Damping of the Trapped-Particle Diocotron Mode

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The damping mechanism of a recently discovered trapped-particle mode is identified as collisional velocity scattering of marginally trapped particles. The mode exists on non-neutral plasma columns that are partially divided by an electrostatic potential. This damping mechanism is similar to that responsible for damping of the dissipative trapped-ion mode. The damping rate is calculated using a Fokker-Planck analysis and agrees with measurement to within 50%. Also, an experimental signature confirms a causal relation between scattering of marginally trapped particles and damping.

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Electric and magnetic field inhomogeneities in plasma containment devices cause a fraction of the particles to remain localized in certain regions. This condition gives rise to a class of low frequency electrostatic oscillations known as trapped-particle modes [1]. In these modes, trapped particles remain isolated from the global mode structure and experience $\mathbf{E} \times \mathbf{B}$ drift oscillations, while passing particles stream along the field lines Debye shielding the trapped-particle charge density perturbations. In this sense, trapped-particle modes resemble drift waves wherein the trapped particles play the role of ions and passing particles the role of electrons. Trapped-particle modes were originally investigated for toroidal geometry, but have been predicted for and observed in other geometries, such as the Columbia Linear Machine [2].

Trapped-particle modes can be stabilized or destabilized by collisions [3]. In the case of the dissipative trapped-ion mode, collisional scattering of the electrons is destabilizing and collisional scattering of the ions is stabilizing. Rosenbluth, Ross, and Kostomarov pointed out that small angle velocity scattering near the phase-space separatrix contributes dominantly to the damping, and that a careful boundary layer analysis is required [4]. Their boundary layer analysis showed that the relative damping rate due to ion collisions scales approximately as $\sqrt{\nu_i/\omega}$ rather than as ν_i/ω. The square root provides a significant enhancement since ν_i/ω is small.

In this Letter, we identify the damping of the recently discovered trapped-particle diocotron mode [5] as being due to velocity scattering in the separatrix boundary layer, and derive a damping rate that also scales as $\sqrt{\nu/\omega}$. This mode exists on single-species plasma columns in which classes of trapped and passing particles have been created by an electrostatic potential barrier. Particles trapped on one side of the barrier experience $\mathbf{E} \times \mathbf{B}$ drift oscillations that are 180° out of phase with those experienced by particles trapped on the other side. Simultaneously, passing particles stream axially Debye shielding the charge perturbations of the trapped particles.

Because the trapped particles and the passing particles experience different dynamics, the mode perturbation in the velocity distribution would be discontinuous were it not for collisions. Small angle scattering provides an essential correction, smoothing the distribution in a boundary layer near the separatrix. Significantly, this correction contains a component that is in phase with the mode electric field, so the mode can exchange energy with the scattered particles and damp as a consequence. We will show that the velocity scattering and damping are intrinsically associated with a kind of neoclassical radial transport.

The mode potential acts nonlinearly to raise and lower the separatrix velocity each cycle, causing transitions from trapped to passing and vice versa. Thus, nonlinearly driven transitions across the separatrix compete with the collisional scattering across the separatrix. However, the collisional scattering dominates — is faster and acts over a wider band in velocity — provided that $\sqrt{\nu/\omega} \gg e\delta\phi/T$, where $\delta\phi$ is the mode potential and T is the plasma temperature [6]. This inequality is the criterion for validity of the linear mode analysis presented here, and the inequality is well satisfied in the experiments reported here. In our linear analysis, the classification of a particle as trapped or passing refers to the equilibrium potential in the absence of the mode.

Experimentally, the mode is excited externally and then damps exponentially in time. The calculated damping rates agree with the measured rates to within 50%, and predict the observed scalings under variation in barrier strength and magnetic field strength. The 50% level of agreement is reasonable considering the coarseness of the theory and the experimental uncertainty in the plasma state. More compelling than this numerical agreement is an experimental signature that shows a causal relation between velocity scattering of marginally trapped particles and damping: the damping is observed to increase dramatically coincident with the artificial enhancement of the scattering. The enhancement is produced by applying an rf electric field that oscillates in resonance with the bounce motion of the marginally trapped particles.

The experiments employ a pure electron plasma column that is confined in a Malmberg-Penning trap configuration [5]. The confinement region is bounded radially by a series of conducting cylinders with radius R_w. The end cylinders are held at negative potential to provide axial confinement of the electron plasma, while radial confinement is provided by a large axial magnetic field $B\hat{z}$. Here, (r, θ, z) is a cylindrical coordinate system with the z axis coincident with the axis of the trap. Because the column is unneutralized, there is a radial space charge electric field and consequent $\mathbf{E} \times \mathbf{B}$ drift rotation of the column at rate $\omega_E(r)$.

A static, θ-symmetric barrier is created by applying a negative "squeeze" voltage to a short cylindrical section of the bounding wall at the axial midpoint of the column. The resulting barrier potential is an increasing function of radius, with more particles trapped near the radial edge of the plasma than near the trap axis. At any given radius, the potential presents an insurmountable barrier to low axial velocity particles, trapping them in the end regions. Particles with high axial velocity pass over the potential and sample the entire length of the column during their bounce motion.

The plasma has time to come into thermal equilibrium along each field line before the mode is launched. Thus, the equilibrium distribution function is given by

$$f_0(r, z, v) = \frac{N(r)\exp\{-\frac{1}{T}[mv^2/2 - e\phi_0(r, z)]\}}{\int dz \int dv \exp\{-\frac{1}{T}[mv^2/2 - e\phi_0(r, z)]\}}, \tag{1}$$

where the z integrated density $N(r)$ and the temperature on axis are known from measurement. The equilibrium potential $\phi_0(r, z)$ is obtained self-consistently by a solution of the Poisson-Boltzmann equation. From the solution, one can see that Debye shielding forces the equilibrium potential to be z-independent except in the barrier region and near the ends. The separatrix velocity, $v_s(r)$, is determined from the potential difference between the main column interior and the center of the barrier region ($z = 0$ cm),

$$v_s(r) \equiv \left\{-\frac{2e}{m}[\phi_0(r, 0) - \phi_0(r, L/2)]\right\}^{1/2}. \tag{2}$$

In the experiments, frequencies are ordered as $\Omega_c \gg \omega_b \gg (\omega, \omega_E) \gg \nu$, where Ω_c is the cyclotron frequency, $\omega_b \equiv 2\pi\bar{v}/2L$ the axial bounce frequency in each half, and ν the collision frequency. Also, length scales are ordered as $r_c \ll (\lambda_D, \Delta, R) \ll L$, where r_c is the cyclotron radius, λ_D the Debye length, Δ the length of the barrier region, R the plasma radius, and $2L$ the length of the column. Thus, the motion transverse to the magnetic field can be described by bounce-average $\mathbf{E} \times \mathbf{B}$ drift dynamics, and the streaming motion parallel to the magnetic field causes Debye shielding of the mode potential.

Because the system has cylindrical symmetry, the modes are taken to be of the form $\delta\phi(r, z, \theta, t) = \delta\phi_\ell(r, z)\exp(i\ell\theta - i\omega t)$. The column admits modes that have even parity in z (the usual diocotron modes) and odd parity in z (the trapped-particle modes). In both cases, Debye shielding forces $\delta\phi_\ell(r, z)$ to be z independent except in the barrier region and near the extreme ends of the column. For the trapped-particle mode, $\delta\phi_\ell(r, z)$ changes sign in the barrier region. As a simple model, we use a step function potential

$$\delta\phi_\ell(r, z) = \begin{cases} -\delta\phi_\ell(r) & z < 0, \\ +\delta\phi_\ell(r) & z > 0, \end{cases} \tag{3}$$

relying on the length scale ordering mentioned above.

Without collisions, the perturbed guiding center drift distribution $\delta f_\ell(r, z, v)$ would develop a discontinuity in v. Since the mode potential has odd parity in z, the bounce-average mode potential is zero for passing particles. These particles experience no bounce-average $\mathbf{E} \times \mathbf{B}$ drift perturbation and develop only the streaming response $\delta f_\ell^{(p)} = (e\delta\phi_\ell/T)f_0$. In contrast, each group of trapped particles experiences $\mathbf{E} \times \mathbf{B}$ drift orbits in a nearly z-independent potential $\delta\phi_\ell(r)$, and develops the response

$$\delta f_\ell^{(t)}(r, v) = \frac{c\ell}{Br}\frac{\delta\phi_\ell(r)}{\omega'(r)}\frac{\partial f_0(r, v)}{\partial r}, \tag{4}$$

where $\omega'(r) \equiv \ell\omega_E(r) - \omega$, and the sign corresponds to the $z > 0$ trapped-particle region. In general, $\delta f_\ell^{(p)}$ and $\delta f_\ell^{(t)}$ differ in value and slope at the separatrix. The Fokker-Planck collision operator contains velocity derivatives that become arbitrarily large at such a discontinuity, so the effect of collisions on δf cannot be ignored even if ν is small.

Including the effect of collisions, the trapped-particle perturbation satisfies the equation

$$\left\{i\omega' - D_v(v_s)\frac{\partial^2}{\partial v^2}\right\}\delta f_\ell^{(t)} = \frac{ic\ell}{Br}\delta\phi_\ell\frac{\partial f_0}{\partial r}, \tag{5}$$

where only the second derivative term in the Fokker-Planck collision operator has been retained. In the separatrix boundary layer region, where derivatives are large, the second velocity derivative term suffices. Here, $D_v(v_s)$ is the coefficient of parallel velocity diffusion evaluated at the separatrix velocity $v_s = v_s(r)$, and is of order $\nu\bar{v}^2$.

For $\ell > 0$, the solution to Eq. (5) is given by

$$\delta f_\ell^{(t)} = \frac{c\ell}{Br}\frac{\delta\phi_\ell}{\omega'}\frac{\partial f_0}{\partial r} + a\exp\left[\frac{1+i}{\sqrt{2}}\sqrt{\frac{\omega'}{D_v(v_s)}}(v - v_s)\right], \tag{6}$$

where a is an arbitrary constant and we have chosen the sign so that the exponential decays as $(v - v_s)$ becomes large and negative. The new term represents a collisional correction in a boundary layer of width $\delta v_s^{(t)} \sim \sqrt{D_v(v_s)/\omega'}$. To understand this width, note that the sign of the perturbation seen by the trapped particles changes on the time scale $1/\omega'$, and velocity diffusion can extend to the width $\delta v_s^{(t)}$ during this time [i.e., $[\delta v_s^{(t)}]^2 \sim D_v(v_s)/\omega'$]. A similar correction is obtained for the passing particles. However, in this case the correction is of width $\delta v_s^{(p)} \sim \sqrt{D_v(v_s)/\omega_b}$, since the passing particles see a sign change on the bounce time scale $1/\omega_b$.

There is a disparity between the width of the boundary layer for trapped and passing particles: $\delta v_s^{(p)} \ll \delta v_s^{(t)}$ since $\omega_b \gg \omega'$. The consequence of this disparity is that only the trapped-particle correction contributes significantly to the damping. Matching the value and slope of the distribution at $v = v_s$ yields $a = \delta\phi_\ell[ef_0/T - (c\ell/Br\omega')(\partial f_0/\partial r)]_{v=v_s}$. Substituting into Poisson's equation then yields an eigenvalue equation for the mode potential

$$\nabla_\perp^2 \delta\phi_\ell = 4\pi e\left(\frac{en_p}{T} + \frac{c\ell}{Br}\frac{\partial n_t}{\partial r}\frac{1}{\omega'}\right)\delta\phi_\ell + \frac{8\pi e}{1+i}\sqrt{\frac{2D_v(v_s)}{\omega'}}\left[\frac{ef_0}{T} - \frac{c\ell}{Br}\frac{\partial f_0}{\partial r}\frac{1}{\omega'}\right]_{v=v_s}\delta\phi_\ell, \tag{7}$$

which is subject to the boundary conditions $\delta\phi_\ell(0) = \delta\phi_\ell(R_w) = 0$. Multiplying both sides of Eq. (7) by $\delta\phi_\ell^*$, integrating over rdr, setting $\omega = \omega_r + i\gamma$, and taking the imaginary part of both sides yields an expression for the growth or damping rate

$$\gamma = \frac{\frac{B}{\ell}\int_0^{R_w} rdr|\delta\phi_\ell|^2\sqrt{\frac{2D_v(v_s)}{\ell\omega_E-\omega_r}}\left[\frac{ef_0}{T} - \frac{c\ell}{Br}\frac{\partial f_0}{\partial r}\frac{1}{\ell\omega_E-\omega_r}\right]_{v=v_s}}{\int_0^{R_w} dr\frac{|\delta\phi_\ell|^2}{(\ell\omega_E-\omega_r)^2}\frac{\partial n_t}{\partial r}}. \tag{8}$$

To understand the energy budget for the damping, we rewrite Eq. (8) as

$$0 = 2\gamma W + \int_0^{R_w} rdr\int_0^{2\pi} d\theta\, 2e\delta\phi(r,\theta,t)\frac{d}{dt}[L\delta n_c(r,\theta,t)] + \int_0^{R_w} rdr\int_0^{2\pi} d\theta\left[-e\frac{\partial\phi_0}{\partial r}\right][\delta v_r(r,\theta,t)\delta n_c(r,\theta,t)2L], \tag{9}$$

where

$$W = \frac{2Lec\omega\ell}{B}\int_0^{R_w} 2\pi dr\frac{|\delta\phi_\ell|^2}{(\ell\omega_E - \omega_r)^2}\frac{\partial n_t}{\partial r} \tag{10}$$

is the mode energy, which turns out to be negative. The first term in Eq. (9) is the rate of change of mode energy, which is positive for damping since both W and γ are negative. In the second term, the quantity $\delta n_c(r,\theta,t)$ is the collisional correction to the trapped electron perturbation, and the convective derivative $d(L\delta n_c)/dt = (\partial/\partial t + \omega_E\partial/\partial\theta)(\delta n_c L)$ is the flux of scattered electrons to the side where the mode potential is $+\delta\phi(r,\theta,t)$. As these electrons transit the squeeze region, the mode does work $2e\delta\phi$ on each electron. The second term is the rate of such work, which turns out to be positive. In the third term, the quantity $\delta v_r(r,\theta,t) = -(c/Br)(\partial\delta\phi/\partial\theta)$ is the radial drift velocity imparted by the mode potential, so the third term is the rate at which the mode does work in moving the scattered particles through the potential gradient $\partial\phi_0/\partial r$. There is a net outward radial flux of scattered particles, so the third term is negative, balancing the other two terms. In summary, the mode potential increases the kinetic energy of the scattered particles by acceleration along the magnetic field and decreases the electrostatic energy of the particles by radial transport outward, the latter effect being dominant. When the liberated energy is added to the negative energy mode, the mode damps.

To understand the sign of the second term, first recall that in the absence of collisions, $\delta f_p(v_s) > 0$ and $\delta f_t(v_s) < 0$ on the side where $\delta\phi > 0$. On this side, the smoothing action of collisions must produce a velocity-space flux from passing to trapped. The situation is reversed on the other side where $\delta\phi < 0$. Thus, the net effect is a spatial flux of trapped particles from the side where $\delta\phi < 0$ to the side where $\delta\phi > 0$. Physically, this is reasonable, since we expect collisions to produce a flux of trapped particles from high to low potential energy. Equivalently, we expect collisions to produce heating, and the second term is the heating rate.

The θ-average radial flux can be written as

$$\frac{1}{2\pi}\int_0^{2\pi} d\theta\,\delta v_r\,\delta n_c = D_r(r)\left[-\frac{\partial n_0}{\partial r} + \frac{e}{T}\frac{\partial\phi_0'}{\partial r}n_0\right], \tag{11}$$

where $D_r(r) = \epsilon(\Delta r)^2\omega'$ is a neoclassical-like diffusion coefficient. To understand this coefficient, note first that $\epsilon = 2\sqrt{2D_v(v_s)/\omega'}\exp[-v_s^2/2\bar{v}^2]/\sqrt{2\pi\bar{v}^2}$ is the fraction of particles in the boundary layer at radius r. In the presence of the mode, the $\mathbf{E}\times\mathbf{B}$ drift orbits for the trapped particles are distorted from circularity by an amount $\Delta r = c\ell|\delta\phi_\ell|/Br\omega'$, and the distortions on the

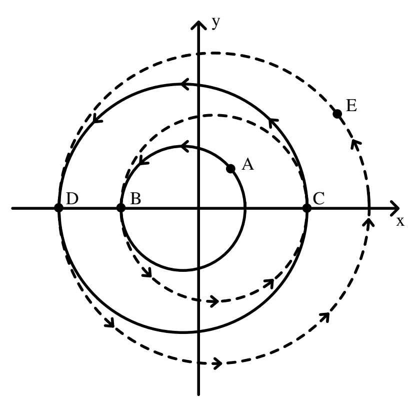

FIG. 1. Potential energy contours in the frame of an $\ell = 1$ wave (solid $z > 0$, dashed $z < 0$). A particle initially at A is transported to E through drift orbits and detrapping/retrapping collisions at B, C, and D.

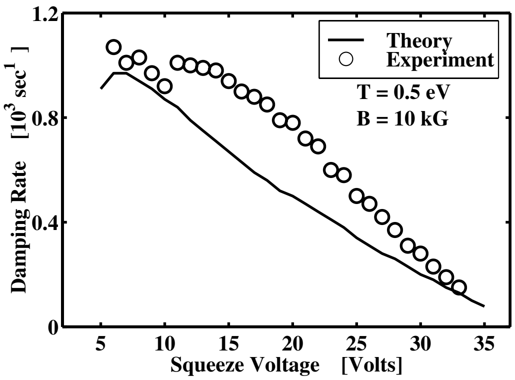

FIG. 2. Mode damping rate vs applied squeeze voltage from theory [Eq. (8)] and experiments.

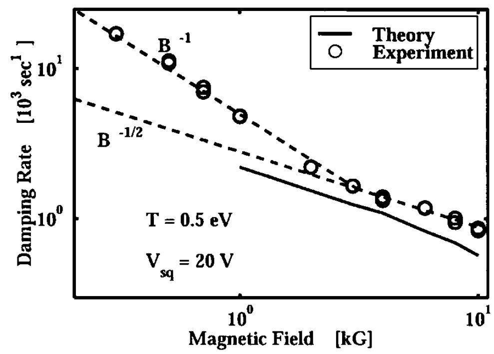

FIG. 3. Mode damping rate vs magnetic field. The theory correctly predicts the $B^{-1/2}$ scaling at high magnetic fields.

two ends are 180° out of phase. When a trapped particle is scattered and changes trapped-particle class (and drift orbit), the particle effectively makes a radial step Δr. For the particles in the boundary layer, the class changes at the rate ω', so $D_r(r) = \epsilon(\Delta r)^2\omega'$ is the expected diffusion coefficient.

Equation (11) is the usual form for the transport flux in the presence of a density gradient and an external force with the diffusion and mobility coefficients related by an Einstein relation [i.e., $\mu = (e/T)D$] [7]. The radial electric field $-\partial\phi_0'/\partial r \equiv -\partial\phi_0/\partial r + (r\omega_\ell/\ell)(B/c)$ is the effective field in the rotating frame of the wave. This is the frame where the mode perturbation is static and the Einstein relation is valid. One can easily understand that diffusion in a negative density gradient ($\partial n_0/\partial r < 0$) produces an outward radial flux. However, the origin of the mobility term is more subtle being intimately connected with the net axial flux of scattered particles from the high potential energy side to the low side.

The preference for outward radial transport is illustrated in Fig. 1. The solid curves represent potential energy contours [i.e., $-e\phi_0'(r) - e\delta\phi(r,\theta) = \text{const}$] on one side of the barrier, and the dashed curves the contours on the other side. The difference between the two arises from the fact that $\delta\phi$ has opposite signs in the two ends. The pattern is stationary in the rotating frame of the wave, except for the slow damping. A particle that is initially trapped in one end at A will $\mathbf{E}\times\mathbf{B}$ drift along the contours until it comes to B where the contour is farthest from the center, that is, where $-e\delta\phi(r,\theta)$ takes its maximum value. Statistically, this is where the particle is most likely to be detrapped and move rapidly to the other end, where this particle or an equivalent particle is scattered and trapped. The particle will then $\mathbf{E}\times\mathbf{B}$ drift along the dashed contour, continuing its motion outward to C. Repeating the process then takes the particle to D and E.

Figures 2 and 3 show damping rates predicted by Eq. (8) compared to measured rates as squeeze voltage and magnetic field strength are varied. The agreement is to within 50% over the expected range of validity for the theory. The damping rate is expected to be a decreasing function of the squeeze voltage, since the denominator in (8) increases with the number of trapped particles. Since ω_r and ω_E are proportional to $1/B$, Eq. (8) implies that γ scales as $1/\sqrt{B}$. This scaling is observed for large field strength ($B \simeq 2$–10 kG), where the theory assumption $\omega_b \gg \omega_E, \omega_r$ is well satisfied. At lower field strength $\gamma \propto 1/B$ is observed, but not understood theoretically. The magnetic field dependence was missed in early experiments [5] because it was counterbalanced by transport-induced temperature changes.

We have direct experimental evidence that velocity scattering of marginally trapped particles is responsible for the mode damping. We artificially enhance the scattering by adding a small oscillating potential $V_{RF} \approx 2\times 10^{-4}V_{sq}$ to the dc squeeze voltage, choosing the frequency to match the bounce motion of marginally trapped particles. This causes an immediate strong increase ($\approx 10\times$) in the damping rate, which returns to its original level after the rf component is switched off. The enhancement in the damping depends sharply on the rf-component frequency, with the peak and width consistent with the bounce frequencies of the marginally trapped particles responsible for the damping.

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