Relativistic Kinetic Equations for Finite Domains and the Freeze-Out Problem

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The relativistic kinetic equations for two domains separated by a hypersurface with both spacelike and timelike parts are derived. The particle exchange between the domains separated by timelike boundaries generates source terms and modifies the collision term of the kinetic equation. The correct hydrodynamic equations for the "hydro + cascade" models are obtained and their differences from existing freeze-out models of the hadronic matter are discussed.

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I. Introduction.—In recent years, essential progress has been achieved in our understanding of the freeze-out problem in relativistic hydrodynamics, i.e., how to convert the hydrodynamic solution into free streaming particles. The solution of this problem [1] required two principal steps: first, the correct generalization of the famous Cooper-Frye result [2] for timelike hypersurfaces given by the *cutoff* formula derived in [1]; second, the extension of the energy-momentum and particle conservation from the expanding fluid alone to a system consisting of a fluid and a gas of free streaming particles [1,3] which are emitted from the freeze-out hypersurface. The advantage of this approach is the absence of logical and causal paradoxes usually arising, if the emission of particles happens from the timelike hypersurfaces [1,3]. At the same time this approach is rather complicated.

Numerous attempts to improve and to develop this approach further using primitive kinetic models [4] were not very successful. Very recently, a more fundamental approach [5] was suggested; however, it cannot be used to describe the phase transition phenomenon. The latter difficulty has been overcome naturally within the "hydro + cascade" models suggested in Ref. [6] [Bass and Dumitru (BD)] and further developed in [7] [Teaney, Lauret, and Shuryak (TLS)]. The latter models assume that the nucleus-nucleus collisions proceed in three stages: hydrodynamic expansion ("hydro") of the quark gluon plasma (QGP), phase transition from the QGP to the hadron gas (HG), and the stage of hadronic rescattering and resonance decays ("cascade"). The switch from hydro to cascade modeling takes place at the boundary between the mixed and hadronic phases. The spectrum of hadrons leaving this hypersurface of the QGP–HG transition is taken as input for the cascade.

Such an approach incorporates the most attractive features of both *hydrodynamics*, which describes the QGP–HG phase transition very well, and *cascade*, which works better during hadronic rescattering. For this reason, it is rather successful in explaining the nuclear collisions data at CERN SPS and BNL RHIC energies. However, both the BD and TLS models face some principal difficulties which cannot be ignored. Thus, within the BD approach the initial distribution for the cascade is found using the Cooper-Frye formula [2], which takes into account particles with all possible velocities, whereas in the TLS model the initial cascade distribution is given by the *cutoff* formula [1,3], which accounts for only those particles that can leave the phase boundary. As shown below, the Cooper-Frye formula will lead to causal and mathematical problems in the present version of the BD model because the QGP–HG phase boundary inevitably has timelike parts. On the other hand, the TLS model from the beginning does not conserve energy, momentum, and number of charges and this, as demonstrated later, is because the equations of motion used in [7] are not complete and, hence, should be modified.

The main difficulty of the "hydro + cascade" approach looks very similar to the difficulty of the freeze-out problem in relativistic hydrodynamics [1,3]. In both cases the domains (subsystems) have timelike boundaries through which the exchange of particles occurs and this should be taken into account. The exchange of particles on the timelike boundary between domains should generate source terms in the kinetic equations. To obtain these terms, we shall consider two semi-infinite domains which are separated by a hypersurface of general type and rederive the kinetic equations for this case in Sect. II. In Sect. III, the modification of the collision terms is found and the relation between the system obtained, and the Boltzmann equation is discussed.

II. Drift term for semi-infinite domain.—Consider two semi-infinite domains, "in" and "out," separated by the hypersurface Σ^* which, for the purpose of presenting the idea, we assume to be given in $3+1$ dimensions by a single valued function $t = t^*(\bar{x}) = x_0^*(\bar{x})$. The distribution function $\phi_{\rm in}(x, p)$ for $t \le t^*(\bar{x})$ belongs to the "in" domain, whereas $\phi_{\rm out}(x, p)$ denotes the distribution function of the "out" domain for $t \ge t^*(\bar{x})$ (see Fig. 1). In this work, it is assumed that the initial conditions for $\phi_{\rm in}(x, p)$ are given, whereas on Σ^* the function $\phi_{\rm out}(x, p)$ is allowed to differ from $\phi_{\rm in}(x, p)$ and this will modify the kinetic equations for both functions. For simplicity, we consider a classical gas of pointlike Boltzmann particles.

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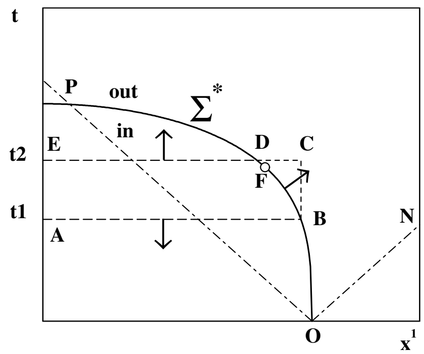

FIG. 1. Schematic 2D picture of the boundary hypersurface Σ^* (solid curve). Arrows show the external normal vectors. The light cone NOP is shown by the dash-dotted line. The point F divides Σ^* into the timelike (OF) and spacelike (FP) parts.

Similarly to Ref. [8], we derive the kinetic equations for $\phi_{\rm in}(x, p)$ and $\phi_{\rm out}(x, p)$ from the requirement of particle number conservation. Therefore, the particles leaving one domain (and crossing hypersurface Σ^*) should be subtracted from the corresponding distribution function and added to the other one. Now we consider the closed hypersurface of the "in" domain, Δx^3 (shown as the $ABDE$ contour in Fig. 1), which consists of two semiplanes σ_{t1} and σ_{t2} of constant time $t1$ and $t2$, respectively, that are connected from $t1$ to $t2 > t1$ by the arc BD of the boundary $\Sigma^*(t1, t2)$ in Fig. 1. The original number of particles on the hypersurface σ_{t1} is given by [8]

$$N_1 = -\int_{\sigma_{t1}} d\Sigma_\mu \frac{d^3p}{p^0} p^\mu \phi_{\rm in}(x, p), \tag{1}$$

where $d\Sigma_\mu$ is the external normal vector to σ_{t1} and, hence, the product $p^\mu d\Sigma_\mu \le 0$ is nonpositive. It is clear that without collisions these particles can cross either hypersurface σ_{t2} or $\Sigma^*(t1, t2)$. The corresponding numbers of particles are as follows:

$$N_2 = \int_{\sigma_{t2}} d\Sigma_\mu \frac{d^3p}{p^0} p^\mu \phi_{\rm in}(x, p), \tag{2}$$

$$N^*_{\rm loss} = \int_{\Sigma^*(t1,t2)} d\Sigma_\mu \frac{d^3p}{p^0} p^\mu \phi_{\rm in}(x, p)\Theta(p^\nu d\Sigma_\nu). \tag{3}$$

The Θ function in the *loss* term (3) is very important because it accounts for the particles leaving the "in" domain (see also discussion in [1,3]). For the spacelike parts of the hypersurface $\Sigma^*(t1, t2)$ which are defined by negative sign $ds^2 < 0$ of the squared line element, $ds^2 = dt^*(\bar{x})^2 - d\bar{x}^2$, the product $p^\nu d\Sigma_\nu > 0$ is always positive and, therefore, particles with all possible momenta can leave the "in" domain through the $\Sigma^*(t1, t2)$. For the timelike parts of $\Sigma^*(t1, t2)$ (with sign $ds^2 > 0$), the product $p^\nu d\Sigma_\nu$ can have either sign, and the Θ function *cuts off* those particles which return to the "in" domain.

Similarly, one has to consider the particles coming to the "in" domain from outside. This is possible through the timelike parts of $\Sigma^*(t1, t2)$, if the particle momentum satisfies the inequality $-p^\nu d\Sigma_\nu > 0$. In terms of the external normal $d\Sigma_\mu$ with respect to the "in" domain [this normal vector is shown as an arrow on the arc BD in Fig. 1 and will be used hereafter for all integrals over the hypersurface $\Sigma^*(t1, t2)$], the number of gained particles,

$$N^*_{\rm gain} = -\int_{\Sigma^*(t1,t2)} d\Sigma_\mu \frac{d^3p}{p^0} p^\mu \phi_{\rm out}(x, p)\Theta(-p^\nu d\Sigma_\nu), \tag{4}$$

is, evidently, non-negative. Since the total number of particles is conserved, i.e., $N_2 = N_1 - N^*_{\rm loss} + N^*_{\rm gain}$, one can use the Gauss theorem to rewrite the obtained integral over the closed hypersurface Δx^3 as an integral over the four-volume Δx^4 (area inside the $ABDE$ contour in Fig. 1) surrounded by Δx^3,

$$\int_{\Delta x^4} d^4x \frac{d^3p}{p^0} p^\mu \partial_\mu \phi_{\rm in}(x, p) = \int_{\Sigma^*(t1,t2)} d\Sigma_\mu \frac{d^3p}{p^0} p^\mu \times [\phi_{\rm in}(x, p) - \phi_{\rm out}(x, p)]\Theta(-p^\nu d\Sigma_\nu). \tag{5}$$

In contrast with the usual case [8], i.e., in the absence of a boundary Σ^*, the right-hand side (rhs) of Eq. (5) does not vanish identically. The rhs of Eq. (5) can be transformed further to a four-volume integral as follows. First, we express the integration element $d\Sigma_\mu$ via the normal vector n^*_μ as ($dx^j > 0$, for $j = 1, 2, 3$)

$$d\Sigma_\mu = n^*_\mu dx^1 dx^2 dx^3; \quad n^*_\mu \equiv \delta_{\mu 0} - \frac{\partial t^*(\bar{x})}{\partial x^\mu}(1 - \delta_{\mu 0}), \tag{6}$$

where $\delta_{\mu\nu}$ denotes the Kronecker symbol. Then, using the identity $\int_{t1}^{t2} dt\, \delta(t - t3) = 1$ for the Dirac δ function with $t1 \le t3 \le t2$, we rewrite the rhs integral in (5) as

$$\int_{\Sigma^*(t1,t2)} d\Sigma_\mu \cdots \equiv \int_{V^4_\Sigma} d^4x\, \delta[t - t^*(\bar{x})] n^*_\mu \cdots, \tag{7}$$

where the four-dimensional volume V^4_Σ is a direct product of the three- and one-dimensional volumes $\Sigma^*(t1, t2)$ and $(t2 - t1)$, respectively. Evidently, the Dirac δ function allows us to extend integration in (7) to the unified four-volume $V^4_U = \Delta x^4 \cup V^4_\Sigma$ of Δx^4 and V^4_Σ (the volume V^4_U is shown as the area $ABCE$ in Fig. 1). Finally, with the help of notations

$$\Theta_{\rm out} \equiv \Theta[t - t^*(\bar{x})]; \qquad \Theta_{\rm in} \equiv 1 - \Theta_{\rm out}, \tag{8}$$

it is possible to extend the lhs integral in Eq. (5) from Δx^4 to V^4_U. Collecting the above results, one gets

$$\int_{V^4_U} d^4x \frac{d^3p}{p^0} \Theta_{\rm in} p^\mu \partial_\mu \phi_{\rm in} = \int_{V^4_U} d^4x \frac{d^3p}{p^0} p^\mu n^*_\mu \times [\phi_{\rm in} - \phi_{\rm out}]\Theta(-p^\nu n^*_\nu)\delta[t - t^*(\bar{x})]. \tag{9}$$

Since the volumes Δx^4 and V^4_U are arbitrary, one gets the collisionless kinetic equation for $\phi_{\rm in}(x, p)$

$$\Theta_{\rm in} p^\mu \partial_\mu \phi_{\rm in}(x,p) = p^\mu n^*_\mu [\phi_{\rm in}(x,p) - \phi_{\rm out}(x,p)] \times \Theta(-p^\nu n^*_\nu)\delta[t - t^*(\bar{x})]. \quad (10)$$

In the general case on the rhs of Eq. (10) there can be an arbitrary function $g(x, p)$ which identically vanishes while being integrated over the invariant momentum measure d^3p/p_0. Such a property is typical for a collision integral [8], but we do not consider it for the collisionless case.

Similarly, one can obtain the equation for the distribution function of the "out" domain

$$\Theta_{\rm out} p^\mu \partial_\mu \phi_{\rm out}(x,p) = p^\mu n^*_\mu [\phi_{\rm in}(x,p) - \phi_{\rm out}(x,p)] \times \Theta(p^\nu n^*_\nu)\delta[t - t^*(\bar{x})], \quad (11)$$

where the normal vector n^*_ν is given by (6). Note the asymmetry between the rhs of Eqs. (10) and (11): For the spacelike parts of hypersurface Σ^*, the source term with $\Theta(-p^\nu n^*_\nu)$ vanishes identically because $p^\nu n^*_\nu > 0$. This reflects the causal properties of the equations above: Propagation of particles faster than light is forbidden, and, hence, no particle can (re)enter the "in" domain.

III. Collision term for semi-infinite domain.—Since in the general case $\phi_{\rm in}(x, p) \neq \phi_{\rm out}(x, p)$ on Σ^*, the rhs of Eqs. (10) and (11) cannot vanish simultaneously on this hypersurface. Therefore, the functions $\Theta^*_{\rm in} \equiv \Theta_{\rm in}|_{\Sigma^*} \neq 0$ and $\Theta^*_{\rm out} \equiv \Theta_{\rm out}|_{\Sigma^*} \neq 0$ do not vanish simultaneously on Σ^* as well. Since there is no preference between "in" and "out" domains, it is assumed that $\Theta^*_{\rm in} = \Theta^*_{\rm out} = \Theta(0) = \frac{1}{2}$, but the final results are independent of this choice.

Now the collision terms for Eqs. (10) and (11) can be readily obtained. Adopting the usual assumptions for the distribution functions [8], one can repeat the standard derivation of the collision terms [8] and get the desired result. We shall not do this, but discuss only how to modify the standard derivation for our purpose. One has to start the derivation in the Δx^4 volume of the "in" domain and then extend it to the unified four-volume $V^4_U = \Delta x^4 \cup V^4_\Sigma$ similarly to the preceding section. Then the first part of the collision term for Eq. (10) is

$$C^I_{\rm in}(x,p) = \Theta^2_{\rm in}(I_G[\phi_{\rm in}, \phi_{\rm in}] - I_L[\phi_{\rm in}, \phi_{\rm in}]), \quad (12)$$

$$I_G[\phi_A, \phi_B] \equiv \frac{1}{2}\int D^9P \phi_A(p')\phi_B(p'_1) W_{p\,p_1|p'p'_1}, \quad (13)$$

$$I_L[\phi_A, \phi_B] \equiv \frac{1}{2}\int D^9P \phi_A(p)\phi_B(p_1) W_{p\,p_1|p'p'_1}, \quad (14)$$

where the invariant measure of integration is $D^9P \equiv [(d^3p_1)/p^0_1][(d^3p')/p'^0][(d^3p'_1)/p'^0_1]$ and $W_{p\,p_1|p'p'_1}$ is the transition rate in the elementary reaction with energy-momentum conservation given in the form $p^\mu + p^\mu_1 = p'^\mu + p'^\mu_1$. The rhs of (12) contains the square of the $\Theta_{\rm in}$ function because the additional $\Theta_{\rm in}$ accounts for the fact that on the boundary hypersurface Σ^* one has to take only one-half of the traditional collision term [due to our choice of $\Theta^*_{\rm in}$ only one-half of Σ^* belongs to the "in" domain]. It is easy to understand that on Σ^* the second part of the collision term is defined by the collisions between particles of "in" and "out" domains:

$$C^{II}_{\rm in}(x,p) = \Theta_{\rm in}\Theta_{\rm out}(I_G[\phi_{\rm in}, \phi_{\rm out}] - I_L[\phi_{\rm in}, \phi_{\rm out}]). \quad (15)$$

Combining (10), (12), and (15), one gets the kinetic equation for the "in" domain ($A \in$ in; $S_{\rm in} = -1$):

$$\Theta_A p^\mu \partial_\mu \phi_A(x,p) = C^I_A(x,p) + C^{II}_A(x,p) + p^\mu n^*_\mu \times [\phi_{\rm in}(x,p) - \phi_{\rm out}(x,p)]\Theta(S_A p^\nu n^*_\nu)\delta[t - t^*(\bar{x})]. \quad (16)$$

The kinetic equation for the "out" domain can be derived similarly and then it can be represented in the form of Eq. (16) with the obvious notation: $A \in$ out, $S_{\rm out} = 1$, $C^I_{\rm out} \equiv \Theta^2_{\rm out}(I_G[\phi_{\rm out}, \phi_{\rm out}] - I_L[\phi_{\rm out}, \phi_{\rm out}])$, and $C^{II}_{\rm out} \equiv \Theta_{\rm in}\Theta_{\rm out}(I_G[\phi_{\rm out}, \phi_{\rm in}] - I_L[\phi_{\rm out}, \phi_{\rm in}])$.

For the continuous distribution functions on Σ^*, i.e., $\phi_{\rm out}|_{\Sigma^*} = \phi_{\rm in}|_{\Sigma^*}$, the source terms in the rhs of system (16) vanish and one recovers the Boltzmann equations. Moreover, with the help of the evident relations

$$-\partial_\mu \Theta_{\rm in} = \partial_\mu \Theta_{\rm out} = n^*_\mu \delta[t - t^*(\bar{x})], \quad (17)$$

$$C^I_{\rm in} + C^{II}_{\rm in} + C^I_{\rm out} + C^{II}_{\rm out} = I_G[\Phi, \Phi] - I_L[\Phi, \Phi], \quad (18)$$

where $\Phi(x,p) \equiv \Theta_{\rm in}\phi_{\rm in}(x,p) + \Theta_{\rm out}\phi_{\rm out}(x,p)$, one can get the following result summing up Eq. (16) for "in" and "out" domains:

$$p^\mu \partial_\mu \Phi(x,p) = I_G[\Phi, \Phi] - I_L[\Phi, \Phi]. \quad (19)$$

In other words, the usual Boltzmann equation follows from the system (16) automatically without *any assumption* about the behavior of $\phi_{\rm in}$ and $\phi_{\rm out}$ on the boundary hypersurface Σ^*. Also, Eq. (19) is valid not only for our choice Θ^*_A, but for *any choice* $0 < \Theta^*_A < 1$ obeying Eq. (8). In fact, the system (16) generalizes the relativistic kinetic equation to the case of strong temporal and spatial inhomogeneity, i.e., for $\phi_{\rm in}(x,p) \neq \phi_{\rm out}(x,p)$ on Σ^*. In what follows, we shall discuss exclusively the spatial inhomogeneities or discontinuities on the timelike parts of Σ^*, since the *timelike shocks* [9] (strong temporal inhomogeneity) might contradict the usual assumptions adopted for distribution functions.

From the system (16), it is possible to derive the macroscopic equations of motion by multiplying the corresponding equation with p^ν and integrating it over the invariant measure. Thus, Eq. (16) for $A \in$ in generates the following expression $[T^{\mu\nu}_A \equiv \int[(d^3p)/p^0]p^\mu p^\nu \phi_A(x,p)]$:

$$\Theta_A \partial_\mu T^{\mu\nu}_A = \int \frac{d^3p}{p^0} p^\nu C^{II}_A(x,p) + \int \frac{d^3p}{p^0} p^\nu p^\mu n^*_\mu \times [\phi_{\rm in} - \phi_{\rm out}]\Theta(S_A p^\rho n^*_\rho)\delta[t - t^*(\bar{x})]. \quad (20)$$

Similarly to the usual Boltzmann equation, the momentum integral of the collision term $C^I_{\rm in}$ vanishes due to its symmetries [8], but it can be shown that the integral of the second collision term $C_{\rm in}^{II}$ does not vanish because it involves two different (and not identical) distribution functions. The corresponding equation for the "out" domain [$A \in$ out in (20)] follows similarly from Eq. (16).

IV. Discussion.—It is clear that Eqs. (16) and (20) remain valid both for finite domains and for a multiple valued function $t = t^*(\bar{x})$ as well. To derive the systems (16) and (20) in the latter case, one has to divide the function $t^*(\bar{x})$ into the single valued parts, but this discussion is beyond the scope of this paper. Using Eqs. (16) and (20), we are ready to analyze the "hydro + cascade" models. In the TLS model, the *cutoff* formula relates $\phi_{\rm in}$ [≡ hydro ≡ "in" Eq. (20)] and $\phi_{\rm out}$ [≡ cascade ≡ "out" Eq. (16)] on Σ^* as follows:

$$\phi_{\rm out}|_{\Sigma^*} = \Theta(p^\rho n^*_\rho)\phi_{\rm out}|_{\Sigma^*} = \Theta(p^\rho n^*_\rho)\phi_{\rm in}|_{\Sigma^*}, \qquad (21)$$

i.e., for the spacelike parts of hypersurface Σ^* these functions are identical, whereas for the timelike parts of Σ^* there are no particles returning to the "in" domain. In this case the source term in the cascade Eq. (16) is zero, while the source term in the hydro Eq. (20) does not vanish on the timelike parts of the boundary Σ^*. Therefore, the main defect of the TLS model is not even the energy-momentum nonconservation, but the incorrect hydrodynamic equations. The absence of the δ-like source term in [7] breaks the conservation laws [evidently, the system (20) obeys the conservation laws], but its inclusion into consideration will inevitably change the hydrodynamic solution of Ref. [7]. The full analysis of the possible solutions of the systems (16) and (20) requires a special investigation. From the negative sign of the TLS source term in the rhs of the hydro Eq. (20) for equal indices $\nu = \mu$, one can deduce that such a correction to the hydro equations should increase the degree of the fluid rarefaction in comparison to the standard hydrodynamic expansion. It is, therefore, quite possible that such a source term will generate a discontinuity between "in" and "out" domains. In the thermodynamically normal media [10], the rarefaction shocks are mechanically unstable. However, it is well known that on the phase transition boundary between QGP and HG the properties of the mixed phase are thermodynamically anomalous [10] and the usual rarefaction shocks are possible. Another possibility is the occurrence of a new type of discontinuity, the *freeze-out shock* suggested in Refs. [1,3], where the post-freeze-out state is described by the *cutoff* distribution and, hence, is very similar to the TLS ansatz. It is clear that in both cases the additional rarefaction will reduce the mean transverse size and the lifetime of the hadronizing QGP.

Let us consider briefly the BD approach. Since in the BD model the hydro and cascade distributions on Σ^* are equal $\phi_{\rm out}|_{\Sigma^*} = \phi_{\rm in}|_{\Sigma^*}$, the corresponding source terms vanish in all equations. Therefore, at first glance the BD approach correctly conjugates the hydro and cascade solutions on the arbitrary hypersurface. For the oversimplified kinetics considered above, this is the case. However, the real situation differs essentially from our consideration. Thus, the hydro part in both the BD and TLS models is assumed to be in the local thermodynamic equilibrium, whereas the matter in the cascade domain can be far from equilibrium [6,7]. Consequently, the local thermal equilibrium inside the "in" domain is the necessary condition of the equivalence of the kinetic Eq. (16) and hydro Eq. (20), i.e., the kinetic Eq. (16) should be homogeneous inside the "in" domain. However, according to the analysis of Refs. [6,7], kaons, ϕ mesons, and Ω hyperons are not in full equilibrium just after the hadronization. For other hadrons the equilibrium dismantles during the cascade stage, i.e., in the "out" domain. Therefore, if for each hadron the inhomogeneous BD cascade solution, that originated at the timelike parts of Σ^* (the arc OBF in Fig. 1), did propagate inside of the "in" domain, then it would not match, in general, the distribution function $\phi_{\rm in}$ found previously from the hydro equations (or homogeneous transport ones) on the causally connected parts of Σ^* (arc FP). Thus, one arrives at a causal paradox, the *recoil problem* [1], and at a mathematical inconsistency. Note that, even if this inconsistency may be small on the spacelike parts of Σ^* (arc FP), it may become sizable at later times for such long interacting hadrons as pions, protons, and lambdas due to the nonlinear nature of the transport equations.

Therefore, the only way out of the discussed problems of the BD and TLS approaches is to find the boundary conditions for $\phi_{\rm in}$ and $\phi_{\rm out}$ on the separating hypersurface Σ^* from the system of kinetic equations (16). Then these boundary conditions should be applied to the system of the hydro Eq. (20) and cascade Eq. (16), which ensures the correct treatment of the relativistic nuclear collisions within the frame of the "hydro + cascade" model.

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