Antisymmetric Spin Filtering in One-Dimensional Electron Systems with Uniform Spin-Orbit Coupling

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We theoretically analyze the possibility to use a ferromagnetic gate as a spin-polarization filter for one-dimensional electron systems formed in semiconductor heterostructures showing strong Rashba spin-orbit interaction. The proposed device is based on the effect of the breaking time-reversal symmetry due to the presence of weak magnetic fields. For a proper strength and magnetic field orientation there appears an energy interval in the electron energy spectrum at which the orientation of spin states is controlled by the direction of the electron velocity. It leads to the natural spin polarization of the electron current if the Fermi energy falls into this energy interval.

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Spin-polarized injection of electrons into semiconductor structures has been a field of growing interest during the last several years. The experiments based on the simplest idea to inject spin-polarized electrons from ferromagnetic metals have shown effects only of less than 1% because of a large resistivity mismatch between ferromagnetic and semiconductor materials [1]. Nevertheless, it has been predicted that atomically ordered and suitably oriented interfaces should allow a much stronger effect [2]. Experimentally more successful has been injection from dilute magnetic semiconductors because of the giant Zeeman splitting which can be utilized to force all current-carrying electrons to align their spin to the lower Zeeman level [3,4].

In the regime of the quantum-coherent transport the Zeeman splitting in combination with a buildup potential barrier can lead to spin filtering [5]. It gives rise to a spin-polarized flow since spin-up and spin-down electrons see different barrier heights. In low-dimensional electron systems a metallic gate across the conducting channel can be used to form the potential barrier. By applied voltage its height can be tuned to allow occupation of states belonging to the lower Zeeman level only to strengthen the spin filtering.

This Letter is devoted to the theoretical description of the possible spin filtering in the systems with relatively strong spin-orbit coupling. It will be argued that spin-filtering effects of the built-in potential barrier within a one-dimensional conducting channel depend on the current direction if the magnetic field of the appropriate strength and orientation is applied. Strictly speaking, electrons with opposite spin orientation will be filtered out if the current direction is changed. This property has its origin in the breaking of the time-reversal symmetry.

Let us first discuss properties of the two-dimensional electron gas confined in the z direction in the presence of the in-plane magnetic field, $\vec{B} \equiv (B_x, B_y, 0) \equiv (B\cos\gamma, B\sin\gamma, 0)$. For the sake of simplicity spin-orbit coupling due to the crystalline anisotropy of the host semiconductor (Dresselhaus term) is neglected and only the so-called "Rashba term" associated with the interfacial electric field [6,7] is considered. The unperturbed single-electron Hamiltonian reads

$$H_0 = \frac{p^2}{2m^\star}\sigma_0 + \frac{\alpha\langle\mathcal{E}_z\rangle}{\hbar}[p_y\sigma_x - p_x\sigma_y] + \frac{\epsilon_Z}{2B}\vec{B}\cdot\vec{\sigma}, \quad (1)$$

where $m^\star$ is the effective mass, σ_0 stands for unit matrix, $\langle\mathcal{E}_z\rangle$ represents the effective strength of the Rashba field perpendicular to the electron layer, ϵ_Z is the Zeeman splitting energy, and the components of the vector $\vec{\sigma} \equiv (\sigma_x, \sigma_y, \sigma_z)$ are Pauli matrices

$$\sigma_x \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_y \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (2)$$

Instead of the canonical momentum $\vec{p}$ entering the Hamiltonian the kinetic momentum $\vec{p} - e\vec{A}$ with vector potential $\vec{A}$ should be used. However, in the weak field limit when the magnetic length is much larger than the width of the confining well its effect can be neglected for the case of in-plane magnetic fields. For this reason the Hamiltonian H_0, Eq. (1), is considered as strictly a two-dimensional one with momentum $\vec{p} \equiv (p_x, p_y) \equiv -i\hbar\vec{\nabla}$. From the Hamiltonian equation the velocity operator $\vec{v}$ including spin contribution can be derived and we get

$$v_x = \frac{p_x}{m^\star}\sigma_0 - \frac{\alpha\langle\mathcal{E}_z\rangle}{\hbar}\sigma_y, \qquad v_y = \frac{p_y}{m^\star}\sigma_0 + \frac{\alpha\langle\mathcal{E}_z\rangle}{\hbar}\sigma_x. \quad (3)$$

The eigenfunctions of the Hamiltonian, Eq. (1), identified by quantum numbers $\vec{k} \equiv (k_x, k_y) \equiv (k\cos\phi, k\sin\phi)$ and $s = \pm 1$ are given as the product of the plane wave and a spin state

$$\phi_{\vec{k}}^{(s)}(\vec{r}) = \frac{1}{2\pi} e^{i\vec{k}\vec{r}} \begin{pmatrix} a_{\vec{k}}^{(s)} \\ b_{\vec{k}}^{(s)} \end{pmatrix}. \quad (4)$$

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Eigenvalues $E^{(s)}(\vec{k})$ and values of the elements $a_{\vec{k}}^{(s)}$ and $b_{\vec{k}}^{(s)}$ are solutions of the following eigenvalue problem:

$$\begin{pmatrix} k^2 - \varepsilon, 2\kappa_Z^2 e^{-i\gamma} + 2ik_\alpha k e^{-i\phi} \\ 2\kappa_Z^2 e^{i\gamma} - 2ik_\alpha k e^{i\phi}, k^2 - \varepsilon \end{pmatrix} \begin{pmatrix} a_{\vec{k}}^{(s)} \\ b_{\vec{k}}^{(s)} \end{pmatrix} = 0, \qquad (5)$$

where the following notation has been used:

$$\varepsilon = \frac{2m^*}{\hbar^2} E - k_\alpha^2, \qquad k_\alpha \equiv \frac{m^\star}{\hbar^2} \alpha \langle \mathcal{E}_z \rangle, \qquad \kappa_Z^2 \equiv \frac{m^\star}{\hbar^2} \frac{\epsilon_Z}{2}. \qquad (6)$$

Instead of eigenenergies $\varepsilon \hbar^2 / 2m^\star$ the results for the energy E shifted by $E_\alpha \equiv \hbar^2 k_\alpha^2 / 2m^\star$ will be presented.

For the case of the zero magnetic field we get [8]

$$E^{(s)}(\vec{k}, B = 0) = \frac{\hbar^2}{2m^\star} (k + sk_\alpha)^2, \qquad s = \pm 1. \qquad (7)$$

The corresponding eigenfunctions

$$\phi_{\vec{k}}^{(s)}(\vec{r}, B = 0) = \frac{1}{2\pi} e^{i\vec{k}\vec{r}} \frac{1}{\sqrt{2}} \begin{pmatrix} ise^{-i\phi/2} \\ e^{i\phi/2} \end{pmatrix} \qquad (8)$$

are eigenfunctions of the ϕ-dependent spin operator $\sigma(\phi) \equiv \sigma_x \cos(\phi - \pi/2) + i\sigma_y \sin(\phi - \pi/2)$ as well, and the spin is thus perpendicular to the direction of the electron velocity. For given energy E we have

$$\langle v_x + iv_y \rangle = \sqrt{\frac{2E}{m^\star}} e^{i\phi}; \qquad \langle \sigma_x + i\sigma_y \rangle = e^{i[\phi - s(\pi/2)]}. \qquad (9)$$

For each velocity there are two states with the opposite spins, and the system is thus not polarized by the effect of the considered spin-orbit coupling.

The above described situation is qualitatively changed if the in-plane magnetic field is applied. Let us assume that it is weak enough to satisfy the inequality $|\kappa_Z| < |k_\alpha|$. As expected the energy dispersion for $\vec{k}$ being perpendicular to the magnetic field direction,

$$E^{(s)}(k_\parallel = 0, k_\perp, B) = \frac{\hbar^2}{2m^\star} (k_\perp + sk_\alpha)^2 + s\frac{\epsilon_Z}{2} \qquad (10)$$

is composed of the two branches quadratic in $k_\perp$ which are shifted in energy by the Zeeman splitting energy and in the $k_\perp$ direction by the $2k_\alpha$. Note that the electron spin is directed along the magnetic field for $s = -1$ in accord with the standard usage.

More complicated is the dispersion for $\vec{k}$ being parallel with $\vec{B}$ $(\gamma = \phi)$,

$$E^{(s)}(k_\parallel, k_\perp = 0, B) = \frac{\hbar^2}{2m^\star} \left(k_\parallel^2 + k_\alpha^2 + 2s \cdot \mathrm{sgn}(k_\parallel) \sqrt{\kappa_Z^4 + k_\alpha^2 k_\parallel^2}\right), \qquad (11)$$

which is shown in Fig. 1. Note that we did not introduce a branch index. We preserve notation $s = \pm 1$ to stress the positive or negative expectation values of the spin operator $\sigma(\phi)$ representing spin projection into the direction perpendicular to $\vec{k}$.

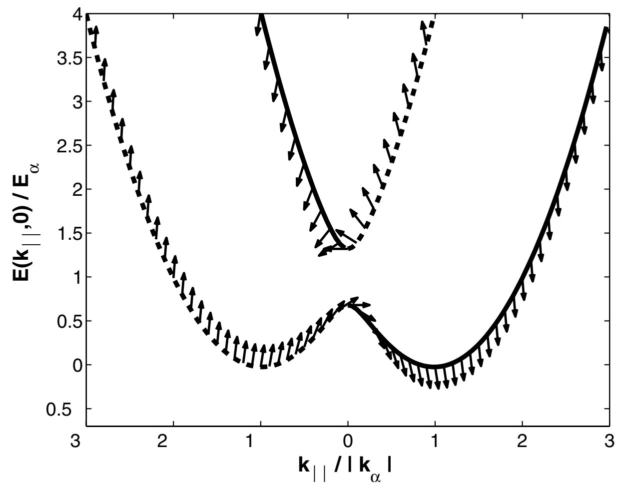

FIG. 1. Energy dispersion $E^{(s)}(k_\parallel, k_\perp = 0, B)$. The full line corresponds to $s = -1$, the dashed line to $s = +1$, and arrows illustrate spin orientation ($\kappa_Z = 0.4 \cdot k_\alpha$).

The energy branches avoid the crossing by forming a local gap which is called "pseudogap" in the following. Its width just equals Zeeman splitting energy ϵ_Z. In the vicinity of pseudogap edges there appears a strong tendency of the electron spin to be parallel with the magnetic field direction. Far from this energy region the spin is oriented approximately in the perpendicular direction, i.e., similarly as in the case of the zero magnetic field.

Energy dispersions along any $\vec{k}$ direction are similar to that presented in Fig. 1. With rising deviation from the direction of the magnetic field the width of the pseudogap decreases and the energy shift of the local minima in opposite directions appears. Finally, in the perpendicular direction, $k_\parallel = 0$, the pseudogap vanishes and the energy difference between minima reaches the Zeeman energy. Corresponding Fermi contours are shown in Fig. 2 for several energies.

For negative values of the Fermi energies the Zeeman splitting dominates and electron spin approximately follows the direction of the applied magnetic field along the Fermi contour. With rising Fermi energy the expanding contour starts to form a cavity. Just at $E_F = \epsilon_Z/2$ ($E_F/E_\alpha = 0.32$ for $\kappa_z = 0.4k_\alpha$) it gives rise to the second contour within the outer one. Internal Fermi contour collapses into the point at $E_F = (1 + \kappa_Z^2/k_\alpha^2)E_\alpha$ ($E_F/E_\alpha = 1.16$ for $\kappa_z = 0.4k_\alpha$) and starts to expand with a further increase of E_F. At high energies spin orientation of eigenstates is dominated by spin-orbit coupling.

Pronounced spin-polarization effects can be expected in the one-dimensional case [9] if the magnetic field is applied along the wire. Within the pseudogap region, where the system has one energy branch only as seen in

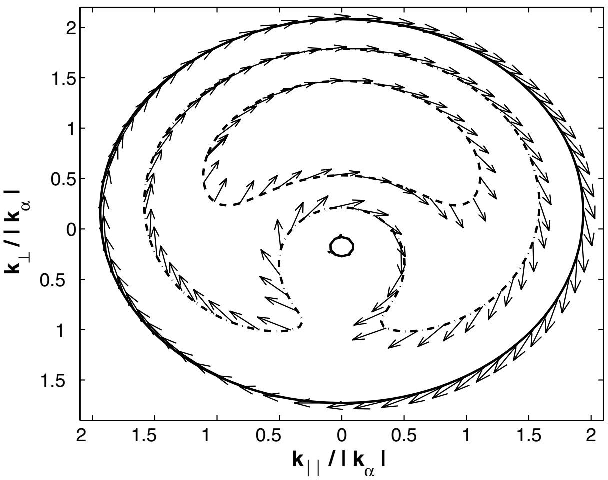

FIG. 2. Fermi contours for $E_F/E_\alpha = -0.1$ (dashed line), $E_F/E_\alpha = 0.3$ (dashed-dotted line), and $E_F/E_\alpha = 0.9$ (full line); $\kappa_Z = 0.4 \cdot k_\alpha$. Spin orientation is shown by arrows.

Fig. 1, the right and left going states are spin-polarized in opposite directions. It means that any current in the system with Fermi energy located within the pseudogap region will be strongly polarized and the sign of the polarization will depend on the current direction. Note that this effect is suppressed at higher magnetic fields, for which $|\kappa_Z|$ becomes greater than $|k_\alpha|$.

The above described property of the current control of the spin polarization can be used to design an antisymmetric spin filter. To show it, let us analyze the scattering at the interface introduced by a potential step of the height V_0 as sketched in Fig. 3. Transmission of electrons incoming from the left is controlled by four partial coefficients: $t_{\downarrow\downarrow}$ describing transmission probability from the left asymptotic states with $s = -1$ to the right asymptotic states with $s = -1$, $t_{\uparrow\uparrow}$ describing transmission probability between states with $s = +1$, and two probabilities allowing the spin-flip process, $t_{\downarrow\uparrow}$ and $t_{\uparrow\downarrow}$ denoting transmission probabilities from $s = -1$ to $s = +1$ and from $s = +1$ to $s = -1$, respectively.

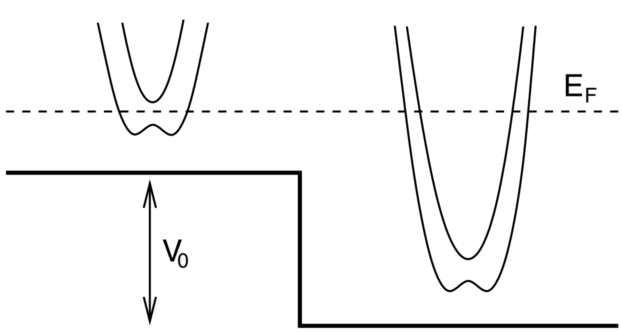

FIG. 3. Potential step of the height V_0 with sketched energy dispersions of the asymptotic states.

Transmission probabilities are defined as squares of the scattering-wave-function amplitudes which satisfy the standard conditions at the potential edge, i.e., the continuity of wave functions and their first derivatives. Within the pseudogap region the eigenvalue problem gives, besides the two real-$k_{\|}$ eigenstates, also two solutions with complex $k_{\|}$. Corresponding eigenfunctions describe states localized at the potential edge. Matching of these states to the extended states on the right-hand side of the step allows the spin-flip process.

The obtained dependence of partial transmission coefficients on V_0 for the fixed Fermi energy E_F which is well above the pseudogap of the right asymptotic states is shown in Fig. 4. For energies well above pseudogaps on both sides of the potential step the ccnductance

$$G = \frac{e^2}{h}(t_{\downarrow\downarrow} + t_{\uparrow\downarrow} + t_{\downarrow\uparrow} + t_{\uparrow\uparrow}) \equiv \frac{e^2}{h}T \tag{12}$$

is dominated by transmissions without spin-flip, i.e., by $t_{\downarrow\downarrow}$ and $t_{\uparrow\uparrow}$. However, if E_F falls within the pseudogap of incoming states the incoming current will be fully polarized and consequently $t_{\uparrow\uparrow} = t_{\uparrow\downarrow} = 0$. Although transmission $t_{\downarrow\downarrow}$ is still dominant the probability $t_{\downarrow\uparrow}$ is not negligible since the spin-flip process is strengthened by the presence of edge states.

Spin polarization of the current injected to the right side can be characterized by the polarization vector

$$\vec{P}^{(R)} = \frac{t_{\uparrow\uparrow} + t_{\downarrow\uparrow}}{T}\langle\uparrow| - \vec{\sigma}|\uparrow\rangle + \frac{t_{\uparrow\downarrow} + t_{\downarrow\downarrow}}{T}\langle\downarrow| - \vec{\sigma}|\downarrow\rangle, \tag{13}$$

where T is the transmission coefficient defined by Eq. (12). The polarization vector is normalized to have unit length for the fully polarized current. Its components perpendicular and parallel to the magnetic field direction, $P_\perp^{(R)}$ and $P_\|^{(R)}$, respectively, are shown in Fig. 5 as functions of V_0.

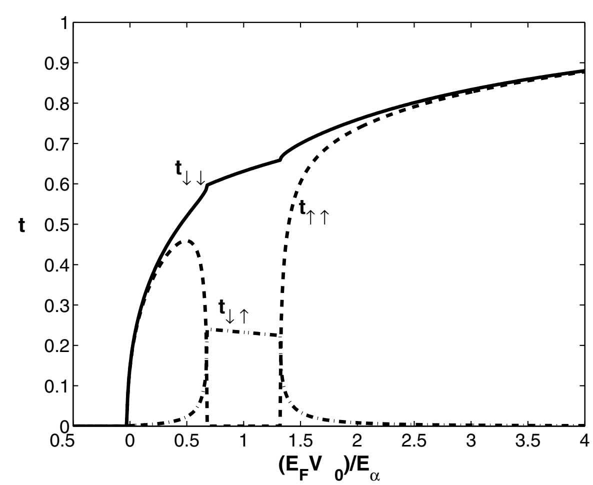

FIG. 4. Dependence of transmission probabilities $t_{\downarrow\downarrow}$, $t_{\uparrow\uparrow}$, and $t_{\downarrow\uparrow}$ on the height of the potential step. The probability $t_{\uparrow\downarrow}$ is negligible ($\kappa_Z = 0.4 \cdot k_\alpha$).

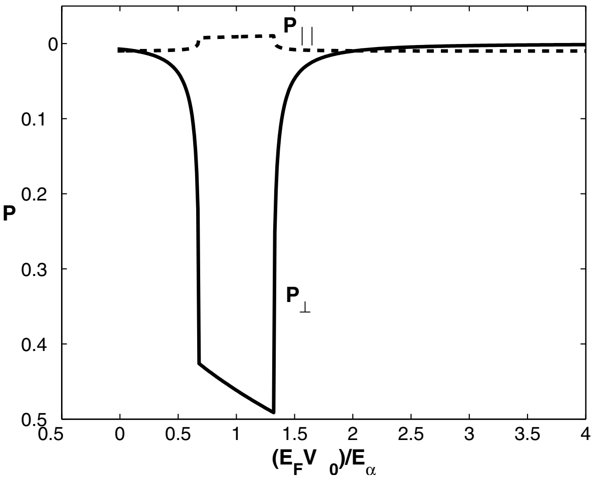

FIG. 5. Polarization components $P_\perp^{(R)}$ (full line) and $P_\parallel^{(R)}$ (dashed line) of the transmitted current as functions of the height of the potential step V_0 ($\kappa_Z = 0.4 \cdot k_\alpha$).

The above analysis leads to the conclusion that the potential barrier of the proper height for which the Fermi energy will fall into the pseudogap can be used as a spin filter. Polarization of the current flowing to the right is approximately given by the value of $P_\perp^{(R)}$, which is of the order of 50% for the presented numerical example. If the current is applied in the opposite direction the polarization has the opposite sign.

With a rising tilted angle between the wire axis and the magnetic field direction the width of the energy region with a pronounced spin-polarization effect decreases due to a decreasing pseudogap width. If the magnetic field is applied perpendicularly to the wire the pseudogap vanishes and no pronounced polarization effect appears.

Up to now we have discussed interface effects at a potential step in the case of the uniform magnetic field. A similar analysis can easily be extended to the case when the Zeeman splitting is nonzero in the region of the potential step only. Concerning filtering effects we have found no qualitative differences. If $|\vec{k}|$ at the Fermi energy out of the barrier region is much larger than $|k_\alpha|$ the differences in the values of transmission coefficients and polarization are negligible.

Quasi-one-dimensional systems have a much richer energy spectrum. Higher subbands avoid the crossing due to the Rashba term originated in the additional confinement even in the absence of the magnetic field [10]. The resulting pseudogap structure can lead to other types of the spin filtering [11]. Also the spin-orbit coupling of the Dresselhaus type, not considered in our treatment, modifies spin properties and under special conditions it can lead even to the cancellation of the Rashba effect [12].

The experimental verification of the discussed antisymmetric spin filtering thus requires a one-dimensional wire prepared from the structure with the dominant Rashba type of spin-orbit coupling. A gated semiconductor heterostructure seems to be promising with a conducting channel located within InAs layer [13]. It shows a relatively large Rashba field ($\alpha\langle\mathcal{E}_z\rangle \sim 4.5 \times 10^{-11}$ eV m) corresponding to $k_\alpha \sim 0.2 \times 10^8\ \mathrm{m}^{-1}$. The conducting channel of the width less than π/k_α will have a one-dimensional character within the pseudogap region. Because of the small effective electron mass ($\sim 0.036 m_e$) the width of the several tenths of nanometers would be satisfactory.

To open the pseudogap the magnetic field of the order 0.3 T should be applied to satisfy the relation $\kappa_Z \sim 0.4 k_\alpha$ used for the presented numerical examples. It is weak enough to be reached by a ferromagnetic gate of the proper thickness. The height of the potential barrier beneath the gate can be tuned by a gate voltage to reach conditions allowing the discussed antisymmetric spin filtering at low temperatures.

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