oscillator, we can apply to the trap electrodes an oscillating potential $ez\mathcal{V}\cos(\omega t - \beta)$ [2,15]. When the driving frequency ω is close to the axial frequency ω_z, the relevant part of the Hamiltonian of the system can be written in IP and RWA as

$$H_z \approx \hbar\frac{\Omega}{2}(a_z^\dagger e^{i\beta} + a_z e^{-i\beta}), \tag{4}$$

where we used the relation $z = \sqrt{\hbar/(2m\omega_z)}(a_z + a_z^\dagger)$ and $\Omega \equiv e\mathcal{V}/\sqrt{2m\hbar\omega_z}$. If the oscillating potential is applied for a time t and has a sufficiently narrow bandwidth centered around the value $\omega_z(k=0) = \omega_z + \delta_e$, it produces the following transformations:

$$|0\rangle_z \rightarrow \cos\left(\frac{\Omega t}{2}\right)|0\rangle_z - ie^{i\beta}\sin\left(\frac{\Omega t}{2}\right)|1\rangle_z, \tag{5}$$

$$|1\rangle_z \rightarrow \cos\left(\frac{\Omega t}{2}\right)|1\rangle_z - ie^{-i\beta}\sin\left(\frac{\Omega t}{2}\right)|0\rangle_z. \tag{6}$$

Hence, the above drive can be used to realize any single-qubit gate, when considering the axial states $|0\rangle_z$ and $|1\rangle_z$ as the logical states $|0\rangle$ and $|1\rangle$. We refer to the above interaction as a $p_z(\Omega t, \beta)$ pulse.

However, in order to perform logic operations on a system storing quantum information in both the axial motion and the electron spin, we need an interaction between these 2 degrees of freedom. A possible way to accomplish this task relies on the application, when required, of an inhomogeneous static magnetic field, the so-called *magnetic bottle*. In fact, with an appropriate dependence on the spatial coordinates, a static magnetic field can induce shifts on the axial transition frequency depending on the spin-cyclotron state and vice versa. The same mechanism is already used to perform the measurement of the electron state [2]. If we consider this additional static magnetic field

$$\mathbf{B}_1 = B_1\left[\left(z^2 - \frac{x^2+y^2}{2}\right)\mathbf{k} - z(x\mathbf{i} + y\mathbf{j})\right] \tag{7}$$

we obtain, treating it as a perturbation, the following transition frequencies [15]:

$$\omega_z(n, s, k) \simeq \omega_z + \delta_e(k+1) + \delta_m\left(n + \frac{1}{2} + \frac{s}{2}\right), \tag{8}$$

$$\omega_s(k) \simeq \omega_s + \delta_m\left(k + \frac{1}{2}\right), \tag{9}$$

where $\delta_m \equiv \hbar\omega_z|e|B_1/(2m^2c\omega_c\omega_m)$, with ω_c and ω_m being, respectively, the cyclotron and the magnetron frequency [2]. The frequency shifts δ_e and δ_m refer, respectively, to the electrostatic corrections and to the effects produced by the inhomogeneous magnetic field. Equations (8) and (9) clearly show the dependence of the transition frequencies on the quantum numbers n, k, and s describing the state of the electron. If during the computation we keep the cyclotron oscillator in its ground state $|n=0\rangle_c$, we can resolve any axial transition. This is a

reasonable assumption, since at the trap temperature of 80 mK the cyclotron oscillator remains in its ground state [3]. Having several electrons, we should also be able to singly address each of them. The individual axial frequencies depend on the applied voltage and can, therefore, be made distinguishable [15]. As far as the single addressability of the spin qubits, one can differentiate among them by inserting a small magnetic field gradient along the z axis [15].

We are now in the position to discuss the implementation of conditional dynamics between the spin and the axial motion of the electron. Rotations of the spin state, controlled by the axial qubit, can be realized with just one pulse. Indeed, if we apply the oscillating magnetic field on resonance with the frequency $\omega_s(k=1)$ the spin state is modified only if the axial state is $|1\rangle_z$. This opens up the possibility to implement a controlled-NOT (CNOT) gate having the spin qubit as a target and the axial qubit as a control. This two-qubit operation requires the following two pulses: (i) a $p_s(\pi, \pi/2)$ pulse on resonance with the frequency $\omega_s(k=1)$, which flips the spin only if the axial state is $|1\rangle_z$, that is $|1\rangle_z|\downarrow\rangle \rightarrow -|1\rangle_z|\uparrow\rangle$ and $|1\rangle_z|\uparrow\rangle \rightarrow |1\rangle_z|\downarrow\rangle$, without affecting the other states of the computational basis; (ii) A $p_z(2\pi, \beta)$ pulse on resonance with the frequency $\omega_z(n=0, k=1, s=1)$ to correct the minus sign $-|1\rangle_z|\uparrow\rangle \rightarrow |1\rangle_z|\uparrow\rangle$. To implement the other CNOT, with the axial qubit as a target and the spin qubit as a control, we have to apply the following two pulses: (i) a $p_z(\pi, -\pi/2)$ pulse on resonance with the frequency $\omega_z(n=0, k=0, s=1)$ which acts on the transition $|0\rangle_z \leftrightarrow |1\rangle_z$ only if the spin state is $|\uparrow\rangle$, i.e., $|0\rangle_z|\uparrow\rangle \rightarrow -|1\rangle_z|\uparrow\rangle$ and $|1\rangle_z|\uparrow\rangle \rightarrow |0\rangle_z|\uparrow\rangle$; (ii) a $p_z(2\pi, \beta)$ pulse on resonance with the frequency $\omega_z(n=0, k=1, s=1)$ which changes only the sign of the state $|1\rangle_z|\uparrow\rangle$.

The schemes described so far make it possible to perform arbitrary one- and two-qubit gates on a single electron. The maximum clock frequency of these gates is limited by the value of the corrections δ_e and δ_m that should be much smaller than the axial oscillation frequency ω_z.

To make our system scalable and computationally universal, we should be able to realize conditional dynamics, i.e., CNOT gates, between qubits belonging to different electrons. This task can be accomplished by considering the electrostatic interaction between neighboring electrons in the array.

By adjusting the external voltage applied to the ring electrodes, we can tune the individual axial frequency and put on and off resonance with the axial motion of two neighboring electrons. When in resonance, the two coupled harmonic oscillators can exchange a quantum of excitation, whereas out of resonance they basically behave as independent systems. Let us consider two electrons e_1 and e_2 separated by an average distance d. By choosing the origin of our coordinates in the center of the trap confining electron e_1, we can write the Hamiltonian of the two particles as $H = H_1 + H_2 + H_{\rm int}$, where H_j, with $j = 1, 2$, is the single-particle Hamiltonian of the jth electron [2] and $H_{\rm int} = e^2/[4\pi\epsilon_0\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}]$, represents the electrostatic interaction between the two electrons. If the oscillation amplitude of the two electrons is much smaller than the average separation d between them, we can expand $H_{\rm int}$ in a series and retain the terms up to the 2nd order. This is true when the cyclotron and the axial oscillators have been prepared in the first Fock states [7,15]. Within this approximation, the dynamics in the z direction is not coupled to that involving the transverse variables x and y. Then, the axial motion of the two electrons can be studied by considering only the axial part of H

$$H_z \simeq \frac{p_{1z}^2}{2m} + \frac{p_{2z}^2}{2m} + \frac{m\omega_{1z}^2}{2} z_1^2 + \frac{m\omega_{2z}^2}{2}(z_2-d)^2 - \frac{e^2}{2\pi\epsilon_0 d^3} z_1(z_2-d), \tag{10}$$

where we checked that the variations produced by the Coulomb interaction on the frequencies and trap distances are negligible. This is true when $\varepsilon \ll 1$, with ε being the ratio of the Coulomb energy, $e^2/4\pi\epsilon_0 d$, to the energy of the second electron with respect to the first trap, $m\omega_{1z}^2 d^2/2$. By introducing the coupling strength $\xi \equiv \varepsilon\omega_{1z}/2$ and the ladder operators

$$\tilde{a}_{1z} = \sqrt{\frac{m\omega_{1z}}{2\hbar}} z_1 + i\sqrt{\frac{1}{2\hbar m\omega_{1z}}} p_{1z}, \tag{11}$$

$$\tilde{a}_{2z} = \sqrt{\frac{m\omega_{2z}}{2\hbar}} (z_2-d) + i\sqrt{\frac{1}{2\hbar m\omega_{2z}}} p_{2z}, \tag{12}$$

we obtain, in IP and RWA, when $\omega_{1z} = \omega_{2z}$

$$H_z^{\rm IP} \simeq -\hbar\xi(\tilde{a}_{1z}\tilde{a}_{2z}^\dagger + \tilde{a}_{1z}^\dagger\tilde{a}_{2z}). \tag{13}$$

This is the Hamiltonian describing the electrostatic interaction between two trapped electrons with the axial frequencies tuned on resonance. The corresponding Schrödinger equation produces, for the lowest Fock states of the axial oscillators of the two particles, the following temporal evolution at the time $t_{ex} \equiv \pi/(2\xi)$:

$$|0\rangle_{z1}|0\rangle_{z2} \rightarrow |0\rangle_{z1}|0\rangle_{z2}, \tag{14}$$

$$|0\rangle_{z1}|1\rangle_{z2} \rightarrow i|1\rangle_{z1}|0\rangle_{z2}, \tag{15}$$

$$|1\rangle_{z1}|0\rangle_{z2} \rightarrow i|0\rangle_{z1}|1\rangle_{z2}, \tag{16}$$

$$|1\rangle_{z1}|1\rangle_{z2} \rightarrow -|1\rangle_{z1}|1\rangle_{z2}. \tag{17}$$

It then gives the wanted swapping operation on the axial qubits. Notice that the inverse operation is realized when the resonant interaction is on for a time $3t_{ex}$. Depending

TABLE I. For given trap distance d and frequency $\omega_z/2\pi$, we give the electron oscillation amplitude Δz relative to d, the swapping time t_{ex}, which is of the same order of the gate operation time for a single electron, and the estimated decoherence time τ_d at $T = 80$ mK and $\omega_s/2\pi \simeq 160$ GHz. The other parameters are as follows: in case A $\varepsilon = 1/60$, $t_{ex}/t_{ad} = 360$, $\Delta\omega_z = 10\xi$; in case B $\varepsilon = 1/600$, $t_{ex}/t_{ad} = 3600$, $\Delta\omega_z = 100\xi$.

	A			B		
d (μm)	500	50	2	500	50	3
$\omega_z/2\pi$ (MHz)	2.5	78	9800	7.8	250	16 900
$\Delta z/d$	1/150	1/86	1/39	1/270	1/150	1/76
t_{ex} (μs)	12	0.38	0.003	38	1.2	0.018
τ_d (s)	5500	300	1	9500	490	1.6

on the electron distance d and trapping frequency ω_z, we can have a swapping time t_{ex} ranging from 38 μs to 3 ns, as shown in Table I. To switch on (off) the resonant interaction between neighboring electrons we have to modify their axial frequencies, so that their detuning $\Delta\omega_z$ becomes much smaller (larger) than ξ. The corresponding energy variation of the system should satisfy the following conditions: (i) it must occur in a time Δt much smaller than the swapping time t_{ex}; (ii) it must be sufficiently slow in order to make the adiabatic theorem valid, i.e., it must occur in a time Δt such that $\Delta t \gg t_{ad}$ with $t_{ad} \equiv 2\pi|\Delta\omega_z|/\omega_z^2$. Hence, the switching operation should take place in a time window with size $t_{ex}/t_{ad} \gg 1$.

To perform conditional dynamics between neighboring electrons, we swap the axial qubits of the two electrons and perform the CNOT gate in one site. Then we swap back the axial qubits into the original electrons. This procedure gives a CNOT gate between the axial and the spin qubits of different electrons. It can be extended to any arbitrary pair of qubits by making use of an additional swapping operation [16] between the axial and the spin qubits of the same electron. Furthermore, by means of a finite number of swapping operations, it is possible to implement any CNOT gate between the ith and the jth electrons in the array. It can be shown [15] that the number of swapping operations, required to implement any CNOT gate, grows linearly with the number of trapped electrons, i.e., with the number of qubits in the network. Therefore, an efficient quantum algorithm, if implemented with this quantum processor, preserves its efficiency. This is a fundamental result because it demonstrates the scalability of our system.

In this Letter we presented an innovative design for a multiple Penning trap able to hold several electrons in a linear array. Our configuration allows for both single-particle addressability and two-particle operations, based on the swapping gate between the axial qubits of neighboring electrons. Starting with this two-particle operation, we showed how to build up a universal set of quantum gates between *all* the particles in the array. According to estimates of the phase decoherence due to the thermal noise in the electrode surface [17], we can perform, with our system, about 10^8 operations within the decoherence time (see Table I). Decoherence effects due to other noise sources are also expected to be very small considered the high field stability reached in present experiments. Recent progress in the fabrication of microtraps [13] makes feasible the realization of our trapping structure. For $d > 20$ μm [18] a crucial step towards the implementation of such a device is the cooling of the axial motion, which, however, is in the reach of present technology [2,4].

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