Single Spin and Chiral Glass Transition in Vector Spin Glasses in Three Dimensions

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Results of Monte Carlo simulations of *XY* and Heisenberg spin glass models in three dimensions are presented. A finite-size scaling analysis of the correlation length of the spins and chiralities of both models shows that there is a single, finite-temperature transition at which both spins and chiralities order.

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There is now a general consensus that a spin glass transition occurs in three-dimensional Ising spin glasses with short range interactions at finite temperature $T_{\rm SG}$. The most convincing work is that of Ballesteros *et al.* [1], who performed a finite-size scaling analysis of the correlation length, ξ_L, in samples of different sizes L. Data for the dimensionless ratio ξ_L/L is found to intersect cleanly at $T = T_{\rm SG}$, as expected at a second order phase transition.

The situation is much less clear, however, for vector spin glasses. Early work on *XY* [2,3] and Heisenberg [2,4] models indicated a zero temperature transition, or possibly a transition at a very low but nonzero temperature. However, following the earlier work of Villain [5], which emphasized the role of "chiralities" (Ising-like variables which describe the handedness of the noncollinear spin structures), Kawamura and Tanemura [6] proposed a chirality transition at $T = T_{\rm CG} (> 0)$, even though the spin glass transition temperature, $T_{\rm SG}$, is assumed to be zero. This scenario requires that spins and chiralities decouple at long length scales. Kawamura and collaborators have given numerical evidence for this scenario both for *XY* [7] and Heisenberg [8,9] models.

However, the absence of a spin glass transition in vector spin glass models has been challenged. For the *XY* case, Maucourt and Grempel [10] and, subsequently, Akino and Kosterlitz [11] found evidence for a possible finite $T_{\rm SG}$ from zero temperature domain wall calculations. Furthermore, by studying the dynamics of the *XY* spin glass in the phase representation, Granato [12] found that the "current-voltage" characteristics exhibited scaling behavior which he interpreted as a transition in the spins as well as the chiralities. For the Heisenberg model, Matsubara *et al.* [13,14], and Nakamura and Endoh [15] have argued that the spins and chiralities order at the same low but finite temperature.

Since the most successful approach to demonstrate a finite-temperature transition in the Ising case has been the scaling of the correlation length [1], it seems useful to perform a similar analysis for vector spin glass models. Furthermore, one can calculate the correlation length for both the spins and chirality, and so perform the *same* analysis for *both* types of ordering. Here, we present results of these calculations for the *XY* and Heisenberg models. For both models, we find a single transition for both spins and chiralities at low but finite temperature.

We take the standard Edwards-Anderson spin glass model,

$$\mathcal{H} = -\sum_{\langle i,j\rangle} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j, \tag{1}$$

where the $\mathbf{S}_i$ are n-component vectors of unit length at the sites of a simple cubic lattice, and the J_{ij} are nearest neighbor interactions with zero mean and standard deviation unity. We consider both the *XY* model ($n = 2$), and the Heisenberg model ($n = 3$). Periodic boundary conditions are applied on lattices with $N = L^3$ spins.

The spin glass order parameter generalized to wave vector $\mathbf{k}$, $q^{\mu\nu}(\mathbf{k})$, is defined to be

$$q^{\mu\nu}(\mathbf{k}) = \frac{1}{N} \sum_i S_i^{\mu(1)} S_i^{\nu(2)} e^{i\mathbf{k}\cdot\mathbf{R}_i}, \tag{2}$$

where μ and ν are spin components, and "(1)" and "(2)" denote two identical copies of the system with the same interactions. From this we determine the wave vector dependent spin glass susceptibility $\chi_{\rm SG}(\mathbf{k})$ by

$$\chi_{\rm SG}(\mathbf{k}) = N \sum_{\mu,\nu} [\langle |q^{\mu\nu}(\mathbf{k})|^2\rangle]_{\rm av}, \tag{3}$$

where $\langle\cdots\rangle$ denotes a thermal average and $[\cdots]_{\rm av}$ denotes an average over disorder. The spin glass correlation length is then determined [1,16] from

$$\xi_L = \frac{1}{2\sin(k_{\min}/2)} \left(\frac{\chi_{\rm SG}(0)}{\chi_{\rm SG}(\mathbf{k}_{\min})} - 1 \right)^{1/2}, \tag{4}$$

where $\mathbf{k}_{\min} = (2\pi/L)(1, 0, 0)$.

For the *XY* model, the chirality of a square is [7]

$$\kappa_i^{\mu} = \frac{1}{2\sqrt{2}} \sum_{\langle l,m\rangle}{}' \operatorname{sgn}(J_{lm}) \sin(\theta_l - \theta_m), \tag{5}$$

where θ_l is the angle characterizing the direction of spin $\mathbf{S}_l$, and the sum is over the four bonds around the elementary plaquette perpendicular to the μ axis and whose "bottom left" corner is site i. The chiral glass susceptibility is then given by

$$\chi_{\rm CG}^{\mu}(\mathbf{k}) = N[\langle |q_c^{\mu}(\mathbf{k})|^2\rangle]_{\rm av}, \tag{6}$$

where the chiral overlap $q_c^{\mu}(\mathbf{k})$ is given by

$$q_c^{\mu}(\mathbf{k}) = \frac{1}{N}\sum_i \kappa_i^{\mu(1)}\kappa_i^{\mu(2)} e^{i\mathbf{k}\cdot\mathbf{R}_i}. \tag{7}$$

We define the chiral correlation lengths $\xi_{c,L}^{\mu}$ by

$$\xi_{c,L}^{\mu} = \frac{1}{2\sin(k_{\min}/2)}\left(\frac{\chi_{\rm CG}(0)}{\chi_{\rm CG}^{\mu}(\mathbf{k}_{\min})} - 1\right)^{1/2}, \tag{8}$$

in which $\chi_{\rm CG}(\mathbf{k}=0)$ is independent of μ. Note that $\xi_{c,L}^{\mu}$ will, in general, be different for $\hat{\mu}$ along $\mathbf{k}_{\min}$ (the $\hat{x}$ direction) and perpendicular to $\mathbf{k}$. We denote these two lengths by $\xi_{c,L}^{\parallel}$ and $\xi_{c,L}^{\perp}$, respectively.

For the Heisenberg spin glass, Kawamura [8] defines the local chirality in terms of three spins on a line as follows:

$$\kappa_i^{\mu} = \mathbf{S}_{i+\hat{\mu}}\cdot\mathbf{S}_i\times\mathbf{S}_{i-\hat{\mu}}. \tag{9}$$

The chiral glass susceptibilities and correlation lengths are then given in terms of the κ_i^{μ} by Eqs. (6)–(8), as for the *XY* model.

We use parallel tempering [17,18] Monte Carlo to go down to the low temperatures that are needed, and study sizes from $L = 4$ to 12. To test for equilibration [19], we require that the following relation [20],

$$[q_l - q_s]_{\rm av} = \frac{2}{z}T[U]_{\rm av}, \tag{10}$$

valid for a Gaussian bond distribution, is satisfied. Here U is the energy per spin, $q_l = (1/N_b)\sum_{\langle i,j\rangle}\langle\mathbf{S}_i\cdot\mathbf{S}_j\rangle^2$ is the "link overlap," $q_s = (1/N_b)\sum_{\langle i,j\rangle}\langle(\mathbf{S}_i\cdot\mathbf{S}_j)^2\rangle$ where $N_b = (z/2)N$ is the number of nearest neighbor bonds, and $z\ (=6$ here) is the lattice coordination number. We averaged over 1000 samples, except for the following cases: *XY*, $L = 12$, 601 samples; Heisenberg: $L = 8$, 436 samples, and $L = 12$, 331 samples. The number of sweeps that each set of spins performed varied from 6000 for the small sizes to 300 000 for $L = 12$.

Since ξ_L/L is dimensionless, it has the finite-size scaling form

$$\frac{\xi_L}{L} = \tilde{X}[L^{1/\nu}(T - T_{\rm SG})], \tag{11}$$

where ν is the correlation length exponent. Note that there is no power of L multiplying the scaling function $\tilde{X}$, as there would be for a quantity with dimensions. There are analogous expressions for the chiral correlation lengths. From Eq. (11), it follows that the data for ξ_L/L different sizes come together at $T = T_{\rm SG}$. In addition, they are also expected to splay out again on the low-T side if there is spin glass order below $T_{\rm SG}$.

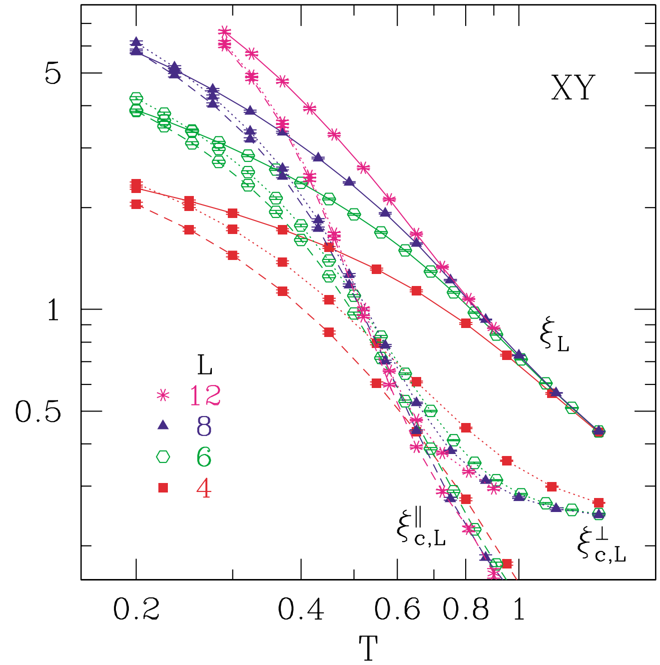

FIG. 1 (color online). A plot of the spin (ξ_L) and chiral ($\xi_{c,L}^{\parallel}$ and $\xi_{c,L}^{\perp}$) correlation lengths for different sizes and temperatures for the *XY* spin glass. The solid lines connect data for ξ_L, the dotted lines connect $\xi_{c,L}^{\perp}$, and the dashed lines connect $\xi_{c,L}^{\parallel}$.

Next we discuss the results, starting with the *XY* spin glass. Data for the various correlation lengths are shown in Fig. 1. One sees that the chiral correlation lengths are

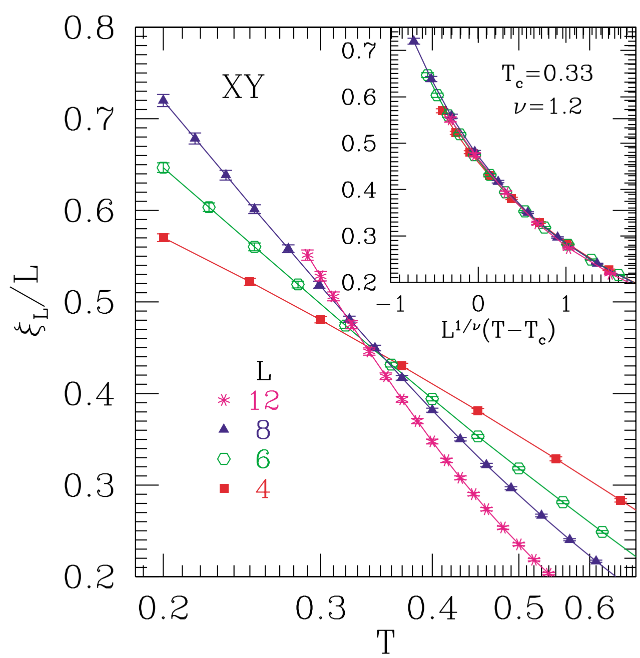

FIG. 2 (color online). Plot of the spin glass correlation length ξ_L divided by L for the *XY* spin glass. The data intersects at $T \simeq 0.34$, implying that there is a spin glass transition at this temperature. The inset shows a scaling plot according to Eq. (11) with $T_{\rm SG} = 0.33$ and $\nu = 1.2$.

smaller than the spin glass correlation length at the higher temperatures, but increase faster on lowering T, such that, for a given L, all the lengths become comparable at the lowest temperatures simulated. Furthermore, the two chiral correlation lengths (parallel and perpendicular to $\mathbf{k}$) become indistinguishable at lower T and larger sizes, as one would expect.

The data for ξ_L/L, shown in Fig. 2, intersects at a well-defined temperature $\simeq 0.34$ and splay out at lower temperatures, which, according to Eq. (11), implies a transition at this temperature. We find

$$T_{\mathrm{SG}} = 0.34 \pm 0.02 \qquad XY \text{ spin glass.} \tag{12}$$

Note the $L = 12$ data intersects at somewhat lower T, implying that corrections to finite-size scaling may still be significant for this range of sizes. Figure 2 provides compelling evidence, in our view, that there is finite spin glass transition temperature in a three-dimensional XY spin glass, in contrast to the claim in most of the literature. The inset of Fig. 2 shows the data collapses well according to Eq. (11) with $\nu = 1.2 \pm 0.2$. In this paper, error bars do not include systematic effects which are hard to estimate. Given the lower intersection point of the $L = 12$ data, it is possible that T_c could be lower than that estimated here, in which case the value of ν would be increased perhaps to the Ising value [1] 2.15 ± 0.15.

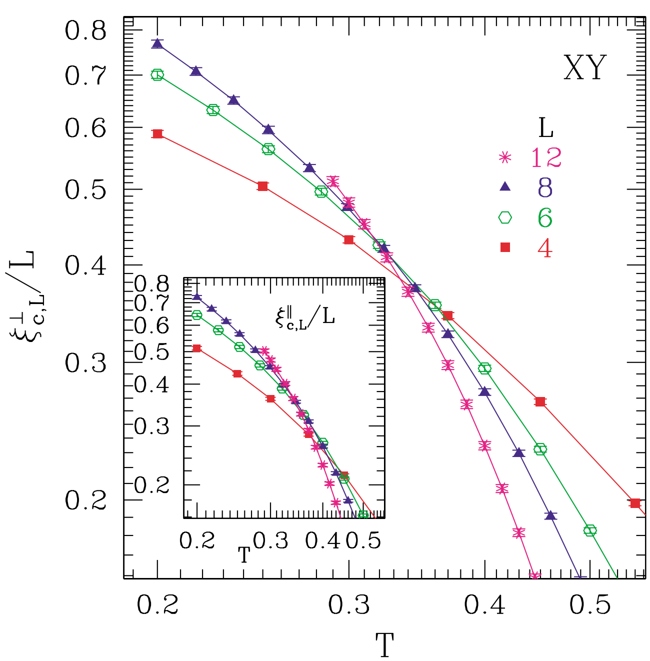

FIG. 3 (color online). The main figure shows data for the perpendicular chiral correlation length $\xi^{\perp}_{c,L}$ divided by L, for sizes $4 \leq L \leq 12$ for the XY spin glass. There are intersections at about the same temperature as that found for the spins in Fig. 2, but evidently with some corrections to scaling. The inset shows analogous data for the parallel chiral correlation length, $\xi^{\parallel}_{c,L}$.

Data for the chiral correlation lengths for the XY model are shown in Fig. 3. Though the intersections are not quite as clean, they occur at about the same temperature as found above for the spin glass correlation length. For both spin and chiral correlations, they occur at a slightly lower T for the $L = 12$ data. Altogether, the evidence is good that there is a finite chiral glass transition at (or very close to) T_{SG}. Collapsing the data, we find the chiral correlation length exponent is $\nu_c = 1.3 \pm 0.3$, which is compatible with our estimate for the spin correlation exponent ν. Note that if the spins order then the chiralities *must* also order, assuming a noncollinear state, and so $T_{\mathrm{CG}} \geq T_{\mathrm{SG}}$.

We are not aware of any estimates of transition temperatures for the XY spin glass with Gaussian couplings, though for the $\pm J$ model Kawamura and Li [7] find $T_{\mathrm{CG}} = 0.39 \pm 0.03$, somewhat higher than ours. Since, for the Ising spin glass, T_c is somewhat higher for the $\pm J$ model than the Gaussian model, our estimate of T_c is probably compatible with Kawamura and Li's. However, we emphasize that, in contrast to them, we find simultaneous ordering of the spins and chiralities.

Next, we go on to our results for the Heisenberg spin glass. As for the XY model, the spin glass correlation length is larger at higher temperatures but the chiral correlation length grows faster and is comparable to the spin correlation length at the lowest temperatures. Figure 4 shows data for ξ_L/L, which intersect at a common temperature indicating a finite spin glass transition temperature T_{SG} which we estimate to be

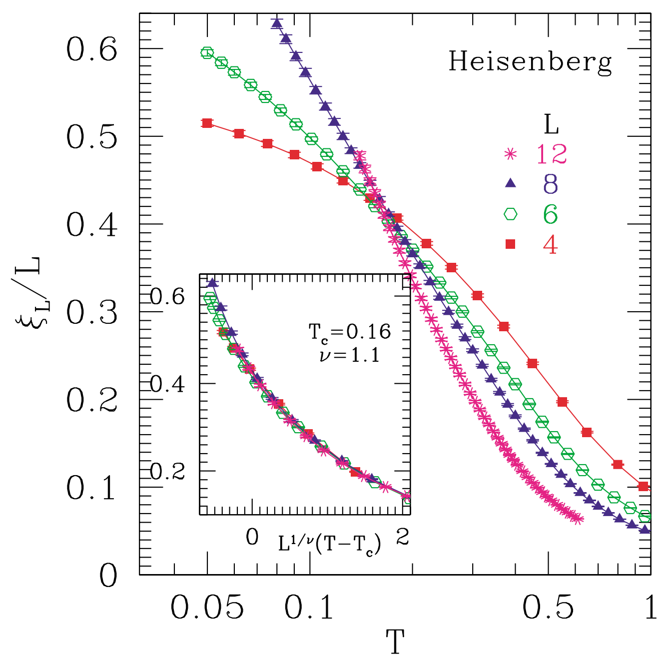

FIG. 4 (color online). Data for the spin glass correlation length ξ_L, divided by L for the Heisenberg spin glass. The intersections imply that $T_{\mathrm{SG}} \simeq 0.16$. The inset shows a scaling plot according to Eq. (11) with $T_{\mathrm{SG}} = 0.16$ and $\nu = 1.1$.

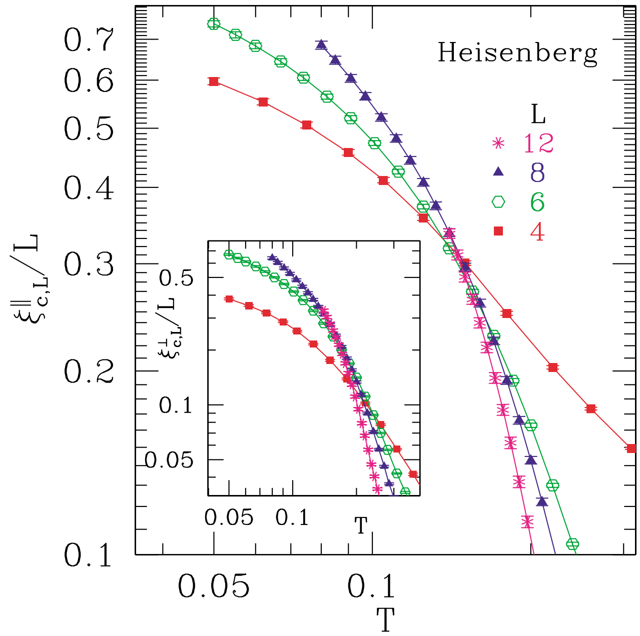

FIG. 5 (color online). The main figure shows data for the parallel chiral correlation length $\xi_{c,L}^{\perp}$ divided by L for the Heisenberg spin glass. There is an intersection at $T \simeq 0.14$, close to that found for the spin glass correlation length ξ_L in Fig. 4. The inset shows analogous data for the perpendicular chiral correlation length, The data intersect but not as cleanly as for $\xi_{c,L}^{\perp}$ or ξ_L.

$$T_{SG} = 0.16 \pm 0.02 \qquad \text{Heisenberg spin glass.} \qquad (13)$$

The inset of Fig. 4 shows the data collapses quite well according to Eq. (11) with $\nu = 1.1 \pm 0.2$.

Figure 5 shows that the data for the chiral correlation lengths indicate a transition at about the same value, with the intersections being cleaner for the parallel than for the perpendicular correlation length. Our estimate for the chiral correlation length exponent is $\nu_c = 1.3 \pm 0.3$. As for the XY spin glass, T_c may be somewhat lower than that found here, which would lead to a larger value of ν.

Our value for the transition temperature agrees well with values of T_{CG} given by Kawamura [8], 0.157 ± 0.01, and Hukushima and Kawamura [9], 0.160 ± 0.005, though, unlike those authors, we claim that the spins, as well as the chiralities, order at this temperature. For the $\pm J$ model, Endoh *et al.* [14] find a spin glass transition at $T_{SG} = 0.19 \pm 0.02$ while Nakamura and Endoh [15] find both chiral and spin glass ordering for $T \simeq 0.21$.

In conclusion, by analyzing data for the spin and chiral correlation lengths, we have argued that there is a single phase transition, at which both spins and chiralities order, in the XY and Heisenberg spin glasses in three dimensions. In our view, the evidence for a spin glass transition is at least as strong as that for a chiral glass transition. Spin-chirality decoupling does not seem to occur. The present work used quite modest work station facilities, so it would be feasible to extend these results to larger sizes by a major computational effort.

Why has this simple picture of a single finite-temperature transition in vector spin glasses in three dimensions not been generally accepted before? One reason is that T_{SG} is very low compared with the mean field value T_{SG}^{MF} ($\simeq 1.22$ for XY and 0.82 for Heisenberg). Until the advent of parallel tempering [17], it was difficult to reach the actual T_{SG} in simulations. Furthermore, the commonly used Binder ratio does not seem to be very useful [21] for vector spins, and $T = 0$ domain wall calculations are plagued by uncertainties over the optimal choice of boundary conditions [11]. We argue that, as for the Ising spin glass [1], finite-size scaling of the correlation length is the optimal technique to use.

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