

One-Shot Quantum Measurement Using a Hysteretic dc SQUID

O. Buisson,¹ F. Balestro,¹ J. P. Pekola,^{1,2} and F. W. J. Hekking³

¹*Centre de Recherches sur les Très Basses Températures, Laboratoire associé à l'Université Joseph Fourier, CNRS, B.P. 166, 38042 Grenoble Cedex 9, France*

²*Low Temperature Laboratory, Helsinki University of Technology, P.O. Box 2200, 02015 HUT, Finland*

³*Laboratoire de Physique et Modélisation des Milieux Condensés, CNRS and Université Joseph Fourier, B.P. 166, 38042 Grenoble Cedex 9, France*

(Received 3 January 2003; published 13 June 2003)

We propose a single shot quantum measurement to determine the state of a Josephson charge quantum bit (qubit). The qubit is a Cooper pair box and the measuring device is a two junction superconducting quantum interference device (dc SQUID). This coupled system exhibits a close analogy with a Rydberg atom in a high Q cavity, except that in the present device we benefit from the additional feature of escape from the supercurrent state by macroscopic quantum tunneling, which provides the final readout. We test the feasibility of our idea against realistic experimental circuit parameters and by analyzing the phase fluctuations of the qubit.

DOI: 10.1103/PhysRevLett.90.238304

PACS numbers: 85.25.Dq, 03.67.Lx, 74.50.+r

A Cooper pair box (CPB) is a controllable macroscopic two-level system [1,2], which is considered as a potential qubit in the context of quantum computing [3]. Coherent Rabi oscillations have been observed in a CPB, using ultrafast pulses [2]. Recently, in a superconducting device named “quantronium,” coherent oscillations were observed using microwave pulses [4]. The observed oscillations lived for almost a microsecond, making superconducting circuits promising for realizing quantum gates.

Despite this progress, quantum measurement on a CPB still remains a challenge. The charge readout circuits, such as the one in Ref. [2] or those using single electron tunneling transistors (see, e.g., Ref. [5]), are far from ideal due to decoherence induced by the unavoidable background charge fluctuations and by backaction during the measurement, e.g., in the form of shot noise. The former problem can be avoided to a large extent if the qubit is operated at the degeneracy point of the two-level system: at this optimum point linear coupling to charge fluctuations is absent. In the quantronium experiment, both problems were eliminated successfully by measuring the two quantum states of a Cooper pair transistor (CPT) at the optimum point by a hysteretic Josephson junction (JJ). This junction, working in its classical regime, measured the persistent current of the two quantum states of the CPT. This method cannot, however, be used to measure the quantum state of a CPB. Moreover, the measurement did not resolve the quantum state in one shot.

In this Letter, we propose a new quantum measurement procedure based on the entanglement between two quantum systems: a CPB coupled to a superconducting resonator. The dynamics of this coupled system has been theoretically investigated recently [6–9]. Here we show that at the degeneracy the two quantum states of the CPB can be resolved in one shot. The readout device is a

current-biased two junction superconducting quantum interference device (dc SQUID) (the resonator) and it is controlled by adiabatic pulses of flux. Very high sensitivity and fast measurement can be reached with this method. A dc SQUID is preferred over a JJ because manipulations using external flux do not suffer from time limitations. In a JJ, nanosecond pulses of bias current cannot be applied because low pass filters are necessary to exclude high-frequency noise [10].

The circuit of a Cooper pair box coupled to a current-biased dc SQUID was inspired by experiments on a Rydberg atom in a high Q cavity [11]. For a bias current I well below the critical current I_c , the CPB and the SQUID play the roles of the Rydberg atom and the high Q cavity, respectively. For example, Rabi oscillations are predicted to occur as a result of spontaneous emission and reabsorption by the CPB of a single oscillation quantum in the SQUID [6]. The device can be considered as a two-level system coupled to a harmonic oscillator. However, for a bias current very close to, but below, the critical current, macroscopic quantum tunneling (MQT) in the SQUID can occur [12]. This quantum escape phenomenon has no equivalent in high Q cavity experiments, and it introduces a new element into the dynamics of the system. We describe how we can benefit from MQT in order to do a very fast one-shot quantum measurement on the CPB. We discuss the performance of this measurement and its backaction on the CPB.

The superconducting quantum circuit that we consider is shown in Fig. 1. It contains three parts which we describe in detail below: a Cooper pair box, a hysteretic dc SQUID, and a coupling capacitance between the two. The circuit is connected to the external classical circuits by three different couplings: a resistance R in parallel to a dc current source and voltmeter, a mutual inductance to a source of flux modulating current pulses, and a gate capacitance to a dc and pulse voltage source.

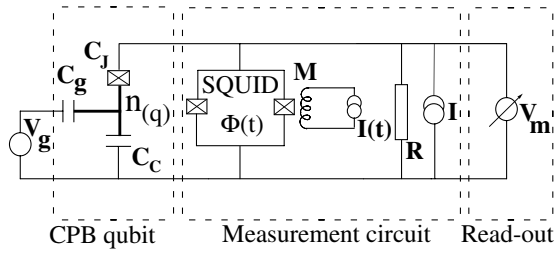


FIG. 1. The CPB qubit is coupled to a measuring SQUID through the capacitance C_c . V_g controlled the CPB states, I and $I(t)$ the measurement procedure. The voltmeter V_m performs the readout.

The *Cooper pair box* consists of a small superconducting island, coupled to a gate voltage V_g by a gate capacitance C_g . The island is furthermore connected capacitively to a superconducting electrode via a JJ with a capacitance C_J and Josephson energy E_J . We are interested in the limit of small Josephson energy, $E_J \ll E_{C,J}$, where $E_{C,J} = 2e^2/C_{J,\text{eff}}$ is the elementary charging energy of the box with $C_{J,\text{eff}} = C_J + [1/C_S + 1/(C_c + C_g)]^{-1}$; C_S is the SQUID capacitance and C_c the coupling capacitance. Let us introduce the basis of charge states $|n_{(q)}\rangle$, where $n_{(q)}$ corresponds to the number of excess Cooper pairs on the island. If the dimensionless gate charge $N_g = -C_g V_g / (2e)$ is close to $1/2$, the charge states $|0_{(q)}\rangle$ and $|1_{(q)}\rangle$ are almost degenerate and the relevant eigenstates of the CPB are superpositions of these charge states. Specifically, the ground state and the first excited state are, respectively, $|-\rangle = (|0_{(q)}\rangle + |1_{(q)}\rangle) / \sqrt{2}$ and $|+\rangle = (|0_{(q)}\rangle - |1_{(q)}\rangle) / \sqrt{2}$, with eigenenergies $E_{\mp} = E_{C,J} \mp \frac{1}{2} E_J$. Thus, the CPB effectively behaves as a quantum-mechanical two-level system [3].

The *hysteretic dc SQUID* consists of a superconducting loop with two underdamped JJs which both have ideally the same critical current I_0 and capacitance $C_S/2$. We neglect the loop inductance and effects due to asymmetry in the SQUID. The damping of the two junctions is also neglected now, but it will be discussed at the end. With these approximations, the SQUID equation of motion is similar to that of a single JJ, describing a particle of mass $m = C_{S,\text{eff}} [\Phi_0 / (2\pi)]^2$ in a tilted washboard potential $U(\varphi) = E_S [-I\varphi / I_c - \cos(\varphi)]$, where φ is the phase difference of the SQUID, I is the bias current through the SQUID, $\Phi_0 = h/2e$ is the superconducting flux quantum, Φ_{dc} is the external flux, and $I_c = 2|\cos(\pi\Phi_{\text{dc}}/\Phi_0)|I_0$, $E_S = I_c \Phi_0 / (2\pi)$, and $C_{S,\text{eff}} = C_S + [1/C_J + 1/(C_c + C_g)]^{-1}$ are the effective critical current, Josephson energy, and capacitance of the SQUID, respectively.

For values of bias current not too far below the critical current, the potential $U(\varphi)$ can be well approximated by a cubic potential. The quantum dynamics of the SQUID using the reduced momentum and position operators $\hat{P} = (1/\sqrt{m\hbar\omega_p})P$ and $\hat{X} = (\sqrt{m\omega_p/\hbar})X$, respectively, is then described by $\hat{H}_0 = \frac{1}{2}\hbar\omega_p(\hat{P}^2 + \hat{X}^2) + \sigma(I)\hbar\omega_p\hat{X}^3$, where $X = \varphi$ is the phase difference, P is its conjugate opera-

tor, and $\omega_p = [2\pi I_c / (\Phi_0 C_{S,\text{eff}})]^{1/2} [2(1 - I/I_c)]^{1/4}$ is the effective plasma frequency of the SQUID. The parameter $\sigma(I) = -1/(6a)[2(1 - I/I_c)]^{-3/4}$, where $a = \hbar^{-1/2} [\Phi_0 / (2\pi)]^{3/4} (C_{S,\text{eff}} I_c)^{1/4}$, gives the relative magnitude of the cubic term as compared to that of the harmonic oscillator term.

For values of I below I_c with $\sigma(I) \ll 1$, the potential barrier is high compared to $\hbar\omega_p$, and the cubic term in \hat{H}_0 can be neglected. Many low-lying states are found near the minimum of the quadratic potential. The broadening of these states due to tunneling can be ignored. Hence, these states are well approximated by harmonic oscillator eigenstates, denoted by $|0\rangle, |1\rangle, |2\rangle, \dots$, corresponding to the presence of $0, 1, 2, \dots$ oscillation quanta in the SQUID, respectively. Thus, at low enough bias current, the SQUID behaves as a superconducting quantum resonator. Since in this limit the phase is localized in a well defined minimum of the potential U , the time-averaged voltage V_m across the SQUID remains zero.

If the bias current is increased such that $I \lesssim I_c$, $\sigma(I)$ is no longer negligible and the cubic term affects the SQUID dynamics. The barrier height, given by $\Delta U = 4\sqrt{2}/3 E_S (1 - I/I_c)^{3/2}$, and ω_p decrease and vanish at the critical current. The remaining energy levels broaden due to quantum tunneling from the metastable wells of the potential U . The broadening of the ground state energy for low damping is given by the tunneling rate $\Gamma_0 = \omega_p 6\sqrt{6}/\pi \sqrt{\Delta U/\hbar\omega_p} \exp[-36\Delta U/(5\hbar\omega_p)]$. Since the excited states $|n\rangle$ are located closer to the top of the barrier, the tunneling rates and hence the broadening of these states increase with increasing energy of the level as $\Gamma_n \sim [\exp(-36/5)]^n \Gamma_0$. Specifically, the tunneling rates from states $|0\rangle$ and $|1\rangle$ of the SQUID are different by approximately a factor of a thousand [13,14]. After a tunneling event, the phase of the SQUID is no longer localized and the time-averaged voltage V_m across the SQUID is finite.

The magnetic flux through the SQUID affects its critical current I_c ; hence, ω_p , $\sigma(I)$, and therefore \hat{H}_0 depend on the flux. For small, time-dependent variations of flux, $\delta\Phi(t) \ll \Phi_0$, and for SQUID parameters such that $a \gg 1$, the total time-dependent Hamiltonian is given by $\hat{H}(t) = \hat{H}_0 - \lambda(I, \Phi_{\text{dc}}) [\delta\Phi(t)/\Phi_0] \hbar\omega_p \hat{X}$, where $\lambda(I, \Phi_{\text{dc}}) = a[2(1 - I/I_c)]^{-3/8} I/I_c \pi \tan(\pi\Phi_{\text{dc}}/\Phi_0)$. Below we consider the effect of a time-dependent perturbation $\delta\Phi(t)$ on the dynamics of the coupled system.

The *coupling capacitance* C_c plays a crucial role in the circuit of Fig. 1 since it couples the qubit and the SQUID to each other. Physically, C_c couples the charge $n_{(q)} - N_g$ on the CPB to the charge on the SQUID. The coupling Hamiltonian can be written as $\hat{H}_c = -iE_{\text{coupl}} [n_{(q)}] \hat{P}$. Here we introduced the characteristic coupling energy $E_{\text{coupl}} = \sqrt{\hbar\omega_p/E_{C,S}E_{C,c}}/4$, where $E_{C,c} = 2e^2/C_{c,\text{eff}}$ and $E_{C,S} = 2e^2/C_{S,\text{eff}}$ with $C_{c,\text{eff}} = C_c + C_g + (1/C_J + 1/C_S)^{-1} [(C_J + C_S)/(C_c + C_g)]/2$. This coupling energy leads to full entanglement between the states of the CPB and the SQUID at the resonance condition $E_J = \hbar\omega_p$ [6].

Having detailed the superconducting quantum circuit, we now describe the measurement procedure. Suppose that, at time $t = 0$, the CPB is in a coherent superposition $\alpha|-\rangle + \beta|+\rangle$, as a result of quantum operations performed at times $t < 0$ using the gate voltage V_g [2]. We propose a way to measure the probability $|\beta|^2$. The measurement procedure is depicted schematically in Fig. 2 and consists of three successive steplike variations of flux through the SQUID. The steps are not sharp, but have a finite rise and fall time δt . The flux steps must be *adiabatic* in terms of the dynamics of the CPB, $\delta t \gg \hbar/E_J$, and of the SQUID, $\delta t \gg 2\pi/\omega_p$. But they must be *instantaneous* in terms of the coupling dynamics, $\delta t \ll \hbar/E_{\text{coupl}}$. The first step at $t = 0$ puts the SQUID into resonance with the CPB during a time T_0 . The second step at T_0 drives the SQUID close to its critical current during a time Δt . The last step sets the SQUID far below the critical current.

In more detail, at $t < 0$, during the quantum manipulation of the CPB, the SQUID must be decoupled. This condition is achieved if $(\hbar\omega_p - E_J) \gg E_{\text{coupl}}$, i.e., off resonance. Thus, in leading order, the eigenstate of the entire system is a product of the eigenstates of the qubit and the SQUID, in spite of the presence of the coupling capacitance. At $t = 0$, suppose the SQUID is in its ground state $|0\rangle$. The quantum state of the entire system is then $|\psi(t=0)\rangle = [\alpha|-\rangle + \beta|+\rangle] \otimes |0\rangle$.

The flux step applied at $t = 0$ reduces the effective critical current, and the resonant condition $E_J = \hbar\omega_p$ is satisfied for a time T_0 . The state $|-\rangle \otimes |0\rangle$ is still stationary at this resonance since $|E_J + \hbar\omega_p| \gg E_{\text{coupl}}$. But $|+\rangle \otimes |0\rangle$ is no longer an eigenstate: the state of the coupled system oscillates in time between $|+\rangle \otimes |0\rangle$ and $|-\rangle \otimes |1\rangle$ at the angular frequency $2E_{\text{coupl}}/\hbar$. Thus, after a time $T_0 = \hbar/(4E_{\text{coupl}})$, $|+\rangle \otimes |0\rangle$ has been transformed into $|-\rangle \otimes |1\rangle$. At this point, a second flux step is applied through the SQUID which drives the system out of resonance. The dynamics is therefore “frozen” in the superposition $|\psi(t=T_0)\rangle = |-\rangle \otimes (\alpha|0\rangle + e^{i\eta}\beta|1\rangle)$, where η is the relative phase arising from the evolution of the initial qubit state during time T_0 . The full entanglement

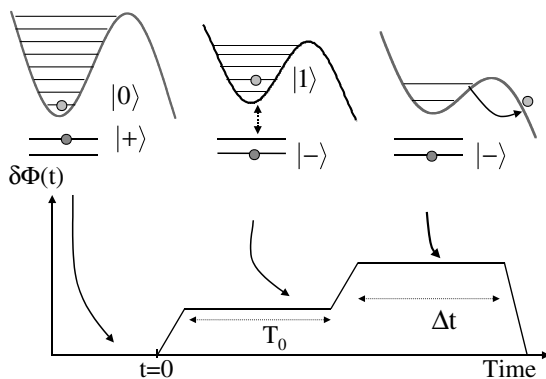


FIG. 2. The quantum measurement procedure is illustrated for an initial state $|+\rangle$.

has transferred the coherent superposition of the CPB to the SQUID; i.e., the information on the initial qubit state is now contained in the SQUID.

The second flux step reduces the effective critical current such that the constant bias current is close to I_c . The barrier is therefore significantly decreased and the tunneling rates Γ_0 and Γ_1 are drastically increased. During the time Δt , the SQUID can escape from the well to the nonzero voltage state by tunneling. If Δt satisfies $1/\Gamma_1 \ll \Delta t \ll 1/\Gamma_0$, and relaxation between the levels is neglected, the SQUID is in its finite-voltage state *if and only if the SQUID was in the state $|1\rangle$* at time T_0 . In other words, the proposed measurement determines the state of the SQUID *in one shot*. The escape probability corresponds to the $|\beta|^2$ amplitude of the initial superposition in the CPB. The lack of perfect contrast between the escape rates from the two states and relaxation processes introduces an intrinsic error in the proposed quantum measurement procedure; the influence of this will be estimated later.

At $t = T_0 + \Delta t$, the flux is switched back to its initial value to prevent further tunneling. As the SQUID is hysteretic, the zero- and finite-voltage states are stable for a sufficiently long time to perform the readout. The first two steps of duration T_0 and Δt perform the quantum measurement. The last step provides the classical readout measurement.

To check the feasibility of our measurement procedure, we use some typical values for parameters of an aluminum superconducting circuit. For the CPB, we choose $E_J = 26.2 \mu\text{eV}$, $C_g = 10 \text{ aF}$, and $C_J = 0.63 \text{ fF}$, and for the SQUID, $I_0 = 1 \mu\text{A}$, $I = 1.1 \mu\text{A}$, $C_S = 1 \text{ pF}$, and $\Phi_{\text{dc}}/\Phi_0 = 0.277$. For $\delta\Phi(t)/\Phi_0 = 0.013$ during the first step, the resonance condition is satisfied with five levels in the well; the escape time from level $|1\rangle$ is much longer than 1 ms. An additional increase of flux $\delta\Phi(t)/\Phi_0$ by the same amount is enough to change $\hbar\omega_p$ by $4 \mu\text{eV}$ ($\gg E_{\text{coupl}}$) and the system is driven out of resonance. The escape time of level $|1\rangle$ drops to about 1 ns. Finally, we choose $C_c = 0.1 \text{ fF}$, yielding $E_{\text{coupl}} = 0.22 \mu\text{eV}$. Using $T_0 = 4.5 \text{ ns}$ and $\Delta t = 5 \text{ ns}$, the one-shot quantum measurement can be performed.

We now turn to the influence of relaxation in the SQUID on the measurement procedure. At low temperature, the rate Γ_R of relaxation between adjacent levels down due to interaction with the environment is dominant. Assuming $\Gamma_1 \gg \Gamma_0, \Gamma_R$, the escape probability at $t = T_0 + \Delta t$ is given in the lowest order by

$$P_{\alpha|0\rangle+\beta|1\rangle}^e = |\beta|^2 + |\alpha|^2[1 - e^{-\Gamma_0\Delta t}] - |\beta|^2[e^{-\Gamma_1\Delta t} + (\Gamma_R/\Gamma_1)(e^{-\Gamma_0\Delta t} - e^{-\Gamma_1\Delta t})]. \quad (1)$$

Neglecting the influence of relaxation in Eq. (1), and assuming an infinite contrast between Γ_0 and Γ_1 , the escape probability gives the $|\beta|^2$ amplitude of the initial superposition in the CPB. The finite contrast between the

two states introduces an intrinsic error in the proposed quantum measurement procedure which is about 0.8%. Taking into account the relaxation processes, the escape probability versus the pulse duration Δt is plotted in Fig. 3 using the experimental parameters listed above for three different initial states: the two states $|-\rangle$ and $|+\rangle$ and the coherent superposition $(|-\rangle + |+\rangle)/\sqrt{2}$. The relaxation rate is defined as $\Gamma_R = \omega_p/Q$, where the Q factor was chosen to be $Q = 500$. The measurement of the escape during $\Delta t = 5$ ns is a direct measurement of the $|\beta|^2$ amplitude. For $Q = 500$, the total error of the one-shot measurement is less than 4%.

It is important to maintain coherence of the qubit. Specifically, we discuss the backaction, i.e., dephasing of the qubit, due to the measuring circuit formed by the SQUID and its electrical environment. To analyze this we need to study the amplitude of the fluctuations of the phase across the CPB. The measurement environment consists of the dc SQUID in parallel with the resistance R , which describes the dissipative part of the circuit. It is straightforward to obtain the phase fluctuations [15], $\langle(\Delta\phi)^2\rangle \equiv \langle[\phi(t) - \phi(0)]^2\rangle$, where $\phi(t)$ is the phase difference across the small junction in the CPB at time t , using the fluctuation-dissipation theorem. These phase fluctuations are determined by the real part of the impedance seen by the small junction. If the environment would be purely dissipative (resistance R), we would obtain fluctuations which diverge in time. On the contrary, the inductively shunted circuit, realized by the SQUID, protects the qubit: asymptotically ($t \rightarrow \infty$), the expectation value of phase fluctuations levels off to [16]

$$\langle(\Delta\phi)^2\rangle_\infty \approx \frac{2\pi}{R_Q} \sqrt{\frac{L_S}{C_{S,\text{eff}}}} \left(\frac{C_c + C_g}{C_J + C_c + C_g} \right)^2. \quad (2)$$

Here $R_Q \equiv h/4e^2$ is the resistance quantum and $L_S = \sqrt{2}(\hbar/2eI_c)(1 - I/I_c)^{-1/2}$ is the Josephson inductance of the SQUID. We assumed low temperatures $k_B T \ll \hbar/(L_S C_{S,\text{eff}})$. According to Eq. (2) the inductance, L_S , provides protection against dephasing: the dissipative part of the environment does not affect the value of $\langle(\Delta\phi)^2\rangle_\infty$. Using the typical experimental values given above, we obtain $\sqrt{\langle(\Delta\phi)^2\rangle} \approx 0.02$, which is much smaller than π , thereby demonstrating the weakness of the residual phase fluctuations. The low temperature condition is verified if $T \ll 1$ K. As we have shown, the SQUID provides protection of the qubit from the decoherence induced by the environment. Finally, the proposed measurement can be realized at the optimum point, where the background charge induced decoherence is largely suppressed.

In summary, we have shown theoretically that a two junction SQUID can perform a single shot quantum measurement of a Josephson charge qubit. We discussed the limits of this detector posed by the finite contrast in measuring the quantum state of the SQUID, the finite quality factor of the SQUID, and coupling of the qubit to environment noise.

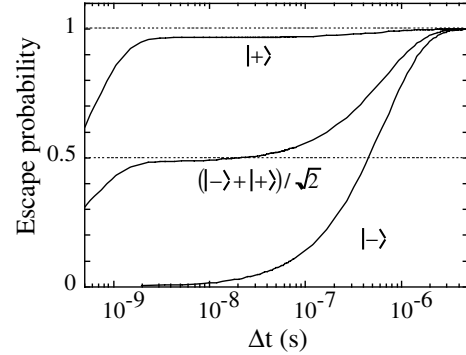


FIG. 3. Escape probability for three different initial states: $|-\rangle$, $|+\rangle$, and $(|-\rangle + |+\rangle)/\sqrt{2}$ for $Q = 500$. The parameters for the circuit used in the calculation are given in the text.

We thank F. Faure, Ph. Lafarge, L. Lévy, M. M. Doria, and P. Maia Neto for useful discussions. This work was supported by the French ACI program. J. P. acknowledges financial support from CNRS and Joseph Fourier University; F. H. is supported by Institut Universitaire de France.

-
- [1] V. Bouchiat *et al.*, Phys. Scr. **T76**, 165 (1998).
 - [2] Y. Nakamura, Yu. A. Pashkin, and J. S. Tsai, Nature (London) **398**, 786 (1999).
 - [3] Yu. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).
 - [4] D. Vion *et al.*, Science **296**, 886 (2002).
 - [5] A. Aassime *et al.*, Phys. Rev. Lett. **86**, 3376 (2001).
 - [6] O. Buisson and F. W. J. Hekking, in *Macroscopic Quantum Coherence and Computing*, edited by D. Averin, B. Ruggiero, and P. Silvestrini (Kluwer Academic, New York, 2001), p. 137.
 - [7] F. W. J. Hekking *et al.*, in *Electronic Correlations: From Meso- to Nano-Physics*, edited by T. Martin, G. Montambaux, and J. Tran Thanh Van (EDP Sciences, Les Ulis, 2001), p. 515.
 - [8] F. Marquardt and C. Bruder, Phys. Rev. B **63**, 054514 (2001).
 - [9] W. A. Al-Saidi and D. Stroud, Phys. Rev. B **65**, 014512 (2002).
 - [10] J. M. Martinis, M. H. Devoret, and J. Clarke, Phys. Rev. B **35**, 4682 (1987).
 - [11] M. Brune *et al.*, Phys. Rev. Lett. **76**, 1800 (1996).
 - [12] Shao-X. Li *et al.*, Phys. Rev. Lett. **89**, 098301 (2002).
 - [13] U. Weiss, *Quantum Dissipative Systems* (World Scientific, Singapore, 1999), 2nd ed.
 - [14] A. I. Larkin and Yu. N. Ovchinnikov, in *Quantum Tunneling in Condensed Media*, edited by Yu. Kogan and A. J. Leggett (Elsevier, New York, 1992), p. 234.
 - [15] G. L. Ingold and Yu. V. Nazarov, in *Single Charge Tunneling*, edited by H. Grabert and M. H. Devoret (Plenum Press, New York, 1992), p. 21.
 - [16] R. Fazio, F. W. J. Hekking, and J. P. Pekola (unpublished).