Can Modern Nuclear Hamiltonians Tolerate a Bound Tetraneutron?

Steven C. Pieper*

Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA

(Received 18 February 2003; published 27 June 2003)

I show that it does not seem possible to change modern nuclear Hamiltonians to bind a tetraneutron without destroying many other successful predictions of those Hamiltonians. This means that, should a recent experimental claim of a bound tetraneutron be confirmed, our understanding of nuclear forces will have to be significantly changed. I also point out some errors in previous theoretical studies of this problem.

DOI: 10.1103/PhysRevLett.90.252501 PACS numbers: 21.30.–x, 21.45.+v, 21.60.Ka, 27.10.+h

An experimental claim of the existence of a bound tetraneutron cluster (^4n) was made last year [1,2]. Since then, a number of theoretical attempts to obtain such bound systems have been made, with the conclusion that nuclear potentials do not bind four neutrons [3–5]. However, these studies have been made with simplified Hamiltonians and only approximate solutions of the four-neutron problem. In this Letter, I use modern realistic nuclear Hamiltonians that provide a good description of nuclei up to $A = 10$ and accurate Green's function Monte Carlo (GFMC) calculations to improve this situation. (Earlier studies, also with generally negative results, are cited in Refs. [3–5].)

A series of papers [6–8] have presented the development of GFMC for calculations of light nuclei (so far, up to $A = 10$) using realistic two-nucleon (NN) and three-nucleon (NNN) potentials. For a given Hamiltonian, the method obtains ground and low-lying excited state energies with an accuracy of 1%–2%. I use this method in the present study; tests similar to those reported in the above papers have verified that the energies reported here have similar accuracies, with two exceptions: (i) when the energies are very close to 0, the error is probably a few 100 keV; and (ii) the ^{4}H calculations contain a technical difficulty that might be introducing systematic errors of up to 1 MeV. (This problem arises from the fact that GFMC calculations are made using a slightly simplified version of the Hamiltonian. The expectation value of the difference of the desired and simplified Hamiltonians is evaluated perturbatively and might have a large relative error. In all cases except ^{4}H, this difference is small; in particular, for ^4n it is less than 0.1 MeV. However, for unknown reasons, the change is up to 2.5 MeV in ^{4}H.) A review of nuclear GFMC is in Ref. [9]; complete details of how the present calculations were made are in Refs. [6–8].

By using the Argonne v_{18} NN potential (AV18) [10] and including two- and three-pion exchange NNN potentials, a series of model Hamiltonians (the Illinois models) were constructed that reproduce energies for $A = 3$–10 nuclei with rms errors of 0.6–1.0 MeV [11]. The best model, the AV18 + Illinois-2 (AV18/IL2) model, is used in the present study.

GFMC starts with a trial wave function Ψ_T, which determines the quantum numbers of the state being computed. For p-shell nuclei studied in the above references, the Jastrow part of Ψ_T contains four nucleons with an alpha-particle wave function and $A - 4$ nucleons in p-shell orbitals. This is multiplied by a product of noncentral two- and three-particle correlation operators. I use Ψ_T for ^4n with the same structure except there are two neutrons in a $^1S_0$ configuration and two in the p shell. The total J^π of the ^4n ground state is assumed to be 0^+. There are two possible symmetry states in the p shell using LS coupling: $^1S[22]$ and $^3P[211]$; both are used in these calculations. I could find no Ψ_T that gave a negative energy for ^4n using the AV18/IL2 model. GFMC calculations, using propagation to very large imaginary time ($\tau = 1.6$ MeV^{-1}), also produced positive energies that steadily decreased as the rms radius of the system increased.

In a second study, I added artificial external wells of Woods-Saxon shape to the AV18/IL2 Hamiltonian and used GFMC to find the resulting total energies of the four neutrons. Figure 1 shows results for wells with radii $R =$ 3, 6, and 9 fm (all have diffuseness parameters of 0.65 fm) and varying depth parameter V_0. It seems clear that four

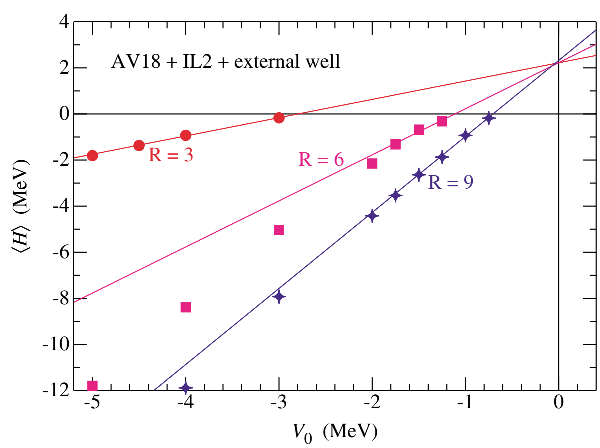

FIG. 1 (color online). Energies of ^4n in external wells versus the well-depth parameter V_0.

neutrons become unbound (have positive energy) significantly before the well depth is reduced to zero. Linear fits to the least-bound energies for each Woods-Saxon radius parameter are also shown; these extrapolate to an energy of +2 MeV when the external well is removed. (These least-bound solutions have large rms radii. A transition from the indicated linear behavior to a steeper linear behavior is observed for deeper wells; this transition is associated with a change to much smaller rms radii solutions. The steeper fits, of course, extrapolate to much larger positive energies.) This suggests that there might be a ^4n resonance near 2 MeV, but since the GFMC calculation with no external well shows no indication of stabilizing at that energy, the resonance, if it exists at all, must be very broad. In any case, the AV18/IL2 model does not produce a bound ^4n.

The authors of Ref. [1] suggest that only small modifications of existing nuclear Hamiltonians may be necessary to bind four neutrons. To study this possibility, I made a number of modifications to the AV18/IL2 model. In each case, the modification was adjusted to bind ^4n with an energy of approximately -0.5 MeV; the consequences of this modification for other nuclei were then computed. Four of the modifications are reported here: long- and moderate-range changes of the NN potential in the $^1S_0$ partial wave; introduction of an additional NNN potential that acts only in total isospin $T = \frac{3}{2}$ triples; and introduction of a $NNNN$ potential that acts only in $T = 2$ quadruples. In all cases, the complete AV18/IL2 Hamiltonian was used with the additional term.

The strong-interaction part of the AV18 NN potential consists of one-pion exchange with the generally accepted value of $f_\pi^2/4\pi = 0.075$, moderate-range terms that are associated with two-pion exchange but which have phenomenologically adjusted strengths, and a short-ranged completely phenomenological part. The potential is written in terms of operators which can be used to produce the potential for any partial wave. By making correlated changes to the radial parts of the different terms, one can change basically only the $^1S_0$ partial wave (the next wave changed is $^1G_0$).

In the first such modification of the AV18, I changed just the two-pion range part of the $^1S_0$ partial wave, so as to leave the theoretically well established one-pion part unaffected. Increasing this two-pion strength by 4.9% results in a ^4n energy of $-0.87(3)$ MeV. (The statistical errors in Monte Carlo computed numbers are shown in parentheses only when they exceed unity in the last quoted digit.) As is shown by the points labeled "mod-$^1S_0$-2π" in Fig. 2, this changes the $^1S_0$ phase shifts by 12° over a large energy range and produces a bound dineutron (the energy is -0.88 MeV, which means that ^4n can still decay into two dineutrons). These changes far exceed those allowed by modern phase shift analysis. A somewhat smaller change that produces a negative-energy ^4n can be made by using the AV1′ potential [12] in the $^1S_0$ partial wave (and AV18 in the other partial waves); this

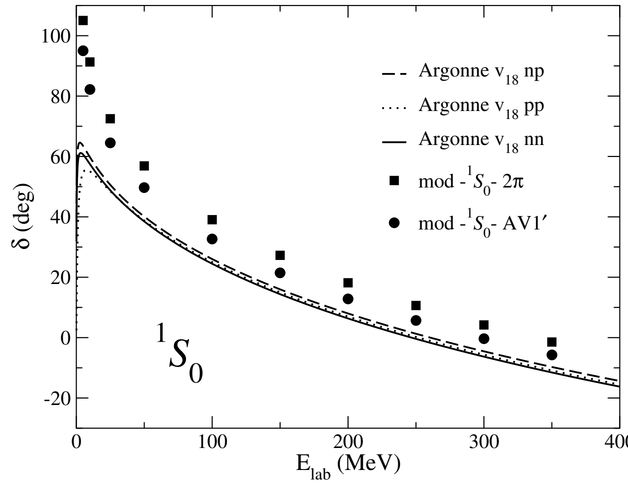

FIG. 2. $^1S_0$ phase shifts from AV18 and modifications to it. The lines show the pp, pn, and nn phase shifts for the unmodified AV18 while the symbols show the modified results.

results in a ^4n energy of -0.52 MeV and about a 50% smaller change in the $^1S_0$ phase shifts (the points labeled "mod-$^1S_0$-AV1′" in the figure). However, again ^2n is bound, this time with an energy of -0.42 MeV and the ^4n is not stable against breakup into two dineutrons. Note that the one-pion-range part of the potential is also changed in mod-$^1S_0$-AV1′. These ^2n and ^4n states are quite diffuse; the rms radii are, respectively, 2.8 and 3.6 fm for the two ^2n cases and 7.3 and 10.3 fm for the ^4n. The ^4n pair distributions have a peak containing about two pairs with a structure close to that of the ^2n pair distribution and a long tail. Thus, the ^4n looks like two widely separated dineutrons.

As noted, these modifications of the $^1S_0$ potential to produce tetraneutrons with negative total energy also produce dineutrons with about the same energies; thus, they are physically unacceptable modifications. Figure 3 shows that they also introduce large changes to the binding energies of other nuclei; for example, ^{3}H is ~50% overbound and ^{5}H is stable or almost stable against breakup into $^3\mathrm{H} + n + n$ as opposed to being a resonance in that channel [13]. Also, six and eight neutrons form bound systems, although three and five do not. Figure 3 also shows that the base Hamiltonian we are using underbinds ^{4,5}H by an amount comparable to the +2 MeV energy that was estimated for ^4n; this might suggest that fixing the Hamiltonian for these cases would result in a bound ^4n. However, as noted, the changes necessary to bind ^4n result in large overbinding for ^{5}H; thus, a change that fixes the ^{5}H energy will make a very small change to the ^4n result. As discussed above, the ^{4}H results could have large systematic errors; the $^1S_0$-AV1′ result for ^{4}H does not seem consistent with all the other nuclei.

The authors of Refs. [3–5] concluded that the nonrealistic Volkov potentials [14] do not bind ^4n. However, these potentials do have bound dineutrons. I made calculations using the first four Volkov potentials in all partial

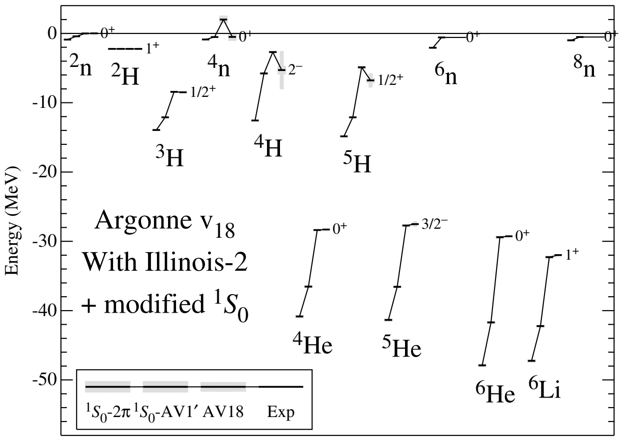

FIG. 3. Energies of nuclei and neutron clusters computed with the AV18/IL2 Hamiltonian with modified NN potentials ($^1S_0$-2π and $^1S_0$-AV1$'$) and with no modification (AV18), compared with experimental values for known nuclei.

waves and no NNN potential. These potentials indeed do give ^4n with energies of -0.91, -1.04, -0.47, and -0.71 MeV, which are considerably more negative than the above studies suggest. However, the ^2n energies are -0.56, -0.60, -0.35, and -0.42 MeV, respectively, so the ^4n can again decay into two dineutrons. The rms radii of the ^4n systems are all about 11.5 fm, which may explain why these states were not discovered in Refs. [3–5]. The variational energies for ^4n with modifications to the AV18/IL2 Hamiltonian are positive; that is, only with GFMC improvement does the energy become negative. However, for the simpler Volkov potentials, the Ψ_T already give negative energies and the GFMC just improves these energies.

It must be emphasized that these almost bound ^4n results do not at all support an experimentally bound ^4n. The more than 35-year-old Volkov potentials are not realistic; they produce bound ^2n, with the same binding energies as their deuterons; they have no tensor or LS terms; and they cannot reproduce modern phase shift analyses in any partial wave. The one thing in their favor is that, by having a space-exchange component, they introduce some saturation in p-shell nuclear binding energies; however, with just one radial form they are even simpler than the space-exchange AVX$'$ introduced in Ref. [12].

The above results show that it is not possible to bind ^4n by modifying the $^1S_0$ potential without severely disrupting other nuclear properties. The next NN possibility is the ^3P_J channel. The net effect of these is a small repulsion in neutron systems. Setting this term to zero had very little effect on ^4n; one would have to introduce significant attraction to bind ^4n and then again many other nuclear properties would be unrealistically changed.

Modifications to the NNN or $NNNN$ potentials, which are experimentally much less constrained than the NN potential, could be used to bind ^4n. Timofeyuk added a central $NNNN$ potential to bind ^4n, but found that it resulted in ^{4}He being bound by about 100 MeV [3,5]. However, as she suggests, one should try less disruptive things. A NNN potential that acts only in $T = \frac{3}{2}$ triples would have the same effect on ^4n as one with no isospin dependence, but no effect on ^{3}H and ^{4}He because they contain only $T = \frac{1}{2}$ triples. A $NNNN$ $T = 2$ potential would also not affect ^{5}He and ^{6}Li.

I added potentials of the forms

$$V_{ijk}\left(T = \frac{3}{2}\right) = V_3 \sum_{\text{cyclic}} [Y(r_{ij})Y(r_{jk})] P\left(T = \frac{3}{2}\right),$$

$$V_{ijkl}(T = 2) = V_4 \sum_{\text{cyclic}} [Y(r_{ij})Y(r_{jk})Y(r_{kl})] P(T = 2),$$

$$Y(r) = \frac{e^{-m_\pi r}}{m_\pi r}[1 - e^{-(m_\pi r)^2}]^2,$$

to the AV18/IL2 Hamiltonian. Here m_π is the pion mass, the P are projectors onto the indicated isospin states, and V_3 and V_4 were chosen to produce ^4n with ~ -0.5 MeV energy. These forms have the longest range that is possible from strong interactions; the cutoff makes the radial forms peak at 1.55 fm. Using more confined radial forms only increases the problems reported below.

It turns out that the couplings must be quite large to produce the minimally bound ^4n: $V_3 = -440$ and $V_4 = -4750$ MeV, which result in ^4n energies of $-0.60(5)$ and $-0.55(6)$ MeV. This can be understood as follows. If the NN potential is used to bind ^4n, the pairs can sequentially come close enough to feel the attraction; this allows the four neutrons to be in a diffuse, large radius, distribution. However, a NNN potential requires three neutrons to simultaneously be relatively close and thus the density of the system must be much higher. Indeed, the rms radii of the ^4n for the $V_{ijk}(T = \frac{3}{2})$ case is only 1.88 fm, while that for $V_{ijkl}(T = 2)$ is 1.61 fm. Such small radii result in kinetic energies that are an order of magnitude more than those for the ^4n systems bound by modified $^1S_0$ potentials; for the $V_{ijk}(T = \frac{3}{2})$ case, the expectation value of the kinetic energy is $\sim$87 MeV, while those of the NN and NNN potentials are -49 and -38 MeV, respectively. (The kinetic energy is found by subtracting GFMC potential values from $\langle H \rangle$ [6].)

The very large coupling constants for the $V_{ijk}(T = \frac{3}{2})$ and $V_{ijkl}(T = 2)$ potentials mean that they have a large, even catastrophic, effect on any nuclear system in which they can act. This is shown in Fig. 4; for example, $V_{ijk}(T = \frac{3}{2})$ doubles the binding energy of ^{6}Li and triples that of ^{6}He, while $V_{ijkl}(T = 2)$, which can have no effect on ^{6}Li, quadruples the binding energy of ^{6}He. As noted before, both of these potentials have no effect on ^{4}He. Both potentials make ^{5}H stable by more than 25 MeV against ^{3}H $+ n + n$. However, the most dramatic result of these potentials is that every investigated pure neutron system with $A > 4$ is extremely bound and, in fact, is the most stable "nucleus" of that A. For $V_{ijk}(T = \frac{3}{2})$ the energies are -62, -220, and -650 MeV, respectively,

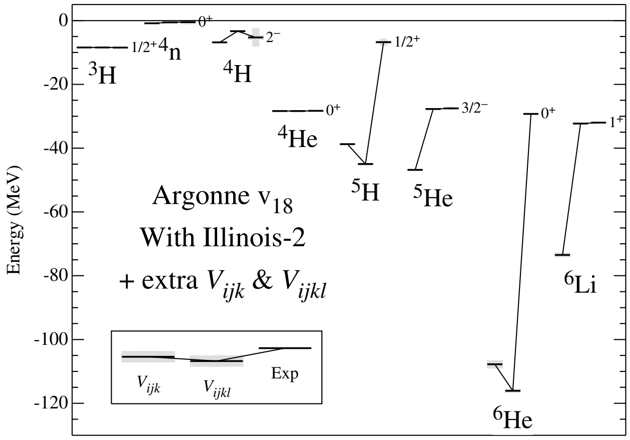

FIG. 4. Energies of nuclei and neutron clusters computed with modified NNN and $NNNN$ potentials.

for $^{5,6,8}n$, while for $V_{ijkl}(T=2)$ they are -358, -1370, and -6690 MeV.

These enormous bindings indicate that matter will collapse with such potentials. This is to be expected for purely attractive many-nucleon potentials. One should add a shorter-ranged stronger repulsion to obtain saturation. Such a repulsion might improve the results for $A \geq 6$ nuclei. I studied this by using a repulsive term with Yukawa radial forms of range $2m_\pi$. However, in order to get any appreciable effect on ^{6}He, the repulsive coupling has to be made quite large; this then requires at least a doubling of the attraction to still bind ^4n; this results in potentials that are so strong that the GFMC starts to become unreliable. The apparent impossibility of correcting the $A = 6$ results by such a term may also be seen from the rms radii of the ^4n reported above; they are smaller than the experimental value for ^{6}Li and reasonable ^{6}He radii. Thus, a short-ranged repulsion that still leaves the ^4n bound will certainly result in $A = 6$ nuclei with too small rms radii.

In all of these cases, I have made isospin-conserving modifications to the AV18/IL2 Hamiltonian; thus, there have been $T = 1$ additions to the NN potential, or a $T = \frac{3}{2}$ addition to the NNN potential, or a $T = 2$ addition to the $NNNN$ potential. One could modify the force only for nn pairs or nnn triples or $nnnn$ quadruples since the nuclear force is least well determined for such systems. Such changes would mean much larger charge-symmetry breaking and charge-independence breaking potentials than are presently accepted. But even so, the changes to the NN force, if limited to just nn pairs, would still bind two neutrons, which would change the experimental scattering length from ~ -18 fm to a positive value. Such a nn potential would still bind ^6n and ^8n. I estimate that it would still increase the binding of ^{3}H by 3 MeV while it would have no effect on ^{3}He. Thus, the Nolen-Schiffer energy for the $A = 3$ system would be some 5 times too large. Many of the devastating effects shown in Fig. 4 would similarly persist even if the potentials were limited to nnn triples or $nnnn$ quadruples.

The GFMC method is presently limited to local potentials while meson-exchange potentials may contain significant nonlocalities; thus, one might wonder if nonlocal NN potentials could produce a bound ^4n without binding ^2n. As discussed, the negative-energy ^4n produced by modifying the NN force have very large (> 7 fm) rms radii and consist of dineutrons with rms radii of ~ 3 fm. These are much larger than the distances over which nonlocalities are significant, so the limitation to local potentials should not matter.

In conclusion, should the results of Ref. [1] be confirmed (Ref. [2] contains additional considerations of background in these types of experiments), our current very successful understanding of nuclear forces would have to be severely modified in ways that, at least to me, are not at all obvious.

I thank R. B. Wiringa for many useful suggestions during the course of this work, especially concerning modifications to the AV18 potential; I also thank V. R. Pandharipande for a critical reading of an early version of the manuscript. The calculations were made possible by generous grants of time on the Chiba City Linux cluster of the Mathematics and Computer Science Division and friendly-user access to the Jazz Linux cluster of the Laboratory Computing Resource Center, both at Argonne National Laboratory. This work is supported by the U.S. Department of Energy, Nuclear Physics Division, under contract No. W-31-109-ENG-38.

*Electronic address: spieper@anl.gov

[1] F. M. Marqués *et al.*, Phys. Rev. C **65**, 044006 (2002).
[2] N. A. Orr and F. M. Marqués, nucl-ex/03030005.
[3] N. K. Timofeyuk, nucl-th/0203003.
[4] C. A. Bertulani and V. Zelevinsky, nucl-th/0212060.
[5] N. K. Timofeyuk, J. Phys. G **29**, L9 (2003).
[6] B. S. Pudliner, V. R. Pandharipande, J. Carlson, S. C. Pieper, and R. B. Wiringa, Phys. Rev. C **56**, 1720 (1997).
[7] R. B. Wiringa, S. C. Pieper, J. Carlson, and V. R. Pandharipande, Phys. Rev. C **62**, 014001 (2000).
[8] S. C. Pieper, K. Varga, and R. B. Wiringa, Phys. Rev. C **66**, 044310 (2002).
[9] Steven C. Pieper and R. B. Wiringa, Annu. Rev. Nucl. Part. Sci. **51**, 53 (2001).
[10] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995).
[11] S. C. Pieper, V. R. Pandharipande, R. B. Wiringa, and J. Carlson, Phys. Rev. C **64**, 014001 (2001).
[12] R. B. Wiringa and S. C. Pieper, Phys. Rev. Lett. **89**, 182501 (2002).
[13] A. A. Korsheninnikov *et al.*, Phys. Rev. Lett. **87**, 092501 (2001).
[14] A. B. Volkov, Nucl. Phys. **74**, 33 (1965).