Interaction Dynamics of a Pair of Vortex Filament Rings

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Vortex filament-filament interactions are believed to underlie lethal arrhythmias in cardiac tissue but their dynamics remain poorly understood. We numerically replicate an experimentally postulated reentrant filament configuration as a pair of adjacent circular filaments (scroll rings) with common symmetry axes and varying initial radii and separation distances. The interaction properties are quantified in terms of the scroll-ring lifetime T_L and direction of initial velocity V_0. Two cases were examined, differing only in the direction of the wave around the filament, and observed drastic differences in T_L between the cases as the separation distance between the rings was decreased. We conclude that ring interactions present unexpected behaviors associated with competing interaction and decay mechanisms.

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Spiral waves have been observed in a wide variety of media, including chemical [1], physical [2], and biological settings [3]. Recently, attention has been directed towards the existence of scroll waves, the three-dimensional analog of spiral waves, which rotate around a line defect known as a filament. It is believed that the initiation and interaction of filaments in the excitable substrate of cardiac tissue underlies ventricular tachycardias and arrhythmias [4]. However, the presence of cardiac filaments must be either inferred from epicardial [4–6] or transmural mapping using voltage-sensitive dyes or electrodes [7,8]. In general, filaments are most often directly observed experimentally in the context of the Belousov-Zhabotinsky (BZ) reaction [9].

Recently, quatrefoil reentry (QR) has been shown as a viable means of reproducing four surface singularities in a controlled, repeatable fashion; these singularities may represent the endpoints of two U-shaped (semicircular) filaments [10]. Previous work has examined 2D phase singularity interaction dynamics in QR [11] and demonstrated that the interaction dynamics of the singularities in the experimental preparation were more complex than could be adequately explained by the existing numerical models. In this study, we reproduce the filament configuration inferred for experimentally observed QR by using a simulated pair of adjacent circular filaments (scroll rings) oriented along their symmetry axes with varying initial radii and separation distances. By symmetry, the dynamics of semicircular filaments in a half-domain are identical to a circular filament in a whole domain.

In order to study the interaction effects of paired scroll rings, it is desirable to utilize an excitable system for which a single insolated scroll ring experiences a minimum of translational movement. In this way, we can then attribute any drifting motion to the effects of interaction. It has been mathematically shown that, in the case of an untwisted scroll ring with equal diffusion coefficients (such as the BZ reaction) for which the curvature is not too great, the ring shrinks without drifting along the symmetry axis according to the equation [12]

$$\frac{d(R^2)}{dt} = -2D, \tag{1}$$

where R is the radius and D is the diffusion coefficient.

We use a two-variable model of the BZ reaction [13] using the Field-Koros-Noyes formulation [14]

$$\frac{dv}{d\tau} = \frac{1}{\epsilon}\left[v(1-v) - \left(2q\alpha\frac{w}{1-w} + \beta\right)\frac{v-\mu}{v+\mu}\right] + \nabla^2 v, \tag{2}$$

$$\frac{dw}{d\tau} = v - \alpha\frac{w}{1-w} + \delta\nabla^2 w, \tag{3}$$

where v is the bromous acid concentration, w is the relative ferroin concentration, and $\delta = D_w/D_v$ is the ratio of the diffusion coefficients of the two variables, taken to be equal to 1 in this case. The numerical values of the model parameters were identical with those used in [15].

The calculations were performed in a three-dimensional domain using a cylindrical coordinate system (z, ρ, θ). Since the rings are axisymmetric, the results are independent of θ and all calculations may be performed on the (z, ρ) plane only. The diffusion coefficient was $2 \times 10^{-5}\mathrm{cm}^2/\mathrm{s}$. To solve Eqs. (2) and (3), we used an explicit Euler scheme on a regular grid of 240×240 nodes with zero-flux boundary conditions.

Figure 1 illustrates the initial conditions used to simulate the two modes of quatrefoil reentry, termed cathodal and anodal break; these terms are used for the purpose of nomenclature only and are not meant to imply that the physiological circumstances have been reproduced. A scroll ring was initiated by setting v and w to appropriate values to produce the needed critical point. In this scheme, the cross section of the filament is a singularity in the (z, ρ) plane; the location of the singularity was

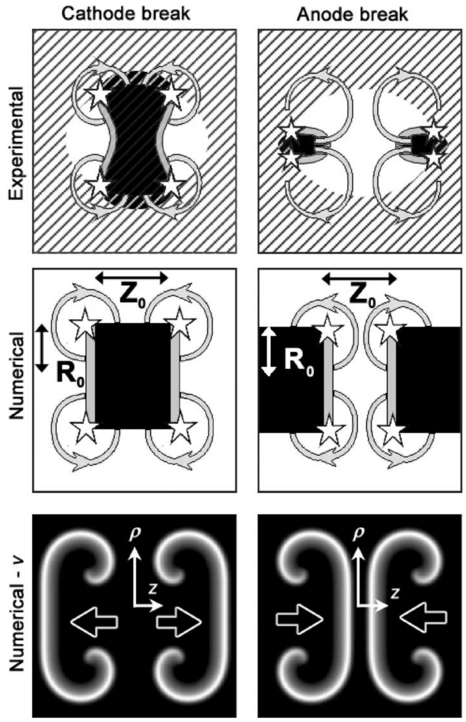

FIG. 1. Quatrefoil reentry produced by cathodal break (left column) and anodal break (right column). Top row depicts the experimental configuration. Black regions are tissue stimulated by cathodal excitation, gray border is initial wave front at the edge of excited tissue, hatched regions are refractory tissue, white regions are unexcited/hyperpolarized tissue, and stars show location of phase singularities. Middle row illustrates initial numerical approximation of the experimental configuration. Black (refractory) is $(v, w) = (0, 0.9)$, gray border (excited) is $(v, w) = (0.8, 0.8)$, and white (unexcited) is $(v, w) = (0, 0.8)$. Bottom row depicts the spatial distribution of the variable v after 160 s. The arrows show the direction of the motion of the wave front as it passes through the plane of the ring.

determined using a topological charge method shown previously [11].

The results of these simulations are shown for the cathodal configuration in Fig. 2 and for the anodal configuration in Fig. 3; T_L is shown as a function of the initial ring radius, R_0, and the inter-ring separation, Z_0, in (a), and the initial trajectories in (ρ, z) for each value of (R_0, Z_0) are shown in (b). We also show the initial scroll-ring velocities of the largest ring radii studied $[R_{0(\max)} = 5.08$ mm] in the z direction in Figs. 2(c) and 3(c), created by generated cubic spline fits to the (z, ρ) trajectories. Not unexpectedly, we observed that scroll rings for which Z_0 was sufficiently large behaved as though they were independent of one another for both configurations. For $Z_0 > 4$ mm, the values of R_0^2/T_L fit the quantity $2D$ to within 5%, although for values of R_0 less than 2.56 mm, the analytical solution of Eq. (1) failed to hold due to the increased curvature of the ring with smaller radii [12].

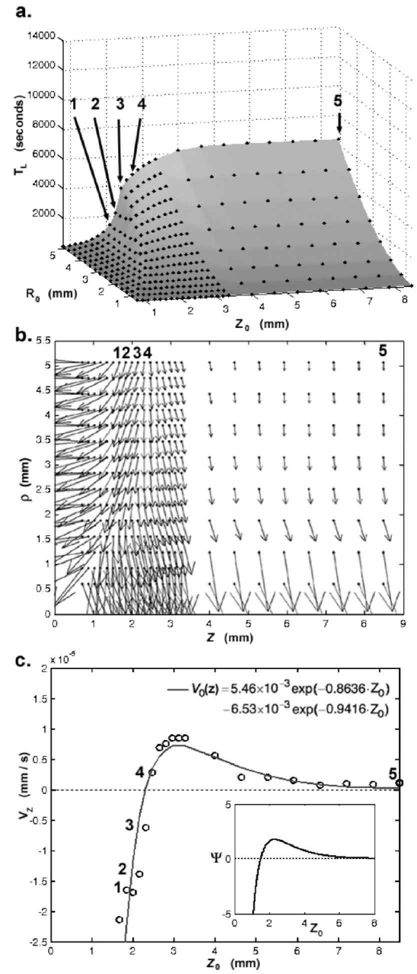

FIG. 2. (a) Scroll-ring lifetime T_L as a function of (R_0, Z_0) in cathodal break simulation. Each point is a single simulation for a given (R_0, Z_0). Points 1–5 refer to (R_0, Z_0) with $R_0 = 5.12$ mm and $Z_0 = 2.08$, 2.24, 2.4, 2.72, and 8.48 mm, respectively, and are used in (b) and (c). (b) Initial trajectories in the (ρ, z) plane. (c) Relationship between the initial separation distance (Z_0) and initial scroll rings velocity in the z direction $[V_0(z)]$. The curve represents a fitting of the numerical data with a difference of Yukawa potentials. A graph of $\Psi(Z_0)$ is shown in the inset.

For cathodal break, as Z_0 is further decreased below 4 mm, T_L decreases rapidly with a $|d(T_L)/d(Z_0)|_{\max}$ at $Z_0 \approx 2.32$ mm for radii close to $R_{0(\max)}$ [Fig. 2(a)]. As can be seen in Fig. 2(c), the velocity in the z direction increases with increasing Z_0 and then subsequently falls off for large Z_0. Note the presence of a zero crossing for $Z_{0(\text{crit})} \approx 2.23$ mm indicating a transition in behavior, from attractive to repulsive as Z_0 increases (velocity is negative where the vectors in Fig. 2(b) point in the $-z$ direction). Upon examination of the filament trajectories in the (z, ρ) plane, we observed that as Z_0 increases, the rings annihilate via mutual collision until $Z_0 = Z_{0(\text{crit})}$; the rings annihilate by individual collapse as Z_0 is further increased after this transition.

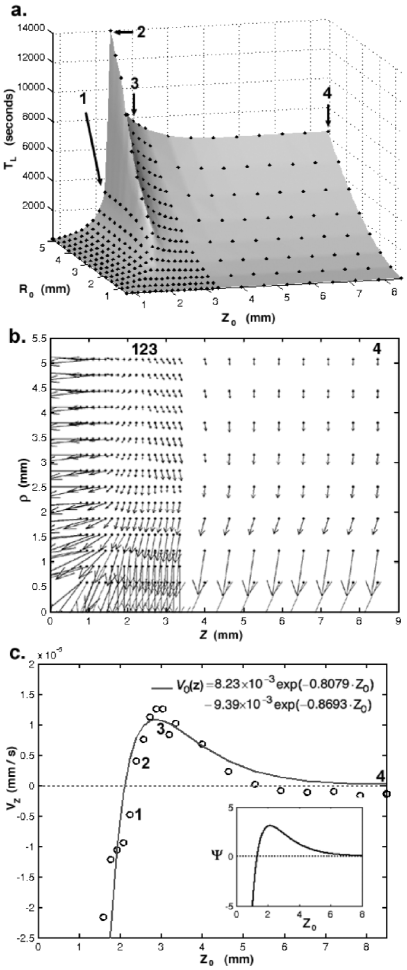

FIG. 3. (a) Scroll-ring lifetime T_L as a function of (R_0, Z_0) in anodal break simulation. Each point is a single simulation for a given (R_0, Z_0). Points 1–4 refer to (R_0, Z_0) with $R_0 = 5.12$ mm and $Z_0 = 2.24$, 2.4, 2.72, and 8.48 mm, respectively, and are used in (b) and (c). (b) Initial trajectories in the (ρ, z) plane. (c) Relationship between the initial separation distance (Z_0) and initial scroll-ring velocity in the z direction $[V_0(z)]$. The curve represents a fitting of the numerical data with a difference of Yukawa potentials. A graph of $\Psi(Z_0)$ is shown in the inset.

For anodal break, as Z_0 is decreased below 4 mm, T_L actually increases with Z_0, reaching a maximum of 1.4×10^5 s at $Z_0 = 2.4$ mm, after which T_L falls off dramatically as Z_0 is further decreased. Much as in cathodal break, the velocity in the z direction increases with increasing Z_0 and then subsequently falls off for large Z_0, with a zero crossing at $Z_{0(\mathrm{crit})} \approx 2.15$ mm [Fig. 3(c)]. Examination of the trajectories in the (z, ρ) plane shows that, for $Z_0 < Z_{0(\mathrm{crit})}$, the rings exhibit attractive behavior and annihilate by collision for radii close to $R_{0(\mathrm{max})}$ and, for $R_0 > 4.12$ mm, the rings expand as they attract one another. For $Z_0 > Z_{0(\mathrm{crit})}$, the rings annihilate by collapse.

The subject of theoretical or numerically simulated 2D spiral wave interaction has been widely treated in the literature. However, due to the analytic intractability of the problem, similar progress in theoretical filament interaction dynamics is only recently forthcoming. Much of the literature deals with topological concerns [16], self-interaction or stability [12,15,17], or motion of a single filament [18]; such experiments have been performed mostly with the complex Ginzburg-Landau equations.

The lifetime of a vortex has been used as a heuristic previously in the case of a spiral pair in which it was found that spirals placed at a certain initial distance apart, $Z_{0(\mathrm{crit})}$, fell into one of two regimes: (i) $<Z_{0(\mathrm{crit})}$, the spirals would attract one another and annihilate, (ii) $> Z_{0(\mathrm{crit})}$, the spirals would remain stable infinitely [19]. In much the same way, we see the same regimes in our data, (i) mutual annihilation and (ii) collapse via shrinkage, the positive filament tension precluding infinite stability. Furthermore, in [20], an unstable solution of the complex Ginzburg-Landau equations for a single pair of vortices is found to correspond to a change of vortex interaction behavior; the transition across a critical distance changes attractive into repulsive behavior. As Figs. 2(c) and 3(c) indicate, this shift is also observed in our data.

A persistent issue in the examination of vortex dynamics is the determination of the parameters involved in modulating the excitable medium which governs close- and long-range interactions [21]. It has been shown that the dynamics of vortex systems are analogous to that of a collection of charged particles [22], and hence the problem of vortex-vortex interaction may be analyzed as a particle-field problem. Determination of the intervortex distance mediated by underlying potential and the associated force has been extensively studied analytically in the Ginzburg-Landau equation [23]. Here we assume the filament may be treated as a point "mass" within a cylindrical coordinate system; by examining behavior for $R_{0(\mathrm{max})}$, the filaments all start with the same (unknown) effective "mass." Because these filaments appear to be moving dissipatively rather than ballistically, the initial velocity in the z direction $V_0(z)$ alone can be used as a measure of the force on the filament and hence the interaction potential, $\Psi(Z_0)$. We have found that the dependence of the initial velocity, and hence force, on the initial separation Z_0 is readily fitted by the difference of two Yukawa potentials [24] of the form

$$V_0(z) = C_1 \frac{e^{-C_2 Z_0}}{Z_0} - C_3 \frac{e^{-C_4 Z_0}}{Z_0}, \tag{4}$$

that describes both the attractive and repulsive behaviors, with the fitted parameter values given in Figs. 2(c) and 3(c). Equation (4) can be integrated trivially to convert force to $\Psi(Z_0)$, where we assume that $\Psi(\infty) = 0$ [graphs of $\Psi(Z_0)$ are shown as insets].

Mathematically, the configuration of the two scroll rings with axial symmetry is equivalent to a single spiral wave with a reflecting (Neumann) boundary condition normal to the symmetry axis. In experiments observing BZ spiral drift along a boundary, it is observed that the direction of drift corresponds to the spiral chirality [25]. This explains the direction of drift once the curvature of the independently rotating ring becomes sufficiently great towards the end of the ring's life [note the direction of the vectors for $R_0 < 1$ mm in Figs. 2(b) and 3(b)]. However, it also explains the increased lifetime of the anodal break ring as the initial separation approaches $Z_{0(\mathrm{crit})}$ since the interaction of the ring with the "boundary" tends to pull the ring in the $+\rho$ direction, offsetting the collapse due to filament tension. By the same token, $Z_{0(\mathrm{crit})}$ in cathodal break is close to that of anodal break (2.23 vs 2.15 mm); in this case, boundary-induced drift cooperates with the filament tension, and hastens ring collapse.

As noted earlier, this is a very generalized approximation to the phenomena of anodal and cathodal break excitation in cardiac tissue. In this study, we have used a chemical oscillatory system rather than an ionic active membrane model due to the BZ reaction's well-defined behavior. We have imposed symmetry constraints for the sake of simplicity; a more extensive study in the absence of such constraints, especially without the reflecting boundary condition, would merit future investigation. Bound spiral pairs may be obtained for singly diffusive systems (of which cardiac models are a subset) [26], in contrast to the doubly diffusive BZ model in which spiral pairs tend to be repulsive in the presence of any symmetry breaking between the spirals [27]. Also, we have ignored such structural properties that would present in cardiac tissue such as anisotropic propagation, fiber rotation with depth, and curvature of the extended tissue as would be found in ventricular myocardium. In actuality, the motion of spiral waves on the cardiac epicardium tends to be characterized by meander and drift upon the surface, possibly arising from the above intrinsic properties of the tissue [28]. However, this present study provides some clues as to the behavior of the filament in the relative absence of additional perturbations.

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