Sandpile Formation by Revolving Rivers

E. Altshuler,[1,2] O. Ramos,[1,2,3] E. Martínez,[1,2] A. J. Batista-Leyva,[1,2,4] A. Rivera,[5] and K. E. Bassler[6]
[1]*Superconductivity Laboratory, IMRE-Physics Faculty, University of Havana, 1 0400 Havana, Cuba*
[2]*"Henri Poincaré" Chair of Complex Systems, Physics Faculty, University of Havana, 10400 Havana, Cuba*
[3]*Physics Department, ELAM, 19100 Havana, Cuba*
[4]*Physics Department, University of Holguín, Holguín 80100, Cuba*
[5]*Zeolite Engineering Laboratory, IMRE-Physics Faculty, University of Havana, 10400 Havana, Cuba*
[6]*Department of Physics, University of Houston, Houston, Texas 77204-5005, USA*
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Experimental observation of a new mechanism of sandpile formation is reported. As a steady stream of dry sand is poured onto a horizontal surface, a pile forms which has a thin river of sand on one side flowing from the apex of the pile to the edge of its base. The river rotates about the pile, depositing a new layer of sand with each revolution, thereby causing the pile to grow. For small piles the river is steady and the pile formed is smooth. For larger piles, the river becomes intermittent and the surface of the pile becomes undulating. The essential features of the system that produce the phenomenon are discussed, and the robustness of the phenomena is demonstrated with experiments using different boundary conditions and sands.

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The conventional understanding of sandpile formation is that as grains of sand are poured onto a horizontal surface, a conical pile develops which grows intermittently through avalanches that "adjust" the angle of repose of the pile about some critical value, or, at least, keep it between two critical values. This mechanism of pile formation has been widely studied, both experimentally and theoretically, in recent years. The statistical properties of avalanches have been measured in conical piles [1–3], as well as in piles sandwiched between two vertical glass plates [4–6], and in rotating drums [7]. Those studies have mainly focused on the scaling properties of the avalanches in order to determine if the ideas of self-organized criticality (SOC) [8] apply. Other experiments concerning pile formation have concentrated on the detailed mechanisms of granular flow instead of the statistics of avalanches [9–12]. Those studies have revealed a variety of phenomena, including both deep and shallow surface flows, and slow creep of particles far below the surface. There have also been a number of recent theoretical studies of granular flow that relate to pile formation [13–20]. However, despite all of this effort, the formation of sandpiles is still not completely understood.

Here, we report the experimental observation of a striking new mechanism of formation of a conical sandpile. As a steady stream of dry sand is poured onto a horizontal surface, a pile begins to form. Initially, the pile grows through randomly distributed avalanches, similar to the conventional mechanism of pile formation described above. But then, once the pile grows to a sufficient size, a river of sand spontaneously develops that flows down one side of the pile from the apex of the pile to its base. The river, which is narrow compared with the radius of the pile, then begins to revolve around the pile depositing an helical layer of sand with each revolution, thereby causing the pile to grow. An example of a revolving river is shown in the photograph of Fig. 1(a). Revolving rivers, once formed, can be very robust. Examples which persisted for dozens of rotations, and that stopped only when the input flow of sand was interrupted, have been observed. Rivers form over a range of experimental parameters and with various boundary conditions on the pile. In all cases, rivers revolve about the pile in both clockwise and counterclockwise directions.

In the remainder of this Letter, we first explain the experimental setup used in our experiments, then we describe our observations in detail. Included in that discussion are the experimental parameters and types of sand for which revolving rivers have been observed. A dynamical instability observed in revolving rivers as the size of the pile grows is also described. Finally, a series of experiments is discussed that has been performed to help characterize and understand revolving rivers.

The basic experimental setup is shown in the upper panel of Fig. 2. A vertical glass tube with a 10 mm inner diameter was initially filled using a funnel. Then a hole was opened in the bottom of the tube, allowing sand to fall out of the tube by its own weight. This arrangement produced steady deposition flow rates in the range 0.07–0.8 cm^3/s, depending on the diameter of the hole in the bottom of the tube. Video cameras recorded both lateral and top views of the piles during the experiment. Three versions of the experiment were performed, each with different boundary conditions imposed on the growing pile. They are illustrated in the lower panel of Fig. 2, and will be referred to as boundary conditions I, II, and III (BCI, BCII, and BCIII) in the text.

In the first version of the experiment (BCI) the pile was formed in a cylindrical container. After the pile grows to

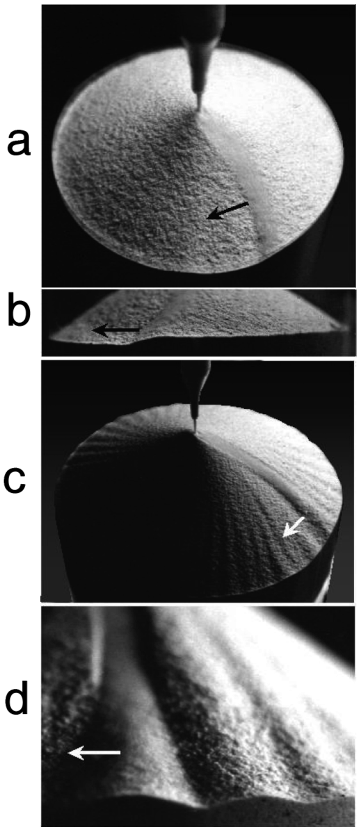

FIG. 1. Formation of a pile of sand by revolving rivers in the case of BCI. The sand is poured vertically in the center of a cylindrical container with a flat, horizontal bottom at a deposition rate of 0.35 cm^3/s, from a constant height of 1.5 cm above the apex of the pile. (a) Top view of a pile growing by a steady revolving river into a 4 cm radius container. (b) Lateral view of the pile shown in (a). (c) Top view of a pile growing by an intermittent revolving river into a 9 cm radius container. (d) Lateral view of the pile shown in (c) (the photo shows about 3 cm of the container's perimeter). In all cases, arrows indicate the revolving direction.

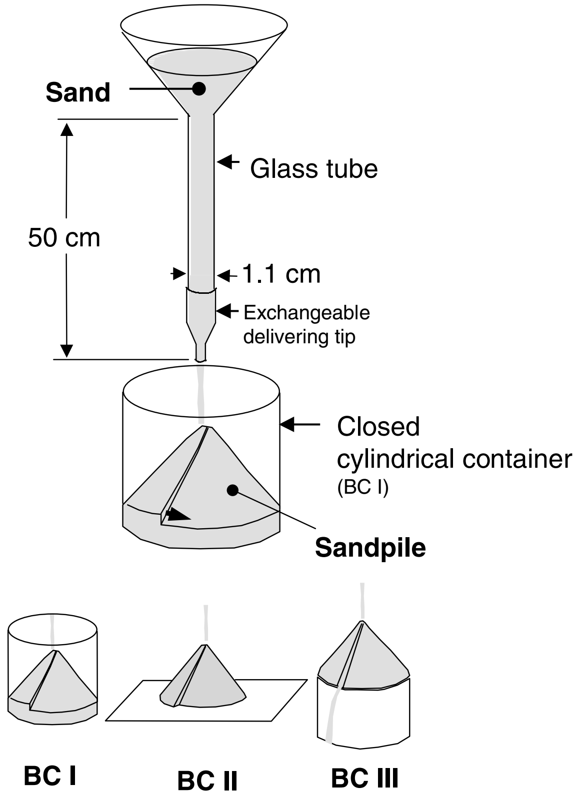

FIG. 2. Upper panel: basic experimental setup. Lower panel: Different boundary conditions used for the piles.

cover the bottom of the container, the boundary condition imposed by the cylinder keeps the radius of the pile fixed. Therefore, once the pile grows to the size of the container, it grows in a "steady state" fashion. Our main observations of the steady state growth of piles are shown in Fig. 1. Figures 1(a) and 1(b) show top and lateral views of the growing pile for a 4 cm radius container, and Figs. 1(c) and 1(d) correspond to a 9 cm radius container. The angle of repose of the piles was $\theta_c \approx 33^\circ$ in both cases. Viewed from above, the rivers were slightly bent, and always revolved around the pile in the direction of their concavity, as seen in Figs. 1(a) and 1(c). For very small cylinders, those with a radius less than 3 cm, no stable revolving rivers were observed. For cylinders with a radius between 3 and 6 cm, "continuous," steady revolving rivers, as shown in Figs. 1(a) and 1(b), were usually observed. In this case, the resulting surface of the pile was smooth. For cylinders with a radius larger than 6 cm, a revolving river still developed, causing the pile to grow as before, but the flow of the river was intermittent rather than continuous. Thus, an instability occurs in the revolving rivers when the size of the pile reaches approximately 6 cm. The intermittent river flow produced an undulating pattern on the pile surface, visible in Figs. 1(c) and 1(d). The undulating pattern resembles, and may be related to, those recently observed for rapid granular flows on an inclined plane [21]. The observed pattern was quite regular for containers with a radius just large enough to observe the instability, but became more irregular as the size of the container grew.

The revolving river mechanism of pile formation has also been observed by simply pouring the sand onto a flat surface (BCII). In that case, a gradual crossover from a continuously flowing revolving river, observed in smaller systems, to an intermittently flowing river, observed in larger systems, occurred as the radius of the pile reached about 6 cm. For piles smaller than 3 cm, and larger than 10 cm, the pile appears to grow through the conventional mechanism, i.e., avalanches more or less randomly distributed around the pile.

Revolving rivers were observed at input flows between 0.08 and 0.7 cm^3/s, and when sand was dropped from heights between 1 and 7 cm. Once formed, the rivers were very robust; they typically performed a few dozen turns around the pile and stopped only if the input flow was interrupted. Outside the experimental ranges, the rivers either do not appear, or they are not robust.

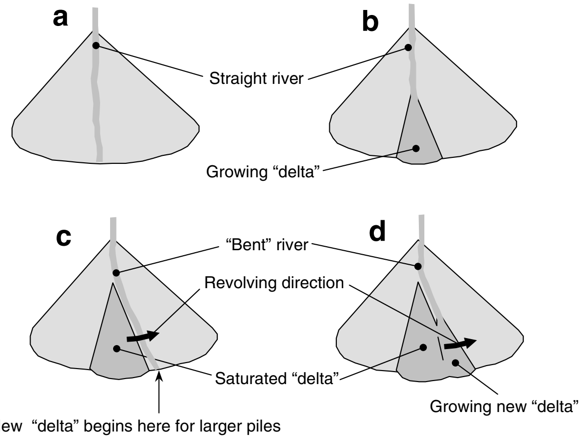

FIG. 3. Development of a revolving river, illustrated for the case of BCII. (a) A river flows straight down the side of the pile, and a delta begins to form at its bottom. (b) The delta continues to grow. (c) When the delta is sufficient size, the river begins to flow down one side and rotate around the pile. (d) For larger piles, a new delta forms intermittently at the bottom of the river, causing the rotation of the river to become intermittent.

The reason for the curved shape of a revolving river and why it rotates in the direction of its concavity can be understood by how it forms. Based on careful observation, revolving rivers appear to form through the following scenario, illustrated in Fig. 3. As sand is poured onto the top of a conical pile it forms a round depression in the top of the pile. Then it spills out of the depression and cascades down a side of the pile. Initially, the cascades occur in random directions. However, when the pile grows to a sufficient height, a "groove" is carved in the edge of the depression at the tip of the pile as the sand spills out of the depression. Once this groove is formed, the sand flows out of the groove in a river down one side of the pile. Sand then begins to build up at the edge of the pile at the bottom of the river, forming a growing inverted V shaped delta of stationary sand [Fig. 3(b)]. The delta grows in size until the river spontaneously chooses to begin to flow down one of the sides of the delta [Fig. 3(c)]. Once it chooses a side, it continues to flow down that side of the delta, depositing sand all along the lower, delta side of the river. As it does so, it rotates about the pile. For smaller piles, the process of rotation was stable. However, for larger piles, it was not. Instead, in that case, a new delta would intermittently begin to form at the bottom of the river [Fig. 3(d)]. When the delta reached a sufficient height, the river would "jump" forward in its rotation, and then begin forming yet another new delta. Note that the axial symmetry of the pile is spontaneously broken when the groove forms at the top of the pile.

This mechanism of revolving river formation above is supported by experiments performed by growing the pile on a cylindrical base of finite size with no walls (BCIII). As soon as the pile grows to reach the radius of the base, the river stops revolving. The sand then drops off the pile when it reaches the edge. In contrast to revolving rivers, the nonrevolving river is *straight*. For BCIII there is no possibility to built up a delta, so no bending of the river nor rotation takes place.

Boundary condition BCIII also allows a simple, direct measurement of the width of the river (w), which in this case is roughly constant for its entire length, and also a rough estimation of the flow speed (v), by a straightforward calculation based on the parabolic motion of the sand that drops off the pile. These two parameters plus the length of the river allows an estimation of the flow depth (d). Our results indicate that w and v increase with the deposition flow rate in the ranges 5–11 mm and 9–19 cm/s, respectively. However, $d \approx 0.35 \pm 0.05$ mm for all deposition rates. This is qualitatively consistent with recent observations of granular flows on rough inclined planes [22]. If we assume an average grain diameter of 140 μm, we estimate that 2–3 layers of grains were involved in the flow of the river. This is also in reasonably good agreement with [11] for flows on granular heaps, and with calculations of the depth of granular flows on heaps based on a model where gravity, intergrain dissipation, and intergrains "trapping" are taken into account [18]. Thus, though the development and rotation of the river is a specific feature of our experiments, the flow of sand in the river itself appears to be consistent with standard models of surface flows.

This allows a plausible explanation for why the crossover from continuous to intermittent rivers occurs. In the

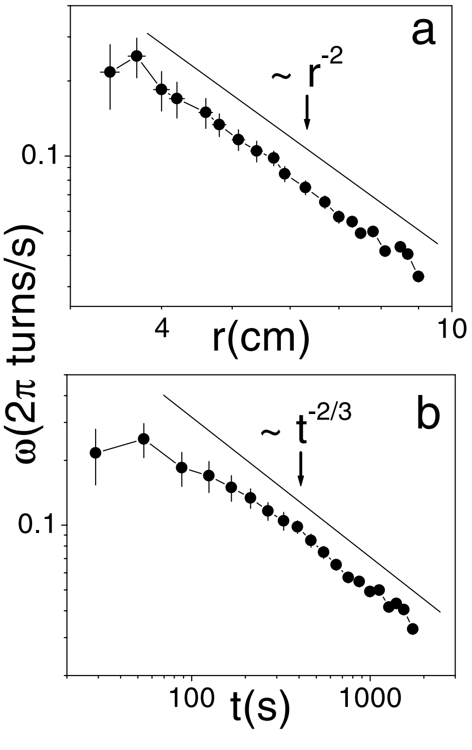

FIG. 4. Radius (a) and time (b) dependence of the angular speed of revolving rivers for BCII with a deposition flow rate of 0.35 cm^3/s and the sand dropped from a height of 1.5 cm above the apex of the pile.

case of a sand heap between two vertical plates, it is well established that there is a transition from continuous to intermittent surface flows as the input flux decreases [12]. Rotating rivers widen slightly as they flow from the tip to the edge of the pile. Thus, toward the edge of the pile the effective input flux of the flowing sand reduces. If the pile is large enough to get a river width above a certain threshold, one observes a transition to the intermittent regime similar to what is observed in the parallel plate experiment as the input flux is decreased.

We also measured the time evolution of the angular velocity of river rotation for both BCI and BCII for a deposition rate of $0.35\ \mathrm{cm}^3/\mathrm{s}$, as well as the relation between the angular velocity and the pile radius in the case of BCII. The angular velocity of river rotation is roughly constant for BCI, while it decreases in time as $t^{-\alpha}$ ($\alpha = 2/3$), and with radius as $r^{-\beta}$ ($\beta = 2$), for BCII, as shown in Fig. 4.

These results can be explained using the following scaling arguments. Assume that a new layer of sand is deposited with rate F on a conical pile of radius r with an angle of repose θ_c. For BCI, the thickness of an added layer, δh, is constant in time. We measured $\theta_c = 33^\circ$, and $\delta h \approx 2.5 \pm 0.5$ mm (δh was measured as the total height of the sand cylinder under the conic pile, divided by the number of turns of the river around the pile, counting from the moment the pile touched the radius of the cylinder the first time). Therefore, the volume of sand deposited in each rotation of the river $V \sim r^2\ \delta h$ is constant in time, and the angular velocity of the river, $\omega = F/V$, is also constant in time. However, for BCII the radius of pile grows in time. In this case, the thickness of each layer is proportional to δr and is also constant in time. We estimated $\delta r \approx 4$ mm± 0.5 mm by dividing the radius of the pile minus the radius where the first river appeared, by the number of full turns of the river needed to cover the difference. Thus, since the volume of sand deposited in a rotation of the river is $V \sim r^2\ \delta r$, $\omega \sim r^{-2}$. The pile radius increases at a rate of $dr/dt = \omega\ \delta r$. Integrating this expression, we get $r \sim t^{1/3}$, and therefore $\omega \sim t^{-2/3}$. Although this argument correctly predicts the scaling of larger piles, very small piles behave differently, presumably due to the fact that our geometrical assumptions are inaccurate for smaller piles. The agreement between the results of our scaling argument and the experiment weakened as the sand was poured from larger heights. This is expected since inertial effects are not accounted for in the model.

Finally, we note that the appearance of revolving rivers is quite sensitive to the type of sand used in the experiments. Nearly 100 sands from all over the world were studied, and the phenomenon was observed in nine of them. The quantitative results reported here were measured in a sand from Santa Teresa, Cuba, which is quite similar to all of the revolving sands. It consists of irregularly shaped grains of size 30–250 μm made of almost pure silicon oxide. Revolving rivers were still observed if the sand was meshed to remove grains smaller than 90 μm and larger than 160 μm, and if the room humidity was stabilized at different values between 60% and 90%.

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