Reduction of Electron Heating in the Low-Frequency Anomalous-Skin-Effect Regime

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It is known that electron thermal motion in the anomalous-skin-effect regime of rf plasma discharges leads to enhancement of rf power absorption by plasma due to the resonant electron-wave interaction, which is a main mechanism of plasma heating in a typical inductively coupled plasma discharge. In this Letter we show, however, that the rf power absorption may be strongly reduced (compared to the Ohmic value) at low frequencies due to the electron thermal motion; an even further reduction occurs due to the nonlinear effects of the rf magnetic field. The absorption reduction occurs for $\omega < \nu < v_{\mathrm{th}}/\delta$ (ω is a driving frequency of rf wave, ν is the collision frequency, v_{th} is the thermal velocity, and δ is characteristic skin depth of the wave) and may be observed in low pressure inductive plasmas driven at low frequency and in a dc plasma with strong field inhomogeneity.

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Collisionless wave absorption is a fundamental phenomenon that is important for a number of applications, in particular, as a mechanism of collisionless heating of magnetically confined, laser and low temperature (industrial) plasmas. The effect was first recognized by Landau in his seminal work [1] where he considered the initial value problem, yielding the temporal collisionless damping of an infinite (in space) wave which is usually referred to as Landau damping. He also considered the boundary value problem, yielding a spatial decay of the longitudinal wave launched from the plasma boundary. The spatial decay of the transverse (electromagnetic) wave due to the wave-particle interaction was studied later [2–6] and is usually referred to as the anomalous skin effect.

Qualitatively, the collisionless dissipation can be described via a simple replacement of the collision frequency ν with the effective frequency of the wave-thermal particle interaction, $\nu \rightarrow v_{\mathrm{th}}/\delta$ [7,8]. Indeed, using the classical skin depth expression $\delta = (c^2\nu/\omega_{\mathrm{pe}}^2\omega)^{1/2}$ (for $\nu > \omega$) and substituting $\nu = v_{\mathrm{th}}/\delta$ results in a qualitatively correct expression for the anomalous skin depth $\delta \sim (c^2 v_{\mathrm{th}}/\omega_{\mathrm{pe}}^2\omega)^{1/3}$. The regime of the anomalous skin effect is characterized by strong spatial dispersion of the plasma conductivity and is often referred to as the nonlocal regime, while the regime of the classical (collisional) skin effect is referred to as the local regime [9].

The resonant absorption of the electromagnetic wave due to the electron thermal motion is an underlying heating mechanism of an inductively coupled plasma (ICP) discharge operating in the regime of the anomalous skin effect. This regime occurs for $v_{\mathrm{th}}/\delta \geq \omega$ and $v_{\mathrm{th}}/\delta > \nu_{\mathrm{en}}$, where ν_{en} is the frequency of electron-neutral atom collisions, which are dominant over all other types of collisions in a typical ICP. It has been shown [10,11] that in a low pressure ICP operating in the nonlocal regime the power absorption significantly exceeds the purely collisional value. Therefore it has been assumed that collisionless absorption always enhances plasma heating in inductively coupled plasma sources, compared to the purely collisional heating [12]. In the present work we show, however, that for low driving frequencies ($\omega < \nu_{\mathrm{en}}$), the collisionless effects reduce the total power absorption below the usual collisional absorption. In other words, the total electron heating due to both collisional and collisionless mechanisms of wave energy dissipation becomes smaller than it would have been if only the collisional mechanism was involved. This reduction occurs already in the linear model. Accounting for the nonlinear effects, such as the influence of the induced rf magnetic field, does not eliminate the heating reduction and can lead to an even stronger reduction of heating, as discussed below.

To consider the absorption of an electromagnetic wave we use a model [4,13] of a semi-infinite plasma occupying the region $z > 0$ with a plane boundary at $z = 0$ and assume that plasma electrons are specularly reflected from the plasma boundary. We consider a TE electromagnetic wave incident perpendicularly on the plasma. In this model, we do not solve the equation for the electric field E_y of the wave in plasma self-consistently but rather assume an exponentially decaying spatial profile $E_y(z) = E_0 \exp(-\gamma z)\exp(-i\omega t)$. Here E_0 is the real amplitude, and γ is the complex wave vector $\gamma = 1/\delta - i\kappa$. Although in the nonlocal regime the electric field profile can be nonmonotonic behind the skin layer, near the plasma boundary (within the skin layer) where the main absorption takes place the electric field profile is close to exponential [4,14], and it is quite accurate to use this assumption for the profile when studying nonlocal heating [15].

To find the electric current in the plasma J_y, induced by the electric field E_y, we employ the electron kinetic equation with a Bhatnagar-Gross-Krook (BGK)-type collision term. The spatial profile of the power absorption in plasma is $w(z) \equiv 1/2\mathrm{Re}(J_y E_y^*)$, where E_y^* is a complex conjugate of E_y. Following [16], for $w(z)$ we obtain

$$w(z) = \frac{\omega_{pe}^2}{8\pi} E_0^2 \mathrm{Re}\left[\frac{\exp(-\gamma^* z)}{\gamma v_{\mathrm{th}}} \times \left(G(z) - \exp(-\gamma z) Z(-is)\right)\right]. \quad (1)$$

Here $Z(p) = 1/\sqrt{\pi}\int_{-\infty}^{\infty}\exp(-x^2)dx/(x-p)$ is the plasma dispersion function, $s = (\omega + i\nu_{\mathrm{en}})/\gamma v_{\mathrm{th}}$, $v_{\mathrm{th}} = \sqrt{2T_e/m_e}$, and $G(z)$ is a complex function of z, $G(z) = 2/\sqrt{\pi}\int_0^{\infty} t/(t^2+s^2)\exp(i\gamma sz/t - t^2)dt$. The parameter s describes the degree of nonlocality of the plasma [4,9]. The condition $|s| = 1$ separates local ($|s| > 1$, classical skin effect) and nonlocal ($|s| < 1$, anomalous skin effect) regimes of ICP.

The total power absorbed by discharge plasma is $S_{\mathrm{tot}} = \int_0^{\infty} w(z)dz$, and after integration

$$S_{\mathrm{tot}} = \frac{\omega_{\mathrm{pe}}^2}{8\pi} E_0^2 \mathrm{Re}\left[\frac{1}{\gamma v_{\mathrm{th}}}\left(\frac{2}{\sqrt{\pi}}\int_0^{\infty}\frac{t^2\exp(-t^2)}{t^2+s^2}\frac{1}{\gamma^* t - i\gamma s}dt - \frac{\delta}{2}Z(-is)\right)\right]. \quad (2)$$

Expressions (1) and (2) describe power absorption in collisional and collisionless regimes [13,16], as well as in intermediate regimes when both thermal and collisional effects are important. Expression (1) also describes the negative power absorption found in [17], as shown in [16].

The total power absorption due to a collisional mechanism alone is given by a standard expression

$$S_{\mathrm{coll}} = \frac{\omega_{\mathrm{pe}}^2}{16\pi} E_0^2 \delta \frac{\nu_{\mathrm{en}}}{\nu_{\mathrm{en}}^2 + \omega^2}. \quad (3)$$

The ratio $\eta = S_{\mathrm{tot}}/S_{\mathrm{coll}}$ can be used as a measure of the influence of the electron thermal motion on the electron heating. As noted above, it is believed that in the nonlocal regime the electron heating usually exceeds the purely collisional heating [10]; however, this has not been investigated at the low driving frequencies, when $\omega < \nu_{\mathrm{en}}$.

It should be noted that in order to correctly estimate the collisional part of the heating in η, the same value of ν_{en} should be used in calculating the collisional heating (3) as that used in calculating the total nonlocal heating (2). In our calculations we use the experimental values of ν_{en} from [10].

Although in the collisionless nonlocal regime the ratio $\kappa/\delta^{-1} \sim 1$, it becomes much smaller if the collisions are taken into account, $\kappa/\delta^{-1} \ll 1$ for $\omega \lesssim \nu_{\mathrm{en}} \sim v_{\mathrm{th}}/\delta$ [9,18,19]. Thus for low frequencies, $\omega < \nu_{\mathrm{en}}$, neglecting the imaginary part $i\kappa$ in γ, the parameter s becomes purely imaginary, $s = i\delta\nu_{\mathrm{en}}/v_{\mathrm{th}}$, and (2) can be reduced to standard integrals so that $\eta = 2/\sqrt{\pi}|s|(-1 + \sqrt{\pi}|s| + \exp(-|s|^2)(1-|s|^2)\{\pi \mathrm{erfi}(|s|) + \mathrm{Re}[\mathrm{Ei}(1, -|s|^2)]\}) < 1$. Here $\mathrm{erfi}(|s|) = -i\,\mathrm{erf}(i|s|)$ is the imaginary error function, $\mathrm{erf}(x) = 2/\sqrt{\pi}\int_0^x \exp(-t^2)dt$, and $\mathrm{Ei}(n, x) = \int_1^{\infty} dt \exp(-xt)/t^n$ is the exponential integral. For low frequencies $\omega < \nu_{\mathrm{en}}$, η is smaller than unity for arbitrary values of $|s|$, contrary to the general belief that the nonlocal heating always exceeds the purely collisional value. For higher collisionality, $|s| \geq 1$, the parameter η approaches unity, which corresponds to the transition into the collisional regime.

To confirm the results of our simple model, we also performed calculations of plasma heating with a self-consistent electromagnetic field [20], thus taking into account any possible nonmonotonic behavior of the electric field in plasma. Following [18,20], the self-consistent rf electric field in plasma is

$$E_y = -\frac{2}{\pi}\left(\frac{dE_y}{dz}\right)\bigg|_{z=0^+}\int_0^{\infty}\frac{\cos(kz)}{k^2 + i\alpha\sigma_a(k)}dk, \quad (4)$$

where $\alpha = 8\sqrt{\pi}ne^2\omega/(mv_{\mathrm{th}}c^2)$, and $\sigma_a(k)$ is the Fourier transform of the nonlocal plasma conductivity defined in [21]. The self-consistent electric current in the plasma is

$$J_y = \frac{\omega_{\mathrm{pe}}^2}{4\pi^{3/2}v_{\mathrm{th}}}\int_{-\infty}^{\infty}\sigma_a(k)E_k\exp(ikz)dk, \quad (5)$$

where $E_k = -2(dE_y/dz|_{z=0^+})/[k^2 + i\alpha\sigma_a(k)]$. The power absorption is calculated as $S_{\mathrm{tot}} = 1/2\int_0^{\infty}\mathrm{Re}[J_y(z)E_y^*(z)]dz$, with E_y and J_y defined by expressions (4) and (5). The ratio of the total to collisional heating $\eta(\omega)$ is now found as a ratio of the total heating calculated self-consistently with warm electrons, to the total heating calculated self-consistently in the collisional limit (cold electrons), keeping ν_{en} the same as in the case of warm electrons.

The comparison of our approximate theory with the self-consistent theory, shown in Fig. 1, demonstrates good agreement between them; the effective reduction of the total absorbed power exists in both cases.

The effect of the heating reduction caused by the electron thermal motion is also illustrated in Figs. 2(a) and 2(b), where the spatial profile of the nonlocal heating rate $w(z)$ is compared to the spatial profile of the purely collisional heating, for two different values of the driving frequency. For $\omega < \nu_{\mathrm{en}}$ [Fig. 2(a)], the total heating rate is reduced compared to the collisional value, while for $\omega > \nu_{\mathrm{en}}$ [Fig. 2(b)], the resonant wave-particle interaction significantly enhances plasma heating. Such an enhancement of plasma heating in ICP as well as negative power absorption have earlier been demonstrated in experiments [10,17].

A simple interpretation of the effective plasma heating reduction by the electron thermal motion can be given on the basis of an expression for the nonlocal electron conductivity in the Fourier space [21,22]

$$\sigma_k = \frac{2n_0e^2}{m_e}\int d^3v\left(\frac{v_y}{v_{\mathrm{th}}}\right)^2\frac{f_m}{\nu_{\mathrm{en}} - i(\omega - kv_z)}, \quad (6)$$

where $k \simeq 1/\delta$. The heating rate is proportional to the real part of the conductivity, $w = \mathrm{Re}(\sigma)|E|^2/2$. One can see that for low frequency, $\omega < \nu_{\mathrm{en}}$, $\omega < v_{\mathrm{th}}/\delta$, the real part of the conductivity, $\mathrm{Re}(\sigma) \sim \nu_{\mathrm{en}}/(v_{\mathrm{th}}^2/\delta^2 + \nu_{\mathrm{en}}^2)$, is reduced by the effect of the thermal motion compared to

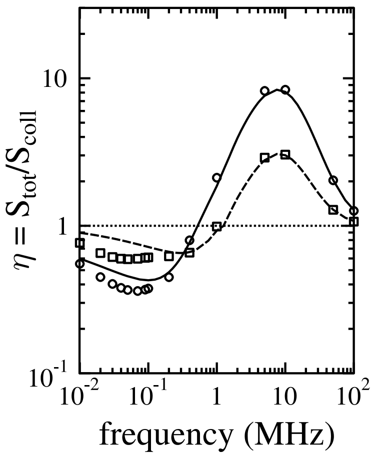

FIG. 1. The ratio of the total to the collisional absorbed power calculated from the approximate model [Eqs. (2) and (3)] and the self-consistent model, for two different sets of plasma parameters corresponding to experimental conditions in Ref. [10]: (i) discharge power 100 W, $p = 0.3$ mTorr, $\nu_{en} = 1.5 \times 10^6\ s^{-1}$, $T_e = 10$ eV, $n_0 = 2.7 \times 10^{10}\ cm^{-3}$ (solid curve: the approximate analytic model; circles: the self-consistent model); (ii) discharge power 100 W, $p = 1.0$ mTorr, $\nu_{en} = 4.6 \times 10^6\ s^{-1}$, $T_e = 5.8$ eV, $n_0 = 3.9 \times 10^{10}\ cm^{-3}$ (dashed curve: the approximate analytic model; squares: the self-consistent model).

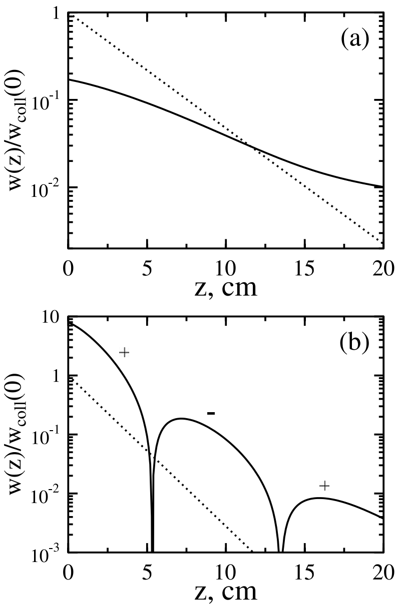

FIG. 2. Spatial profiles of the total (solid lines: warm electrons) and purely collisional (dotted lines: cold electrons) absorbed power $w(z)$ normalized to collisional absorbed power density at the boundary $w(0)$, for two different wave frequencies: $f = 0.1$ MHz (a), and $f = 5$ MHz (b). Other parameters are $T_e = 10$ eV, $n_0 = 2.7 \times 10^{10}\ cm^{-3}$, and $\nu_{en} = 1.5 \times 10^6\ s^{-1}$. Sign $(-)$ indicates the region of negative power absorption.

the cold plasma expression $\mathrm{Re}(\sigma_{coll}) \sim \nu_{en}/(\omega^2 + \nu_{en}^2)$. For relatively high frequencies approaching the resonant condition $\omega \simeq v_{th}/\delta$, the heating is enhanced due to the wave-particle interaction. For even larger frequencies, in the local regime when $\omega > v_{th}/\delta$, thermal effects can be neglected. Our results are also applicable to the case of $\omega = 0$. Note that the expression for $\mathrm{Re}(\sigma)$ implicitly assumes a finite spatial localization of the electric field with a characteristic inhomogeneity scale δ, so that the thermal modification of a dc ($\omega = 0$) electric conductivity can be detected only in a situation with strongly inhomogeneous electric field, $\delta < v_{th}/\nu_{en}$.

So far, we have not taken into account the nonlinear effect of the induced rf magnetic field assuming it to be small. However, at low frequencies, the rf magnetic field can significantly influence the electron heating [23–30]. To account for the effects of rf magnetic field and further confirm our analytical model we conducted a particle-in-cell (PIC) simulation of the electron heating in ICP in the nonlocal regime with a prescribed exponential spatial profile of the rf electromagnetic field in plasma. The simulation assumed immobile ions and thermal electrons and was carried out in a plasma slab of the width $L \gg \delta$, where δ is the electromagnetic field skin depth, with electrons reflecting specularly from the walls. The skin depth δ was found from the plasma surface impedance, calculated self-consistently [18,20]. In the simulation we included the electron-atom collisions (implemented into the PIC code using the direct Monte Carlo method) as the principal collisions in weakly ionized discharge plasma, and the electron-electron collisions (implemented via the Langevin equation [31]) as a mechanism for the "Maxwellization" of the electron distribution function. The simulation has been carried out for two different values of the electric field: (a) a weak field, when the electrons oscillatory velocity $eE_y/m\omega$ is much smaller than their thermal velocity, $eE_y/m\omega \ll v_{th}$ and (b) a strong field, when $eE_y/m\omega \sim v_{th}$. To test the predictions of our linear theory, we also conducted the simulations of the nonlocal electron heating in the linear case neglecting the rf magnetic field. The simulation results are shown in Fig. 3. The parameter $\eta = S_{tot}/S_{coll}$ was calculated as a ratio of the measured electron heating (averaged over the rf field period) to the purely collisional heating (3). We have found that, in the linear case (no magnetic field, circles in Fig. 3) the results of the simulation are in very good agreement with the results of our linear theory. In the case of the weak fields the rf magnetic field at low frequencies leads to a small enhancement of heating compared to the prediction of the linear theory (shown in Fig. 3 by squares). Note that the value of the electric field in the case of a weak field is smaller than the typical experimental values; however, we used such a field to illustrate the effect of weak nonlinearity on electron heating. In the case of a strong field (shown in Fig. 3 by triangles), which is close to the typical experimental values at low frequencies, the rf magnetic field leads to the opposite effect of that of the case of a weak field—a further significant reduction of plasma heating at low frequencies when the nonlinearity parameter ω_c/ω is

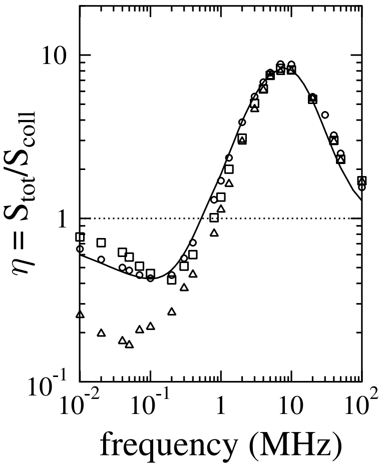

FIG. 3. The ratio of the total to the collisional absorbed power as a function of rf frequency. The solid curve represents the linear model, circles represent the PIC simulations without rf magnetic field, squares represent the simulations with a weak magnetic field ($E_0 = 0.05$ V/cm, $T_e = 10$ eV), and triangles represent the simulations with a strong magnetic field ($E_0 = 0.5$ V/cm, $T_e = 10$ eV). Other parameters are $n_0 = 2.7 \times 10^{10}$ cm^{-3} and $\nu_{en} = 1.5 \times 10^6$ s^{-1}.

large (ω_c is the electron cyclotron frequency in the rf magnetic field). At larger frequencies, ω_c/ω is small and the effect of the rf magnetic field is insignificant. A further (compared to the linear theory) reduction of plasma heating due to the effects of the rf magnetic field could be related to several nonlinear effects, in particular, plasma density depletion in the skin region caused by the ponderomotive effect [23,24] and electron trapping by the electromagnetic field in the skin region [30]. Note that in this illustrative example we have neglected the effect of the self-consistent ambipolar field, which can also influence the electron heating [23,24].

In this work we investigated the nonlocal electron heating in the regime of the anomalous skin effect. We have shown that at low driving frequencies, $\omega < \nu_{en}$, the total absorption of the wave power by plasma electrons becomes smaller than the purely collisional value. This effect has been obtained by using a simple approximate model with a prescribed profile of the electric field and validated by using a more complete theory of the electron heating with a self-consistently calculated electric field, as well as by the "linear" PIC simulation (without rf magnetic field). We have also studied the nonlinear effects of the rf magnetic field by using the PIC simulation. It was found that a strong magnetic field further reduces the electron heating at low frequencies. The reduction of heating by the electron thermal motion could be significant in low pressure inductive plasmas at low frequencies and in dc plasmas with strong field inhomogeneity when the condition $\omega < \nu < v_{th}/\delta$ is met.

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