Efficient Engineering of Multiatom Entanglement through Single-Photon Detections

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We propose an efficient scheme to engineer multiatom entanglement by detecting cavity decay through single-photon detectors. In the special case of two atoms, this scheme is much more efficient than previous probabilistic schemes, and insensitive to randomness in the atom's position. More generally, the scheme can be used to prepare arbitrary superpositions of multiatom Dicke states without the requirements of high-efficiency detection and separate addressing of different atoms.

There is a large current interest in generation and engineering of quantum entanglement, with applications for fundamental tests of quantum mechanics [1], for high-precision measurements [2], and, in particular, for implementation of quantum communication and computation [3]. Although quantum entanglement is typically fragile to practical noise and technical imperfections, there exist elegant ways to overcome this sensitivity by designing schemes with inherent robustness to diverse sources of noise. Some schemes with this property have been known for entangling two single atoms [4–9] as well as for entangling macroscopic atomic ensembles [10,11]. In these schemes, feedback is typically applied to the system of interest based upon the outcome of certain measurements. The protocols are thereby probabilistic, succeeding only conditionally for particular measurement results. Imperfections and noise in these schemes decrease the success probability, but have no influence on the fidelity of the intended state generation for the "successful" subset of trials. In this way, a high-fidelity entangled state can be obtained simply by repeating the scheme successively.

Here, we propose a robust scheme to produce and engineer entanglement between multiple atoms in optical cavities. Compared with the previous robust schemes [4–11], our protocol has the following favorable features. (i) It is much more efficient in the sense that the success probability can be close to unity, whereas in the previous schemes [4,5,8–11], the success probability is required to be much smaller than 1 to have the property of inherent robustness. (ii) It is more insensitive to certain practical sources of noise, such as randomness in the atom's position, atomic spontaneous emission, or detector inefficiency. (iii) Individual addressing of atoms is not required [6], nor are single-photon states as initial resources [7]. (iv) Most importantly, our scheme is not limited to generation of two-atom entanglement. Indeed, we show that based on current experimental technology, it should be possible to generate any superposition of the Dicke states [12] between multiple atoms in an optical cavity.

These Dicke states and their superpositions, including the multiparty GHZ states as special cases, are typically highly entangled, with many applications in quantum information science [2,13–15]. Their entanglement can be directly detected without separate addressing [2].

As the scheme here is inherently robust to noise, it works in principle for entangling atoms (or ions) both in free-space configurations and in high-Q cavities, albeit in the free-space case one has a much smaller efficiency to collect the emitted photons. In this Letter, for a close relation with the current experimental efforts [16–18], we assume that there is a standing-wave high-Q cavity around the atoms [19,20] to improve the collection efficiency.

To explain the scheme, let us start from the simplest case with two atoms trapped in two different cavities. The schematic setup is shown in Fig. 1(a), with the relevant atomic levels depicted in Fig. 1(b). The states $|g\rangle$, $|0\rangle$, $|1\rangle$ correspond to the hyperfine and the Zeeman sublevels of alkali atoms in the ground-state manifold, and $|e\rangle$ corresponds to an excited state. The atom is initially prepared in the state $|g\rangle$, but the basis vectors of a qubit are represented by the states $|0\rangle$ and $|1\rangle$. The transition $|g\rangle \to |e\rangle$ is driven adiabatically through a classical laser pulse with the corresponding Rabi frequency denoted by $\Omega(t)$ [21]. With the driving pulse, the atom is transferred with probability $p_c \simeq 1$ to the $|0\rangle$ and $|1\rangle$ states by emitting a photon from the transitions $|e\rangle \to |0\rangle$ or $|e\rangle \to |1\rangle$.
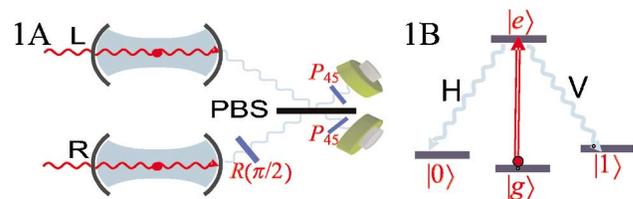
FIG. 1 (color online). (a) The schematic setup to generate entanglement between two atoms in different cavities **L** and **R**. (b) The relevant atomic level structure and the laser configuration.

Without loss of generality, we assume that the transitions $|e\rangle \to |0\rangle$ and $|e\rangle \to |1\rangle$ are coupled to two degenerate cavity modes a^h and a^v with different polarizations h and v. The decay pulses from the two cavities are interfered at a polarization beam splitter (PBS), with the outputs detected by two single-photon detectors after a 45° polarizer [denoted as P_{45} in Fig. 1(a)]. The small fraction of the transmitted classical pulse can be easily filtered based on the frequency selection as detailed in Ref. [21]. For the decay pulse from the **R** cavity, a polarization rotator $R(\pi/2)$ is inserted before the PBS which exchanges h and v polarizations of the incoming photon. Conditioned upon registering one photon from *each* of the detectors, the two atoms in the cavities **L** and **R** will be prepared into the maximally entangled state

$$|\Psi_{LR}\rangle = (|01\rangle_{LR} + |10\rangle_{LR})/\sqrt{2}. \tag{1}$$

To see this, we write down the interaction Hamiltonian in the rotating frame, which, for each of the cavities, has the form (setting $\hbar = 1$)

$$H = \Omega(t)|e\rangle\langle g| + g_0|e\rangle\langle 0|a^h + g_1|e\rangle\langle 1|a^v + \text{H.c.}, \tag{2}$$

where g_0 and g_1 are the corresponding coupling rates. The cavity outputs a_{out}^μ ($\mu = h, v$) are connected with the cavity modes a^μ through the standard input-output relations $\dot{a}^\mu = -i[a^\mu, H] - \kappa a^\mu/2 - \sqrt{\kappa}a_{\text{in}}^\mu(t)$ and $a_{\text{out}}^\mu(t) = a_{\text{in}}^\mu(t) + \sqrt{\kappa}a^\mu$ [22], where κ is the cavity decay rate, and $a_{\text{in}}^\mu(t)$, with the commutation relation $[a_{\text{in}}^\mu(t), a_{\text{in}}^{\mu\dagger}(t')] = \delta(t - t')$, denotes the vacuum cavity input. We are interested in the limit for which the variation rate of $\Omega(t)$ is significantly smaller than the cavity decay rate κ. In this limit, we can define an effective single-mode bosonic operator a_{eff}^μ from the cavity output operator $a_{\text{out}}^\mu(t)$ as $a_{\text{eff}}^\mu = \int_0^T f(t)a_{\text{out}}^\mu(t)dt$ (see Refs. [21,23]), where T is the pulse duration and $f(t)$ is the output pulse shape, which is determined by the shape of $\Omega(t)$ as $f(t) = \sqrt{\kappa}\sin\theta(t)\exp[-(\kappa/2)\int_0^t \sin^2\theta(\tau)d\tau]$ with $\sin\theta(t) = \Omega(t)/\sqrt{|g_0|^2 + |g_1|^2 + |\Omega(t)|^2}$. After the driving pulse, for each of the cavities λ ($\lambda = \mathbf{L}, \mathbf{R}$), the final state between the atom and the corresponding cavity output has the form

$$|\Psi\rangle_\lambda = (g_0|0\rangle_\lambda|h\rangle_\lambda + g_1|1\rangle_\lambda|v\rangle_\lambda)/\sqrt{|g_0|^2 + |g_1|^2}, \tag{3}$$

where $|\mu\rangle = a_{\text{eff}}^{\mu\dagger}|\text{vac}\rangle$, ($\mu = h, v$), and $|\text{vac}\rangle$ denotes the vacuum state of the optical modes.

If the driving pulses have the same shape $\Omega(t)$ for the **L** and **R** cavities, the output single-photon pulses from the two cavities will also have the same shape $f(t)$, and they will interfere with high visibility at the polarization beam splitter (PBS). If one gets a "click" from each of the detectors at the outputs of the PBS, the two incoming photons can be either both in h polarizations or both in v polarizations, and these two possibility amplitudes are coherently superposed when the incoming photon pulses

overlap with each other with the same shape. Therefore, the measurement in Fig. 1(a), together with the polarization rotator $R(\pi/2)$, corresponds to projecting the whole state $|\Psi\rangle_L \otimes |\Psi\rangle_R$ between the atoms and the photons onto a subspace with the projection operator given by $P_s = |hv\rangle_{LR}\langle hv| + |vh\rangle_{LR}\langle vh|$. Within this measurement scheme, the state $|\Psi\rangle_L \otimes |\Psi\rangle_R$ is effectively equivalent to the four-particle GHZ state

$$|\Psi_{\text{eff}}\rangle \propto P_s|\Psi\rangle_L \otimes |\Psi\rangle_R \propto (|01\rangle_{LR} \otimes |hv\rangle_{LR}$$
$$+ |10\rangle_{LR} \otimes |vh\rangle_{LR})/\sqrt{2}. \tag{4}$$

The 45° polarizers in Fig. 1(a) project the photon polarizations to the $(|h\rangle + |v\rangle)/\sqrt{2}$ state. It immediately follows from Eq. (4) that after this measurement the two atoms will be prepared in the maximally entangled state (1). If one rotates the angles of the polarizers in Fig. 1(a), corresponding a measurement of the incoming photon polarizations either in the $\{|h\rangle, |v\rangle\}$ basis or in the $\{(|h\rangle + |v\rangle)/\sqrt{2}, (|h\rangle - |v\rangle)/\sqrt{2}\}$ bases, one can further demonstrate four-particle GHZ-type of entanglement between the atoms and the photons as indicated by the effective state (4) [24]. The 45° polarizer can also be replaced by a PBS with both of its outputs detected by single-photon detectors. The measurement success probability is then increased by a factor of 2 for each side, and the overall success probability of this scheme becomes $p_s = 2|g_0g_1|^2/(|g_0|^2 + |g_1|^2)^2$.

Before introducing the multiatom entangling scheme, we offer a few remarks about this two-cavity scheme. First, it is evident that the scheme is inherently robust to atomic spontaneous emission, output coupling inefficiency, and detector inefficiency, all of which contribute to loss of photons. Since a click from each of the detectors is never recorded if one photon is lost, these processes simply decrease the success probability p_s by a factor of η^2 (where $1 - \eta$ denotes the loss for each of the photons), but have no influence on the fidelity of the final state $|\Psi_{LR}\rangle$. Second, our scheme does not require localization of the atom in the cavity to the Lamb-Dick limit. For the standing-wave cavity shown in Fig. 1(a) and with the collinear pumping configuration proposed in Ref. [21], $\Omega(t)$, g_0, and g_1 depend on the atom's position through approximately the same cavity mode function. The pulse shape $f(t)$, which is determined by the ratios $\Omega(t)/g_0$ and $\Omega(t)/g_1$, thus becomes basically independent of the random variation in the atom's position. For a traveling-wave cavity or for a free-space configuration, the atom's position affects only the common phase of the coupling rates g_0 and g_1, and in this case, a transverse pumping configuration also suffices since the randomness in the common phase of g_0 and g_1 has no influence on the final entangled state $|\Psi_{LR}\rangle$. Finally, the success probability of our scheme is $p_s \sim 1/2$ in the ideal case with $g_0 \sim g_1$ and $\eta \sim 1$, which shows that the present scheme is significantly more efficient than the previous schemes

[4,5,8–11], where the success probability is required to be much smaller than 1 even if $\eta \rightarrow 1$.

We next extend our basic scheme to entangle multiple atoms in the same optical cavity. The schematic setup is shown by Fig. 2, with each of the N_a atoms taken to have the same level structure as depicted in Fig. 1(b) and with the atoms not separately addressable [16–18]. The initial state of the system has the form $|G\rangle = \bigotimes_{i=1}^{N_a} |g\rangle_i$ with all the atoms prepared to the ancillary state $|g\rangle$. The driving laser, incident from one side mirror, is now divided into M sequential pulses, with $M \geq N_a/2$. We assume that the intensity of the pulse is controlled so that for each of the M pulses, an approximate fraction $1/M$ of the atomic population is transferred adiabatically from the $|g\rangle$ state to the $|0\rangle$ or $|1\rangle$ states, by emitting on average N_a/M photons with h or v polarizations. The output photons from the cavity decay are split by a PBS according to their polarizations, and then registered through two single-photon detectors (called h and v detectors, respectively). For each driving pulse, we may or may not get a click from the h or v detectors, which are assumed *not* to distinguish one or more photons. For the whole M pulses, we can count the total number of "clicks" (n_h, n_v) registered from the (h, v) detectors, respectively. Of course, $n_h + n_v \leq N_a$ since there are only N_a atoms. If it turns out that $n_h + n_v = N_a$, the following Dicke state results for the N_a atoms:

$$|N_a, n_h\rangle = c(n_h)(s_0^\dagger)^{n_h}(s_1^\dagger)^{N_a - n_h}|G\rangle. \qquad (5)$$

Here, the collective operators $s_\mu^\dagger$ ($\mu = 0, 1$) are defined as $s_\mu^\dagger = \sum_{i=1}^{N_a} |\mu\rangle_i\langle g|$, and the normalization coefficient $c(n_h) = [N_a! n_h! (N_a - n_h)!]^{-1/2}$. Except the trivial cases with $n_h = 0, N_a$, clearly the Dicke state $|N_a, n_h\rangle$ is entangled. The multiatom Dicke states and the GHZ states in general belong to different classes of entangled states, and the Dicke states are relatively more robust to the influence of noise [13]. The Dicke states have some interesting applications in quantum information processing and in high-precision measurements [14,15].

To understand why a Dicke state results conditioned upon the above type of measurement, we note that each atom has an equal probability to emit a photon with the same pulse shape for each driving pulse for the assumed sequence of adiabatic passages. Hence, each driving pulse
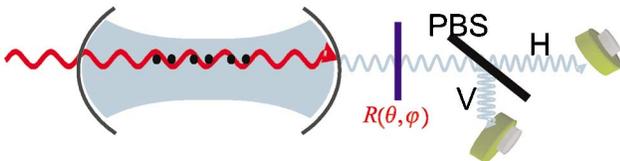
FIG. 2 (color online). The schematic setup to generate entanglement between multiple atoms in the same cavity. The polarization rotator $R(\theta, \varphi)$ is only required for generation of superpositions of the Dicke states.

involves a collective excitation of the atoms to the $|0\rangle$ or $|1\rangle$ levels with homogeneous superposition coefficients. For the subset of measurements for which we register N_a photoelectric events in total from the h and v detectors for the whole M pulses, each click of the detectors should correspond exactly to the emission of one and only one photon by the atoms. This is the case even if there are photon loss and detector inefficiencies, because we post select only the trials with exactly N_a photoelectric events. Therefore, for each click registered from the h or v detectors for these trials, we should apply correspondingly the collective operators $s_0^\dagger$ or $s_1^\dagger$ to the atomic state. After registering n_h h-polarized photons and $(N_a - n_h)$ v-polarized photons, we get exactly the state of Eq. (5). Similar to the two-cavity scheme, this multiatom entangling scheme is also robust to practical imperfections, such as a moderate randomness in the atoms' positions and various sources of photon loss. Again, photon loss reduces the success probability instead of the state fidelity.

To calculate the success probability of the multiatom entangling scheme, we note that the stepwise driving method described above is actually equivalent to the following one-step driving method: we transfer all the atomic population to the $|0\rangle$ and $|1\rangle$ levels with a single driving pulse, but both of the h and v polarized photons after the PBS need to be further split equally into M paths through a series of beam splitters, with separate photoelectric detection for each path. The state in Eq. (5) corresponds to the case when n_h h detectors and $(N_a - n_h)$ v detectors register a photoelectric event. When two or more photons go to the same path, the number of detector events is certainly less than N_a. So, for overall success with N_a events, we require that each photon follow a distinct path, for which the success probability is given by $p_{si} = (2M)!/[(2M - N_a)!(2M)^{N_a}]$ (in total there are $2M$ paths. For simplicity, we have assumed $g_0 \approx g_1$ so that one has equal probability to get h or v photons.) All photon loss processes simply contribute to an undercount probability $1 - \eta$ for each photon. Hence, the success probability to generate one of the Dicke states of Eq. (5) is $p_{\text{succ}} = \eta^{N_a} p_{si}$, while the probability to obtain a specific Dicke state $|N_a, n_h\rangle$ is $p_{n_h} = p_{\text{succ}} 2^{-N_a} N_a!/[n_h!(N_a - n_h)!]$. Excluding the trivial cases with $n_h = 0, N_a$, we then find that the success probability to obtain an entangled state from this scheme is $p_{en} = p_{\text{succ}}(1 - 2^{-N_a+1})$, which tends to unity in the case $2M \gg N_a$ if we neglect contributions from photon loss (i.e., $\eta \rightarrow 1$). This scheme could thus be quite efficient. For instance, with $\eta = 0.70\,(0.20)$ and $M = 50\,(10)$ pulses, $p_{en} = 0.018\,(1.9 \times 10^{-4})$ for $N_a = 10\,(5)$ atoms, so that repeating this scheme on average $1/p_{en} \approx 56$ (5.4×10^3) times leads to a high-fidelity entangled state between 10 (5) atoms. In current experimental setups [16,17], the typical duration Δt of the adiabatic pulse is a few hundred nanoseconds, so that the total duration

$(M/p_{en})\Delta t \simeq 1$ (20) ms for entangling 10 (5) atoms with $\eta = 0.70$ (0.20). The currently available trapping time of atoms in high-Q cavities is about 1 sec [16].

Finally, we would like to extend further the above scheme to generate any superposition of the Dicke states $|N_a, n_h\rangle$. For this purpose, we simply insert a polarization rotator $R(\theta, \varphi)$ before the PBS as shown in Fig. 2, which transforms the photon polarizations according to $|h\rangle \rightarrow \cos\theta|h\rangle + \sin\theta e^{i\varphi}|v\rangle$ and $|v\rangle \rightarrow -\sin\theta e^{-i\varphi}|h\rangle + \cos\theta|v\rangle$. We assume that the parameters θ, φ can be separately controlled for each driving pulse, and are denoted by θ_m, φ_m for the mth pulse. As before, we consider only the subset of cases for which exactly N_a photoelectric events are registered from the whole M-pulse sequence. If the h (or v) detector clicks for the mth pulse with the control parameters θ_m, φ_m, the corresponding atomic excitation operator P_{m0} (or P_{m1}) is expressed by the collective operators $s_\mu^\dagger$ as $P_{m\mu} = \cos\theta_m s_\mu^\dagger - (-1)^\mu \sin\theta_m e^{i\varphi_m(-1)^\mu} s_{1-\mu}^\dagger$ ($\mu = 0, 1$). So, after N_a registered events, the final atomic state has the form $|\Psi_F\rangle = \prod_{i=1}^{N_a} P_{m_i\mu}|G\rangle$, where m_i ($i = 1, 2, \ldots, N_a$) denote the set of driving pulses for which we register a photon. Each operator $P_{m_i\mu}$ introduces two real parameters $\theta_{m_i}, \varphi_{m_i}$, so there are $2N_a$ independently controllable real parameters in the state $|\Psi_F\rangle$. The state $|\Psi_F\rangle$ can be written in general in the form

$$|\Psi_F\rangle = \sum_{n_h=0}^{N_a} b(n_h)|N_a, n_h\rangle, \tag{6}$$

where the Dicke states $|N_a, n_h\rangle$ are defined by Eq. (5), and the complex superposition coefficients $b(n_h)$ are functions of $\theta_{m_i}, \varphi_{m_i}$. Superpositions of the Dicke states have $2N_a$ degrees of freedom, which exactly equals to the number of control parameters $\theta_{m_i}, \varphi_{m_i}$.

Actually, we can prove that an *arbitrary superposition* of the Dicke states $|N_a, n_h\rangle$ [i.e., the state $|\Psi_F\rangle$ with any coefficients $b(n_h)$] is obtainable by choosing an appropriate set of control parameters $\theta_{m_i}, \varphi_{m_i}$. For the proof, we write the state (6) in the form $|\Psi_F\rangle = b(N_a)c(N_a) \times \sum_{n_h=0}^{N_a} b'(n_h)(s_0^\dagger)^{n_h}(s_1^\dagger)^{N_a-n_h}|G\rangle$, where $b'(n_h) = c(n_h)b(n_h)/[b(N_a)c(N_a)]$, and without loss of generality we have assumed $b(N_a) \neq 0$. Each of the atomic excitation operators $P_{m_i\mu}$ can be expressed as $P_{m_i\mu} \propto (s_0^\dagger - r_{m_i\mu}s_1^\dagger)$, where the complex coefficient $r_{m_i\mu}$, determined by the real parameters $\theta_{m_i}, \varphi_{m_i}$, is the relevant control parameter. To prepare a desired state $|\Psi_F\rangle$ with the superposition coefficients $b'(n_h)$, we need to choose the parameters $r_{m_i\mu}$ to satisfy the algebraic equation $\prod_{i=1}^{N_a}(s_0^\dagger - r_{m_i\mu}s_1^\dagger) = \sum_{n_h=0}^{N_a} b'(n_h)(s_0^\dagger)^{n_h}(s_1^\dagger)^{N_a-n_h}$. It immediately follows from this equation that the parameters $r_{m_i\mu}$ should be the N_a solutions of the N_ath-order algebraic equation $\sum_{n_h=0}^{N_a} b'(n_h)x^{n_h} = 0$, where x denotes the variable. In the complex domain, there always exist N_a

solutions to the N_ath-order algebraic equation, and the parameters $r_{m_i\mu}$ are uniquely determined from these solutions if we do not care about the order of the excitation operators $P_{m_i\mu}$ (note that they commute with each other). This finishes the proof.

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