# Observation of a Quantized Hall Resistivity in the Presence of Mesoscopic Fluctuations

E. Peled and D. Shahar
*Department of Condensed Matter Physics, Weizmann Institute, Rehovot 76100, Israel*

Y. Chen
*Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA*

D. L. Sivco and A. Y. Cho
*Bell Laboratories, Lucent Technologies, 600 Mountain Avenue, Murray Hill, New Jersey 07974, USA*
(Received 17 May 2002; published 18 June 2003)

We present an experimental study of mesoscopic, two-dimensional electronic systems at high magnetic fields. Our samples, prepared from a low-mobility InGaAs/InAlAs wafer, exhibit reproducible, sample specific, resistance fluctuations. Focusing on the lowest Landau level, we find that, while the diagonal resistivity displays strong fluctuations, the Hall resistivity is free of fluctuations and remains quantized at its $\nu = 1$ value, $h/e^2$. This is true also in the insulating phase that terminates the quantum Hall series. These results extend the validity of the semicircle law of conductivity in the quantum Hall effect to the mesoscopic regime.

DOI: 10.1103/PhysRevLett.90.246802 PACS numbers: 73.43.–f, 71.30.+h, 72.80.Sk, 73.23.–b

At high magnetic fields ($B$s), two-dimensional disordered systems evolve through a series of integer quantum Hall (QH) states. These states are characterized by the quantization of the Hall resistivity ($\rho_{xy}$) in values of $h/ie^2$ ($h$ is Planck's constant, $e$ is the charge of the electron, and $i$ is an integer) and the concomitant vanishing of the longitudinal resistivity ($\rho_{xx}$). These zero-resistance states support current flow without dissipation. In contrast, in the transition regions separating adjacent QH states, where $\rho_{xy}$ is not quantized, $\rho_{xx}$ exhibits a sharp peak and dissipation is present. This link, between the absence of dissipation and the high degree of quantization of $\rho_{xy}$, is at the heart of the theoretical works that established our understanding of the QH effect [1,2].

In this Letter, we report on a study of disordered, mesoscopic, samples in the limit where only a single Landau level (LL) is occupied (filling factor $\nu < 1$). Because of the low mobility of our samples, no fractional QH states are present and the $\nu = 1$ state is followed by an insulating phase [3–6]. As expected, our samples exhibit strong mesoscopic fluctuations in $\rho_{xx}$. Surprisingly, $\rho_{xy}$ maintains its quantization and is essentially free of fluctuations. This renders possible a new state, where $\rho_{xy}$ is quantized even in the presence of dissipation ($\rho_{xx} \neq 0$). The fluctuation-free $\rho_{xy}$ also establishes the existence of strong correlations between the fluctuations of the conductivity-tensor components, $\sigma_{xx}$ and $\sigma_{xy}$, conveniently expressed in terms of a semicircle relation [7,8].

Previous studies of mesoscopic samples, where the QH effect is disturbed by finite-size effects [9–20], reveal a different picture: The appearance of fluctuations in $\rho_{xx}$ is always accompanied by the disruption of the quantization of $\rho_{xy}$, whereas deep in the QH states where $\rho_{xx}$ vanishes $\rho_{xy}$ remains quantized [9,10,14]. These earlier studies were conducted at intermediate $B$ values where two, or more, LLs are occupied. The apparent discrepancy between our results and earlier studies will be discussed below.

Our data were obtained from two samples, T2Ga and T2C, wet etched from the same InGaAs/InAlAs wafer that contained, after illumination with an LED, a two-dimensional electronic system in a 200 Å quantum well. We defined a Hall-bar geometry with lithographic widths of $W = 10\ \mu$m for sample T2Ga and $W = 2\ \mu$m for sample T2C, and voltage-probe separation of $L = 2 \times W$, maintaining an identical aspect ratio. The samples were cooled in a dilution refrigerator with a base temperature ($T$) of 12 mK. The mobility and density of sample T2Ga were $\mu = 22\,000\ \mathrm{cm^2/V\,s}$ and $n_s = 1.5 \times 10^{11}\ \mathrm{cm^{-2}}$. Sample T2C was cooled down twice, with $\mu = 23\,850\ \mathrm{cm^2/V\,s}$, $n_s = 1.83 \times 10^{11}\ \mathrm{cm^{-2}}$ and $\mu = 13\,700\ \mathrm{cm^2/V\,s}$, $n_s = 1 \times 10^{11}\ \mathrm{cm^{-2}}$ at the first (T2Cc) and second (T2Ci) cooldowns, respectively. Four-probe measurements were done using standard AC lock-in techniques with excitation currents of 0.1–1 nA and frequencies of 1.7–11 Hz. Temperatures below 30 mK are nominal, since the resistivity does not change in the $T$ range 12–30 mK.

In Fig. 1, we plot $\rho_{xx}$ and $\rho_{xy}$ for sample T2Cc, taken over a broad range of $B$. Although the sample is relatively small ($W = 2\ \mu$m), integer QH plateaus in $\rho_{xy}$ (designated by their filling factor) and the associated minima in $\rho_{xx}$ are clearly observed. As mentioned earlier, fractional QH features are not seen down to the lowest $T$ due to the low mobility of our samples. In addition to the QH features, $\rho_{xx}$ and $\rho_{xy}$ exhibit reproducible fluctuations, which maintain their pattern as long as the sample is kept cold,

  

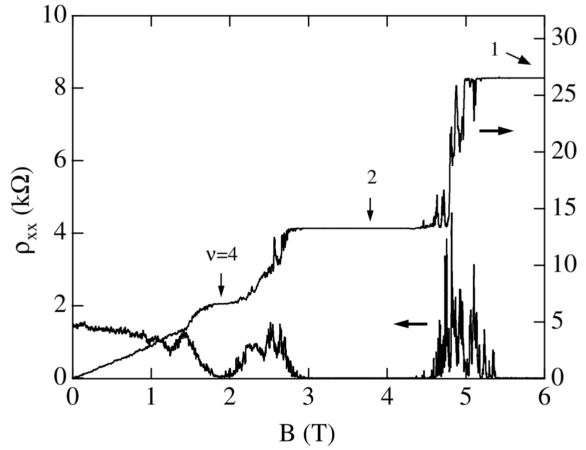


FIG. 1. $\rho_{xx}$ and $\rho_{xy}$ vs $B$ for sample T2Cc, $T = 12$ mK. QH plateaus are designated by their filling factor.

attesting to the mesoscopic nature of our samples. The fluctuations decrease in magnitude when $T$ is increased. At the low-$B$ range, we associate the fluctuations with the theory of universal conductance fluctuations (UCF) [21]. According to this theory, the coherence length ($L_\phi$) of the electrons can be deduced from the amplitude and correlation-$B$ of the fluctuations. $L_\phi$ in our samples is found to be between 1.2 and 3 $\mu$m at base $T$, varying from sample to sample and between cooldowns. The larger sample, T2Ga, also exhibits reproducible fluctuations, but of reduced amplitude, due to averaging of subunits of size $L_\phi$ over the sample area. At $T = 12$ mK and near $B = 0$, the rms of the fluctuations in samples T2Cc and T2Ga is 35 and 9.6 Ω, respectively, corresponding to conductivity fluctuations $\delta\sigma = 0.54$ and $0.067\ e^2/h$.

In the remainder of this Letter, we will focus on data taken at $\nu < 1$, where only the lowest LL contributes to the transport. In Fig. 2, we examine the resistivity obtained from sample T2Ga, in the $B$ range of 8–12 T, corresponding to $\nu = 0.775$–$0.517$. We begin by focusing on $\rho_{xx}$ traces taken at $T = 12$, 500, 700, and 900 mK in Fig. 2(a). Two different transport regimes can be distinguished according to the $T$ dependence of $\rho_{xx}$: the QH liquid regime (low-$B$ side), where $\rho_{xx}$ increases with increasing $T$, and the insulating regime (high-$B$ side), where $\rho_{xx}$ decreases with $T$. The transition $B$, $B_c = 10.62$ T ($\nu_c = 0.58$), where $\rho_{xx}$ is $T$ independent, is indicated by the dotted vertical line. The value of $\rho_{xx}$ at the transition, $\rho_{xxc} = 0.77\,h/e^2$, is in agreement with previous experimental results where $\rho_{xxc} \sim h/e^2$ [3,22–24]. The $\rho_{xx}$ fluctuations here are much larger than at low $B$, and have an amplitude as high as 2840 Ω on the insulating side.

Before we proceed with the main result of our work, which stems from the analysis of the $\rho_{xy}$ data, a word of caution is in order. An experimental determination of $\rho_{xy}$ in the lowest LL is difficult, because an admixture of $\rho_{xx}$ into the data is unavoidable. This is especially important

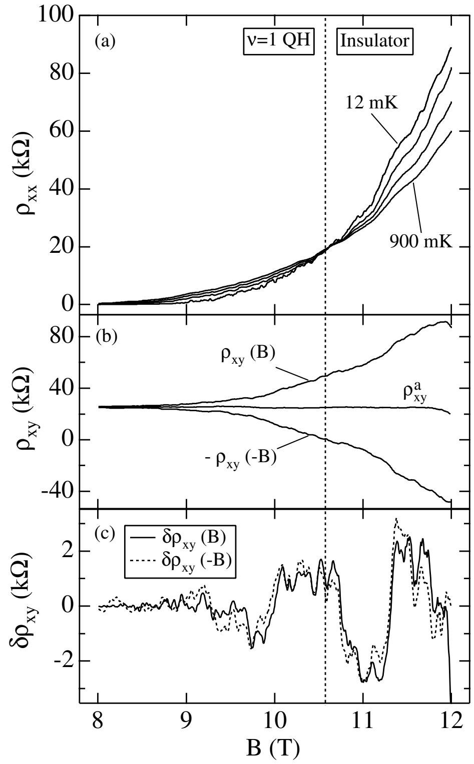


FIG. 2. The resistivity of sample T2Ga at $\nu < 1$. (a) $\rho_{xx}$ traces. Temperatures are 12, 500, 700, and 900 mK. The crossing point of the traces is indicated by the dotted vertical line. (b) $\rho_{xy}(B)$, $-\rho_{xy}(-B)$, and the antisymmetric part, $\rho^a_{xy}$, at $T = 16$ mK. (c) Comparison of the $\pm B$ fluctuations of $\rho_{xy}$.

in the insulating regime where $\rho_{xx}$ is large, and even a small fraction of $\rho_{xx}$ admixture can result in a significant change in the measured $\rho_{xy}$. The traditional way to overcome this problem is by using the $B$ symmetry of the resistivity components: $\rho_{xx}$ is expected to be symmetric in $B$, while $\rho_{xy}$ is antisymmetric. The true $\rho_{xy}$ is obtained, in principle, by an antisymmetrization of the measured $\rho_{xy}(B)$ and $\rho_{xy}(-B)$: $\rho^a_{xy} = \frac{1}{2}[\rho_{xy}(B) - \rho_{xy}(-B)]$. In practice, because the admixture is a result of both contact misalignment and current nonuniformities in the sample, $\rho_{xx}$ itself is not entirely $B$ symmetric, limiting the accuracy with which $\rho_{xy}$ can be determined.

We now turn to the analysis of the $\rho_{xy}$ data at $\nu < 1$. In Fig. 2(b), we plot $\rho_{xy}(B)$, $-\rho_{xy}(-B)$, and the antisymmetrized $\rho^a_{xy}$ taken at $T = 16$ mK. As in previous literature [3–6], we find that $\rho_{xy}(B)$ and $\rho_{xy}(-B)$ show a strong, symmetric, $B$ dependence that is cancelled out of $\rho^a_{xy}$, resulting in a Hall coefficient that remains nearly constant and quantized at its $\nu = 1$ value, $h/e^2$. This holds into the insulating phase, which has been termed the quantized Hall insulator [5]. The novelty in our data is that, while $\rho_{xy}(\pm B)$ include fluctuations, these are symmetric in $B$ and therefore do not appear in $\rho^a_{xy}$. To demonstrate this symmetry more clearly, we compare in Fig. 2(c) the positive and negative $B$ fluctuation patterns, $\delta\rho_{xy}(\pm B)$, after subtracting a smooth background from the original traces. $\delta\rho_{xy}(\pm B)$ are very similar in shape and magnitude, and overlap to within 92% [25]. The maximum amplitude of the $\rho_{xy}(\pm B)$ fluctuations is 2825 $\Omega$, while that of $\rho^a_{xy}$ is 735 $\Omega$. We attribute these remaining $\rho^a_{xy}$ fluctuations to an asymmetric component of $\rho_{xx}$ in sample T2Ga. Measurements of $\rho_{xx}(\pm B)$ reveal an asymmetric component whose magnitude is consistent with the fluctuations of $\rho^a_{xy}$. We stress that, even if sample inhomogeneities were not present, obtaining perfectly reproducible resistivity measurements at opposite $B$ polarities is not practical due to the stringent requirements this will place on $T$ stability when the $B$ field is swept over nearly 20 T. This problem becomes even more severe in light of the strong $T$ dependence of $\rho_{xx}$ in the insulating phase.

In order to enhance the fluctuating part of the data, we repeated our $\nu < 1$ measurements with the smaller ($W = 2\ \mu$m) sample, T2Ci, which was fabricated with utmost care to reduce contact misalignments to the minimum. The results are shown in Fig. 3, where we plot $\rho_{xx}$ and $\rho^a_{xy}$ vs $B$ taken at $T = 100$, 295, 500, and 698 mK. We start by examining the $\rho_{xx}$ data, which now appear to be dominated by fluctuations that are as large as 188 000 $\Omega$ on the insulating side. Nevertheless, by fitting a smooth curve through the data [26], the average $\rho_{xx}$ can be determined and the QH and insulating regimes properly identified (see inset). The fitted curves cross at $B_c = 7.34$ T ($\nu_c = 0.572$) and $\rho_{xxc} = 0.67\, h/e^2$.

In stark contrast with the $\rho_{xx}$ data, $\rho^a_{xy}$ appears to be free of fluctuations. An upper-bound estimate of the magnitude of the $\rho^a_{xy}$ fluctuations is 160–220 times less than the amplitude of $\delta\rho_{xx}$ at similar background values. Similar results are obtained when switching to other voltage probes. Comparing these data to the data obtained from the 10 $\mu$m sample, we see that reducing the sample size by a factor of 5 resulted in an increase of $\delta\rho_{xx}$ by nearly 70 while $\delta\rho_{xy}$ increased by only a factor of 2.

We now turn to a discussion of the consequences of the quantization of $\rho_{xy}$ in the presence of large reproducible fluctuations in $\rho_{xx}$ for $\nu < 1$, in the context of models pertaining to fluctuations in the QH regime. Several known mechanisms can lead to the resistance fluctuations in our samples. Fluctuations as a result of the modification of the interference between many possible electron paths across the sample are at the heart of the theory of UCF [21]. Although this theory is not expected to be valid at high $B$, modified UCF theories have been suggested [27,28] to take into account the influence of $B$ on the electron trajectories, and several experiments were interpreted in terms of UCF at high $B$ [10,11].

Two recent experimental studies suggest a different point of view: Cobden *et al.* [12], and Machida *et al.* [13] have found that in the QH regime fluctuation patterns follow straight lines in the magnetic field–carrier-density plane, parallel to integer $\nu$ lines. These results were attributed to charging of electron puddles in the sample [12], or to changes in a compressible-strip network configuration [13].

Another possible source of fluctuations, expected to dominate when $B$ is high enough such that only a small number of LL are occupied, is resonant tunneling [29]. In this process, fluctuations arise when an electron scatters from one edge of the sample to the other through a bulk impurity. This model was found to be consistent with observed high $B$ fluctuations [14], and has been used in measurements of fractional charge [15,16]. Applying this model to the case when only the lowest LL is occupied, Jain and Kivelson [29] argued that the fluctuations will be limited to $\rho_{xx}$, leaving $\rho_{xy}$ quantized. Although this conjecture was contended by Bütikker [30], our measurements of a fluctuation-free, quantized, $\rho_{xy}$ are consistent with this model.

There are two additional models for conduction in the QH regime that predict a quantized $\rho_{xy}$, although they do not directly address the properties of the mesoscopic fluctuations of the resistivity. In the first model, Ruzin and his collaborators [7,8] treat the electron system in the QH transition region as a random mixture of two phases, and find a semicircle relation for the conductivity components. For the QH-insulator transition, this relation is $\sigma^2_{xx} + [\sigma_{xy} - (e^2/2h)]^2 = [e^2/(2h)]^2$, which is mathematically equivalent to the quantization of $\rho_{xy}$ [5]. Since we are unable to measure the conductivity components directly, we obtain them by inverting the resistivity tensor, $\rho_{xx}$ and $\rho^a_{xy}$. In Fig. 4, we plot the resulting $\sigma_{xx}$ and $\sigma_{xy}$ of sample T2Ci at the QH-insulator transition. In contrast with the resistivity tensor, both $\sigma_{xx}$ and $\sigma_{xy}$ display fluctuations, stemming from their mutual dependence on $\rho_{xx}$ as well as on $\rho^a_{xy}$. Nevertheless, $\sigma_{xx}$ and $\sigma_{xy}$ obey the semicircle relation, as shown in the inset of Fig. 4, indicating the special correlation that exists between the fluctuations of the conductivity-tensor components. Our work extends the validity of the semicircle relation to the mesoscopic regime of transport.

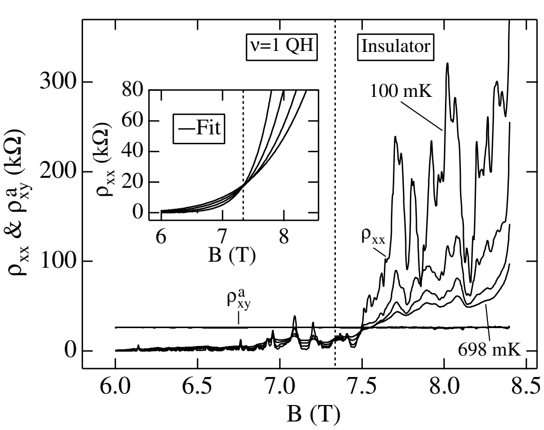


FIG. 3. $\rho_{xx}$ and $\rho^a_{xy}$ of sample T2Ci at high $B$. $T = 100$, 295, 500, and 698 mK. Inset: We identify $B_c$ as the crossing point of the smooth $\rho_{xx}$-fit curves [26].

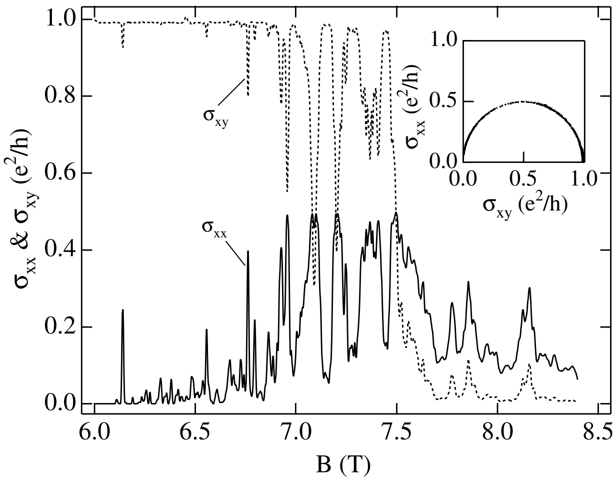


FIG. 4. $\sigma_{xx}$ and $\sigma_{xy}$ of sample T2Ci near the QH-insulator transition, calculated by inverting the resistivity tensor. $T = 100$ mK. Inset: The semicircle relation, $\sigma_{xx}$ vs $\sigma_{xy}$.

The second model pertaining to the quantization of $\rho_{xy}$ near the transition, and in the insulating regime, has been proposed by Shimshoni and Auerbach [31]. They showed that transport in a random Chalker-Coddington-based network of puddles produces a quantized $\rho_{xy}$ when $L_\phi$ is smaller than the puddle size. However, $\rho_{xy}$ is expected to diverge when $L_\phi$ is larger than the puddle size [32,33]. It would be interesting to see how this model can be extended to accommodate samples that exhibit large resistance fluctuations.

Finally, we wish to emphasize that the absence of fluctuations in $\rho_{xy}$ near the QH-insulator transition is not inconsistent with the results of previous experiments (and the present work, see Fig. 1), which show large $\rho_{xy}$ fluctuations in higher LLs. We recall the mapping [34] that exists between transitions at higher LLs and the "basic," QH-insulator, transition at the lowest LL: In this mapping, one regards the higher transitions as a QH-insulator transition occurring in the presence of a number of full and inert LLs. A simple calculation shows that the $\rho_{xy}$ of higher LL transitions will include components proportional to $\rho_{xx}$ of a QH-insulator transition [35]. We leave for future work a more direct verification of this mapping to the fluctuating part of the resistivity.

In conclusion, we have shown that the quantization of the Hall effect in two-dimensional electron systems can be maintained in the mesoscopic regime, even when the diagonal resistivity, $\rho_{xx}$, is nonzero and exhibits large fluctuations. These results are in agreement with the predictions of Jain and Kivelson [29] and with the semicircle relation for the conductivity components [7,8].

We wish to thank A. Auerbach, Y. Oreg, E. Shimshoni, A. Stern, and D. C. Tsui for useful discussions. This work is supported by the BSF and by the Koshland Fund. Y. C. is supported by the (U.S.) NSF.

[1] R. B. Laughlin, Phys. Rev. B **23**, 5632 (1981).
[2] B. I. Halperin, Phys. Rev. B **25**, 2185 (1982).
[3] R. B. Dunford *et al.*, Physica (Amsterdam) **6E**, 297 (2000).
[4] M. Hilke *et al.*, Europhys. Lett. **46**, 775 (1999).
[5] M. Hilke *et al.*, Nature (London) **395**, 675 (1998).
[6] D. T. N. de Lang *et al.*, Physica (Amsterdam) **12E**, 666 (2002).
[7] A. M. Dykhne and I. M. Ruzin, Phys. Rev. B **50**, 2369 (1994).
[8] I. Ruzin and S. Feng, Phys. Rev. Lett. **74**, 154 (1995).
[9] A. M. Chang *et al.*, Solid State Commun. **67**, 769 (1988).
[10] G. Timp *et al.*, Phys. Rev. Lett. **59**, 732 (1987).
[11] F. Hohls, U. Zeitler, and R. J. Haug, Phys. Rev. B **66**, 073304 (2002).
[12] D. H. Cobden, C. H. W. Barnes, and C. J. B. Ford, Phys. Rev. Lett. **82**, 4695 (1999).
[13] T. Machida *et al.*, Phys. Rev. B **63**, 45318 (2001).
[14] P. C. Main *et al.*, Phys. Rev. B **50**, 4450 (1994).
[15] J. A. Simmons *et al.*, Phys. Rev. Lett. **63**, 1731 (1989).
[16] V. J. Goldman and B. Su, Science **267**, 1010 (1995).
[17] D. H. Cobden and E. Kogan, Phys. Rev. B **54**, R17 316 (1996).
[18] S. Cho and M. P. A. Fisher, Phys. Rev. B **55**, 1637 (1997).
[19] T. Ando, Phys. Rev. B **49**, 4679 (1994).
[20] Z. Wang, B. Jovanović, and D. H. Lee, Phys. Rev. Lett. **77**, 4426 (1996).
[21] P. A. Lee, A. D. Stone, and H. Fukuyama, Phys. Rev. B **35**, 1039 (1987).
[22] R. J. F. Hughes *et al.*, J. Phys. Condens. Matter **6**, 4763 (1994).
[23] D. Shahar, D. C. Tsui, and J. E. Cunningham, Phys. Rev. B **52**, R14 372 (1995).
[24] D. Shahar *et al.*, Phys. Rev. Lett. **74**, 4511 (1995).
[25] We define the overlap as overlap $= 100 \times \mathrm{cor}[\delta\rho_{xy}(B), \delta\rho_{xy}(-B)]/\sqrt{\mathrm{cor}[\delta\rho_{xy}(B), \delta\rho_{xy}(B)]}/\sqrt{\mathrm{cor}[\delta\rho_{xy}(-B), \delta\rho_{xy}(-B)]}$, where cor stands for the value of the correlation function at $\Delta B = 0$.
[26] We fit $\rho_{xx}$ to $A \cdot \exp\{-[(\nu - \nu_c)/\nu_0]\}$. See D. Shahar *et al.*, Solid State Commun. **107**, 19 (1998).
[27] S. Xiong and A. D. Stone, Phys. Rev. Lett. **68**, 3757 (1992).
[28] D. L. Maslov and D. Loss, Phys. Rev. Lett. **71**, 4222 (1993).
[29] J. K. Jain and S. A. Kivelson, Phys. Rev. Lett. **60**, 1542 (1988); see also P. Streda *et al.*, Phys. Rev. Lett. **59**, 1973 (1987).
[30] M. Büttiker, Phys. Rev. Lett. **62**, 229 (1989).
[31] E. Shimshoni and A. Auerbach, Phys. Rev. B **55**, 9817 (1997).
[32] L. P. Pryadko and A. Auerbach, Phys. Rev. Lett. **82**, 1253 (1999).
[33] U. Zülicke and E. Shimshoni, Phys. Rev. B **63**, 241301 (2001).
[34] S. Kivelson, D. H. Lee, and S. C. Zhang, Phys. Rev. B **46**, 2223 (1992).
[35] D. Shahar *et al.*, Phys. Rev. Lett. **79**, 479 (1997).