Neutrino-^{12}C Scattering in the *Ab Initio* Shell Model with a Realistic Three-Body Interaction

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We investigate cross sections for neutrino-^{12}C exclusive scattering and for muon capture on ^{12}C using wave functions obtained in the *ab initio* no-core shell model. In our parameter-free calculations with basis spaces up to the $6\hbar\Omega$ we show that realistic nucleon-nucleon interactions, like, e.g., the CD-Bonn, underpredict the experimental cross sections by more than a factor of 2. By including a realistic three-body interaction, Tucson-Melbourne TM$'$(99), the cross sections are enhanced significantly and a much better agreement with experiment is achieved. At the same time, the TM$'$(99) interaction improves the calculated level ordering in ^{12}C. The comparison between the CD-Bonn and the three-body calculations provides strong confirmation for the need to include a realistic three-body interaction to account for the spin-orbit strength in p-shell nuclei.

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The Gamow-Teller (GT) transition from the ground state of ^{12}C to the 1^+ $T = 1$ isobar triplet [^{12}B$_{gs}$, ^{12}C (15.11 MeV), ^{12}N$_{gs}$] is a very sensitive test of nuclear structure models for mass 12 and, particularly, of the strength of the spin-orbit interaction. The two most common p-shell approximations for the structure of the ground state of ^{12}C [(a) the p-shell equivalent of a $L = 0$, $S = 0$ three alpha-cluster structure and (b) the closed $p_{3/2}$ shell structure] give very different (indeed opposite) predictions for the B(GT) strength to the $T = 1$ 1^+ triplet. In the p-shell alpha-cluster limit the ground state of carbon has good SU(4) symmetry [444] and the Gamow-Teller transition is forbidden because there does not exist a 1^+ $T = 1$ state with [444] symmetry and the $\sigma\tau$ operator cannot change SU(4) symmetry. This translates into an exact cancellation between the different $p_{1/2}$ and $p_{3/2}$ transition amplitudes. The observed transition strength requires the inclusion of higher SU(4) components in the wave functions, and the breaking of the cancellation is quite sensitive to the assumed spin-orbit interaction. In the the jj-coupling limit, where one assumes that the ground state of ^{12}C is described by a closed $p_{3/2}$ shell, the transition to the $T = 1$ 1^+ state is pure $p_{3/2} \rightarrow p_{1/2}$. No cancellations between different transition amplitudes are allowed, and the transition strength is overestimated by almost a factor of 6. When random phase approximation correlations are included in the initial and final states the situation improves somewhat, but the transition remains overestimated by about a factor of 4 [1,2]. The strong contrast between the predictions of the pure jj coupling and the pure SU(4) limits makes this Gamow-Teller transition an ideal test case for the strength of the spin-orbit interaction and for model wave functions of mass 12.

In this Letter we present the predictions of no-core shell-model (NCSM) [3] calculations of ^{12}C for the $T = 1$ 1^+ transition in ^{12}C. We examine inelastic electron scattering to the 15.11 MeV state of ^{12}C, muon capture to the ground state of ^{12}B, and neutrino scattering to the ground state of ^{12}N. These different electroweak reactions probe different momentum transfers, and comparisons between theory and experiment allow us to test the convergence of the no-core shell model with increasing basis size, up to $6\hbar\Omega$. We also investigate the contributions of a three-nucleon force since it is now well established [4–6] that realistic nucleon-nucleon interactions alone account for only half the observed p-shell splitting, while the rest arises from two-pion exchange between three or more nucleons. In the present calculations we include a realistic chiral-symmetry-based three-nucleon interaction (TNI), Tucson-Melbourne TM$'$(99) [7].

A detailed description of the NCSM approach was presented, e.g., in Refs. [3,8]. Here, we simply present extensions and modifications needed when a genuine TNI is included. The starting Hamiltonian is $H_A = \frac{1}{A}\sum_{i<j}\frac{(\vec{p}_i-\vec{p}_j)^2}{2m} + \sum_{i<j}^{A} V_{\mathrm{NN},ij} + \sum_{i<j<k}^{A} V_{\mathrm{NNN},ijk}$, where $V_{\mathrm{NN},ij}$ is the nucleon-nucleon (NN) interaction and $V_{\mathrm{NNN},ijk}$ is the TNI. We employ a large but finite harmonic-oscillator (HO) basis. Because of properties of the realistic nuclear interaction we have to derive an effective interaction appropriate for the selected finite basis space. To facilitate this, we modify the Hamiltonian by adding to it the center-of-mass (c.m.) HO Hamiltonian $H_{\mathrm{c.m.}} = T_{\mathrm{c.m.}} + U_{\mathrm{c.m.}}$, where $U_{\mathrm{c.m.}} = \frac{1}{2}Am\Omega^2\vec{R}^2$, $\vec{R} = \frac{1}{A}\sum_{i=1}^{A}\vec{r}_i$. The effect of the HO c.m. Hamiltonian will later be subtracted in the final many-body calculation. The modified Hamiltonian can be cast into the form $H_A^{\Omega} = H_A + H_{\mathrm{CM}} = \sum_{i=1}^{A} h_i + \sum_{i<j}^{A} V_{ij}^{\Omega,A} + \sum_{i<j<k}^{A} V_{\mathrm{NNN},ijk}$, where $h_i = \frac{\vec{p}_i^2}{2m} + \frac{1}{2}m\Omega^2\vec{r}_i^2$ and $V_{ij}^{\Omega,A} = V_{\mathrm{NN},ij} - \frac{m\Omega^2}{2A}(\vec{r}_i - \vec{r}_j)^2$. Next we divide the A-nucleon infinite HO basis space into the finite active space (P) comprising all states up to $N_{\max}$ HO excitations above the unperturbed ground state and the excluded

spaces ($Q = 1 - P$). The basic idea of the NCSM approach is to apply a unitary transformation on the modified Hamiltonian, $e^{-S}H_A^{\Omega}e^S$ such that $Qe^{-S}H_A^{\Omega}e^SP = 0$. If such a transformation is found, the effective Hamiltonian that exactly reproduces a subset of eigenstates of the full space Hamiltonian is given by $H_{\rm eff} = Pe^{-S}H_A^{\Omega}e^SP$. This effective Hamiltonian contains up to A-body terms, and it is essentially as difficult to construct it as to solve the full problem. Therefore, we apply this approach with a cluster approximation. When a genuine TNI is considered, the simplest cluster approximation produces a three-body effective interaction. The NCSM calculation is then performed in four steps:

(i) We solve a three-nucleon system for all possible three-nucleon channels with the Hamiltonian H_A^{Ω}, i.e., using $h_1 + h_2 + h_3 + V_{12}^{\Omega,A} + V_{13}^{\Omega,A} + V_{23}^{\Omega,A} + V_{\rm NNN,123}$. Consequently, the three nucleons feel a pseudo-mean field of the spectator nucleons generated by the HO c.m. potential. It is necessary to separate the three-body effective interaction contributions from the TNI and from the two-nucleon interaction. Therefore, we need to find three-nucleon solutions for the Hamiltonian with and without the $V_{\rm NNN,123}$ TNI term. The three-nucleon solutions are obtained by procedures described in Ref. [9] (without TNI) and [10] (with TNI).

(ii) We construct the unitary transformation corresponding to the choice of the active basis space P from the three-nucleon solutions using the Lee-Suzuki procedure [11,12]. Then we calculate the three-body effective interactions and separate their contributions from the TNI and the two-nucleon interaction.

(iii) As the three-body effective interactions are derived in the Jacobi-coordinate HO basis but the $A = 12$ calculations will be performed in a Cartesian-coordinate single-particle Slater-determinant m-scheme basis, we need to perform a suitable transformation of the interactions.

(iv) We solve the Schrödinger equation for the $A = 12$ nucleon system using the Hamiltonian $H_{A,\rm eff}^{\Omega} = \sum_{i=1}^{A} h_i + \frac{1}{A-2}\sum_{i<j<k}^{A} V_{3{\rm eff},ijk}^{\rm NN} + \sum_{i<j<k}^{A} V_{3{\rm eff},ijk}^{\rm NNN}$, where the $\frac{1}{A-2}$ factor takes care of overcounting the contribution from the two-nucleon interaction. At this point we also subtract the $H_{\rm c.m.}$. The $A = 12$ nucleon calculation is then performed using the many-fermion dynamics shell-model code [13] generalized to handle three-body interactions.

Detailed ^{12}C NCSM calculations using realistic two-nucleon interactions were reported in Ref. [3]. Here we extend those calculations by including the TNI and reach the $4\hbar\Omega$ ($6\hbar\Omega$) basis in calculations with (without) the TNI. In Table I we summarize some of our results. In general, in addition to an increase of binding energy, we observe a substantial sensitivity of the low-lying spectra to the presence of the TNI and a trend toward level-ordering and level-spacing improvement in comparison to experiment. The sensitivity is the largest for states where the spin-orbit interaction strength is known to play a role. Note the correct ordering of the $1^+0 \leftrightarrow 4^+0$ states and ordering and spacing improvement of the lowest $T = 1$ states.

The significant increase in the spin-orbit splitting obtained from the inclusion of the TNI is seen most strikingly in the predicted cross sections to the $T = 1$ 1^+ states. In all our electron scattering and weak interaction results here, only one-body currents are included and the bare operators are used. Dubach and Haxton [17] have shown that at high-momentum transfers it is necessary to include two-body meson-exchange currents to describe the transverse magnetic electron scattering form factor for excitation of the 15.11 MeV state.

TABLE I. Experimental and calculated properties of ^{12}C. The units are e^2 fm^4 (μ_N^2) for $B(E2)$ ($B(M1)$). Three-body effective interactions derived from the AV8′ [14] and AV8′ + TM′(99) and two-body effective interactions derived from the CD-Bonn [15] NN potential and a HO frequency of $\hbar\Omega = 15$ MeV were used. The TM′(99) parameters are given in Refs. [7,10] with the cutoff set to $\Lambda = 4.7$. The experimental values are from Ref. [16]. By extrapolating our results we predict the CD-Bonn ^{12}C binding energy to be $\approx 80 \pm 2$ MeV.

	^{12}C	AV8′ + TM′(99)	AV8′		CD-Bonn		
Basis space	⋯	$4\hbar\Omega$	$4\hbar\Omega$	$6\hbar\Omega$	$4\hbar\Omega$	$2\hbar\Omega$	$0\hbar\Omega$
$\lvert E_{\rm gs}\rvert$ [MeV]	92.162	91.963	85.944	85.630	88.518	92.375	104.947
Q_{2^+} [e fm^2]	+6(3)	4.288	4.613	4.717	4.532	4.430	4.253
$E_x(2^+0)$ [MeV]	4.439	3.603	3.427	3.612	3.697	3.837	3.734
$E_x(1^+0)$ [MeV]	12.710	11.280	13.926	13.930	14.140	14.524	13.866
$E_x(4^+0)$ [MeV]	14.083	13.517	12.272	13.110	13.356	13.638	12.406
$E_x(1^+1)$ [MeV]	15.110	16.221	16.364	16.064	16.165	16.291	15.290
$E_x(2^+1)$ [MeV]	16.106	16.467	17.712	17.409	17.717	17.945	15.970
$E_x(0^+1)$ [MeV]	17.760	17.116	16.213	16.534	16.619	16.493	14.698
B(E2;$2^+0 \to 0^+0$)	7.59(42)	4.146	4.765	5.019	4.624	4.412	4.092
B(M1;$1^+1 \to 0^+0$)	0.951(20)	0.645	0.305	0.384	0.355	0.280	0.158
B(E2;$2^+1 \to 0^+0$)	0.65(13)	0.430	0.247	0.309	0.283	0.015	0.002

However, a reasonable description of the form factor up to momentum transfers of about 200 MeV/c can be obtained with a one-body current.

Figure 1 shows a comparison between the form factors predicted by the NCSM and experiment. The experimental data are represented by the circles, which represent a fit to the data assuming only a one-body current obtained by Dubach and Haxton [17]. The theoretical curves shown are NCSM results for $2\hbar\Omega$, $4\hbar\Omega$, $6\hbar\Omega$ using the CD-Bonn NN interaction and a $4\hbar\Omega$ calculation using the AV8$'$ plus the TM$'$(99) realistic TNI. The qualitative features of our results are seen by looking at the height of the first maximum and the position of the minimum. With two-body interactions alone, the change in transition form factor from $4\hbar\Omega$ to $6\hbar\Omega$ is small compared to the differences between theory and experiment. The magnitude of the form factor is too low and minimum occurs too far out in momentum. When the TNI is included a significant improvement is seen in both the shape and magnitude of the theoretical form factor. The magnitude of the form factor up to the first maximum is close but somewhat lower than experiment. The shape of the form factor is also improved, but it is still stretched out too far in momentum. Comparing the $4\hbar\Omega$ three-body calculation with the $4\hbar\Omega$ CD-Bonn calculation, the magnitude of the form factor at the peak has increased by about 75%, and the position of the first minimum has shifted from $q_{\min} \sim 400$ to 360 MeV/c. This improvement is almost entirely due to the improved strength of the spin-orbit splitting when the TNI is included.

The 15.11 MeV state was included in the fit to the p-shell interaction by Cohen and Kurath (CK) [18], and the CK interaction probably represents the best description of this transition using a globally fitted p-shell interaction. Our AV8$'$ + TM$'$(99) form factor agrees well with the CK prediction, when, for consistency between the two calculations, we use $b = 1.663$ fm.

The conclusion drawn from the transverse magnetic form factor results is further supported by our $B(M1; 1^+1 \to 0^+0)$ results presented in Table I and Fig. 2. The calculations with two-body forces show saturation and underpredict the experiment by almost a factor of 3. By including the TNI, the $B(M1)$ value increases dramatically. We fully expect that further increases in the basis size will produce results with TNI close to experiment. For smaller basis sizes, effective transition operators may be important, and work in this direction is underway.

Table II shows the comparison between the theoretical and experimental neutrino scattering cross section for the same selections of Hamiltonians and basis spaces. These results show a similar trend to the electron scattering results above. In this case, the neutrino spectrum for electron neutrinos from decay at rest (DAR) of the pion peaks around 30 MeV and the average momentum transfer is about 40 MeV/c. The CD-Bonn interaction (without TNI) results indicate an approach to convergence by $6\hbar\Omega$ but experiment is underpredicted by about a factor of 2.4. When the TNI is included with the AV8$'$ interaction the predicted cross section is only 30% lower than experiment. Based on the similarity of trends with the electron scattering results, we anticipate that when the model space is eventually expanded to $6\hbar\Omega$ theory would be within 15% of experiment. The substitution of AV8$'$ for CD-Bonn in the calculations with TNI is expected to be of minor consequence.

Muon capture involves a higher momentum transfer than the (ν_e, e^-) reaction and the average momentum transfer is $q \sim 100$ MeV/c. By $6\hbar\Omega$ the CD-Bonn calculations show signs of converging, yet the experiment is underestimated by a factor of 2.6. The inclusion of the TNI shows a significant improvement and, for $4\hbar\Omega$, theory is 34% lower than experiment. Again extrapolating using the trends of the inelastic electron scattering

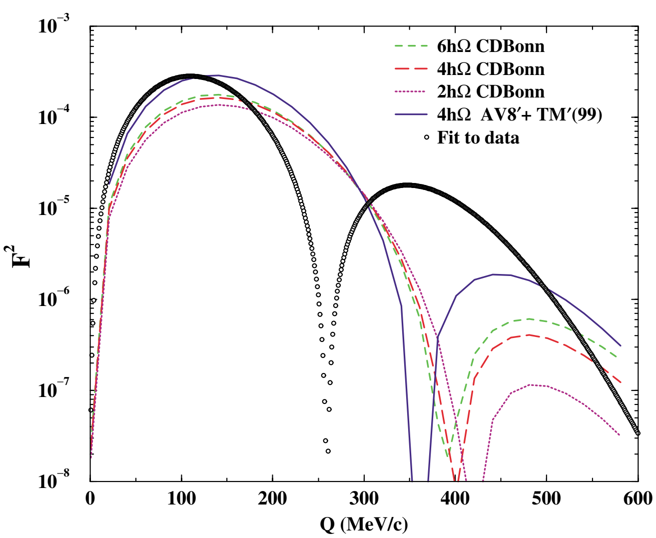

FIG. 1 (color online). The transverse magnetic electron scattering form factor for the 15.11 MeV, $T = 1\ 1^+$ state in ^{12}C.

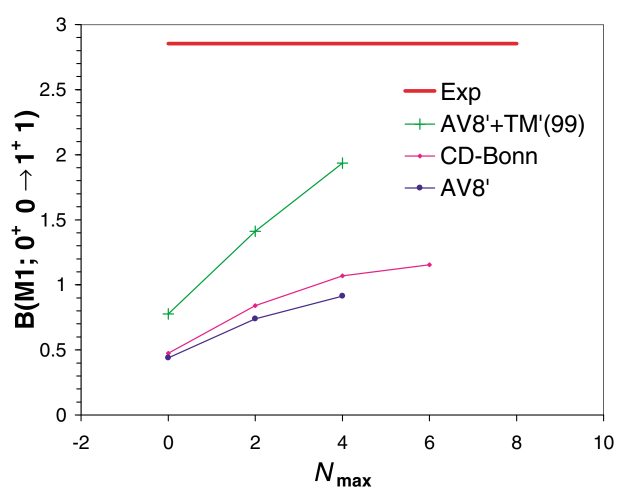

FIG. 2 (color online). $B(M1)$ values, in μ_N^2, of the $0^+0 \to 1^+1$ transition in ^{12}C. For details see Table I.

TABLE II. Predicted weak interaction rates for the $^{12}C \rightarrow T = 1\ 1^+$ transitions. The units are 10^{-42} cm^2 for the (ν_e, e^-) DAR cross section, 10^{-40} cm^2 for the (ν_μ, μ^-) DIF cross section, and 10^3 sec^{-1} for muon capture.

	CD-Bonn			AV8′ + TM′(99)	
Interaction	$2\hbar\Omega$	$4\hbar\Omega$	$6\hbar\Omega$	$4\hbar\Omega$	Experiment
(ν_e, e^-)	2.27	3.2	3.69	6.8	$8.9 \pm 0.3 \pm 0.9$ [19]
(ν_μ, μ^-)	0.168	0.275	0.312	0.537	$0.56 \pm 0.08 \pm 0.1$ [20]
μ-capture	1.46	2.07	2.38	4.43	6.0 ± 0.4 [21]

results suggests that a $6\hbar\Omega$ calculation that included a realistic TNI would come within 20% of experiment.

The (ν_μ, μ^-) neutrino cross section to $^{12}N_{gs}$ corresponds to the LSND muon neutrinos from decay in flight (DIF) of the pion. This spectrum involves neutrinos up to about 250 MeV, with an average neutrino energy of about 150 MeV and an average momentum transfer of about 200 MeV$/c$. In this case the $6\hbar\Omega$ CD-Bonn calculation is off by a factor of 1.8 compared with experiment. The $4\hbar\Omega$ calculation that includes the three-body TM′(99) interaction is, in fact, in agreement with experiment. However, based on the trends established above, this suggests that a larger model space may overpredict experiment. Examining the electron scattering form factor suggests that the problem lies in the fact that at 200 MeV$/c$ the predicted form factor is too large. Of course, as the model space is increased we expect the form factor to be shifted down in momentum.

In conclusion, the transition from the $^{12}C_{gs}$ to the $T = 1\ 1^+$ states in mass 12 is very sensitive to the strength of the spin-orbit interaction. We have investigated neutrino-^{12}C exclusive cross sections and muon capture on ^{12}C as well as inelastic electron scattering using wave functions obtained in the *ab initio* NCSM. In our parameter-free calculations with basis spaces up to $6\hbar\Omega$ we show that realistic NN interactions underpredict the experimental weak interaction cross sections by more than a factor of 2. At high momentum transfers around $q \sim 200$ MeV$/c$ the electron scattering form factor is overpredicted and the position of the predicted minimum is close to 400 MeV$/c$, compared to the experimental minimum at $q \sim 260$ MeV$/c$. By including a realistic TNI the weak interaction cross sections are enhanced significantly, which considerably improves agreement with experiment. The shape of the electron scattering form factor is also significantly improved, but the predicted form factor still peaks at too large a momentum transfer and is too large at the momentum transfers relevant to the LSND DIF cross section. The difference between the observed and predicted shape for the (e, e') form factors and the very different momentum transfers involved in the three weak processes examined here imply that a single experiment/theory scale factor cannot be defined for all three. The comparison between the CD-Bonn and the three-body calculations discussed here provide a strong confirmation of the need to include a realistic three-body interaction to account for the spin-orbit strength in p-shell nuclei.

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