# Centrality Scaling of the $p_T$ Distribution of Pions

Rudolph C. Hwa[1] and C. B. Yang[1,2]

[1]Institute of Theoretical Science and Department of Physics, University of Oregon, Eugene, Oregon 97403-5203, USA

[2]Institute of Particle Physics, Hua-Zhong Normal University, Wuhan 430079, China

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From the preliminary data of PHENIX on the centrality dependence of the $\pi^0$ spectrum in $p_T$ at midrapidity in heavy-ion collisions, we show that a scaling behavior exists that is independent of the centrality. It is then shown that $\langle p_T \rangle$ degrades with increasing $N_{\rm part}$ exponentially with a decay constant that can be quantified. A scaling distribution in terms of an intuitive scaling variable is derived that is analogous to the Koba-Nielsen-Olesen scaling. No theoretical models are used in any part of this phenomenological analysis.

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In a recent paper [1], we reported on the finding of a scaling property of the $p_T$ distribution of pions produced in heavy-ion collisions that is independent of the collision energy. Here we present an extension of that scaling property to include centrality variations and show that a Koba-Nielsen-Olesen(KNO)-type [2] scaling behavior exists over the entire range of $p_T$ measured. The investigation is primarily a phenomenological analysis with no assumptions about the hard and soft collisions, nor about the parton energy losses.

Recently, a scaling behavior of the transverse-mass spectrum has been reported in [3]. That work was motivated by color glass condensate and the saturation of the gluon density in nuclear collisions. Our investigation has no theoretical motivation other than the search for the simplest form that can represent the data. The dynamical origin of the $p_T$ distribution is complicated. At low $p_T$, the statistical model seems to work well, as does the hydrodynamical description up to $p_T = 3$ GeV/$c$ [4]. At high $p_T$, hard parton scattering will create jets, which can lose energy due to multiple scatterings of partons in the dense medium [5]. A universal description of the hadron distribution over all $p_T$ is nonexistent, if not meaningless from the point of view of the sectarian nature of the dynamical theories that claim validities in different domains. However, if a universal scaling behavior can be found phenomenologically, it can serve as a common goal for different dynamical approaches to aim at.

From the preliminary PHENIX data of $\pi^0$ produced in Au + Au collisions at the relativistic heavy-ion collider [6], we have the pion distribution, $(2\pi p_T)^{-1} dN_\pi/d\eta dp_T$, at midrapidity for $\sqrt{s} = 200$ GeV and for a wide range of centrality that has nine bins from 0%–10% to 80%–92%. To unify the nine distributions, it is necessary to define a scaling variable $z$. First, we use the number of participants, $N_{\rm part}$, to quantify centrality; those numbers for different bins are taken from [7], which agree well with those given by PHENIX [8]. Next, we define, for fixed $\sqrt{s}$ (at 200 GeV),

$$z = p_T/K(N), \tag{1}$$

where $K$ depends on $N_{\rm part}$, for which we use the abbreviated notation $N = N_{\rm part}$ hereafter. For every centrality bin, we vary $K$ by plotting the data of $(2\pi p_T)^{-1} dN_\pi/d\eta dp_T$ in terms of $z$ and adjusting the normalization so that all data points lie on a universal curve. That is, we define

$$\Phi(z) = A(N)\, K^2(N)\, \frac{1}{2\pi p_T} \frac{dN_\pi}{d\eta dp_T}, \tag{2}$$

and find $A(N)$ and $K(N)$ such that $\Phi(z)$ has no explicit dependence on $N$. That turns out to be possible, as evidenced by Fig. 1. For clarity, we show only five bins of centrality in that figure. It is a remarkable property of the centrality dependence of the pion spectra that such a universal scaling distribution exists.

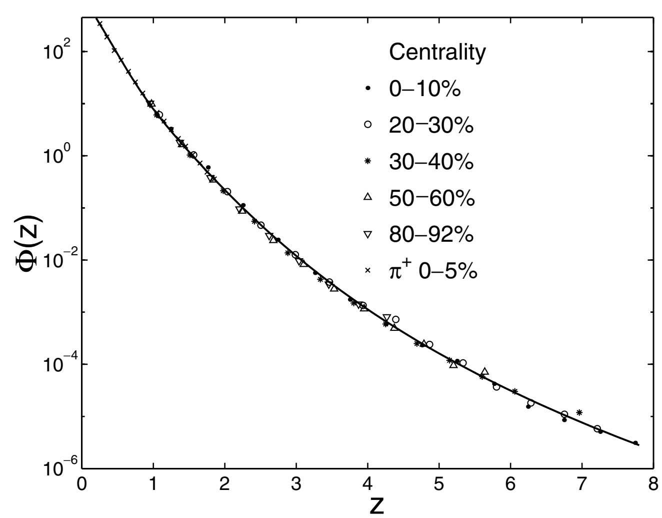


FIG. 1. Scaling distribution $\Phi(z)$ showing the coalescence of five centrality bins of the preliminary data from PHENIX on $\pi^0$ production in Au + Au collisions at $\sqrt{s} = 200$ GeV [6]. The points labeled by $\times$ are for $\pi^+$. The solid line is a fit parametrized by Eq. (5).

  

The values of $K(N)$ used to obtain the scaling behavior are shown in Fig. 2(a) in units of GeV$/c$. The dependence of $K(N)$ on $N$ can be well fitted by

$$K(N) = 1.226\text{–}6.36 \times 10^{-4} N, \tag{3}$$

such that $K(N_{\max}) = 1$ at $N = N_{\max} = 350$. The effects of the degradation of parton momenta are hidden in this formula. Any change of the overall scale of $K(N)$ is trivial and does not affect the scaling behavior that we have found. Although the normalization factor $A(N)$ does not have a simple dependence on $N$, it turns out to depend simply on the number of binary collisions $N_c$. The values of $A(N_c)$ needed to achieve the scaling $\Phi(z)$ are shown in Fig. 2(b) in a log-log plot. They can be fitted by

$$A(N_c) = 530 N_c^{-0.9}. \tag{4}$$

From the tables listed in Refs. [7,8], $N_c$ and $N$ can be related by $N_c = 0.44 N^{1.33}$. Note that the normalization of $\Phi(z)$ is set by the most central bin by choosing $A(N) = 1$ at $N = N_{\max}$. If $A(N_c)$ were to behave as $N_c^{-1}$, it would suggest that the average multiplicity of pions at midrapidity is proportional to $N_c$, which is a variable that measures the number of hard collisions. Thus, the factor $N_c^{-0.9}$ in Eq. (4) is an indication that the centrality dependence of the midrapidity multiplicity scales as $N_c^{0.9}$ from the $pp$ collisions, revealing the effect of suppression of $p_T$ in the nuclear medium.

To fit the scaling curve, the $\pi^0$ data are insufficient to give us guidance in the small $z$ region, since they do not extend below $p_T = 1$ GeV$/c$. For $0 < p_T < 1$ GeV$/c$, we use the $\pi^+$ data of PHENIX for 0%–5% centrality [9] shown in Fig. 1. The combined $\pi^0$ and $\pi^+$ data can be well fitted by

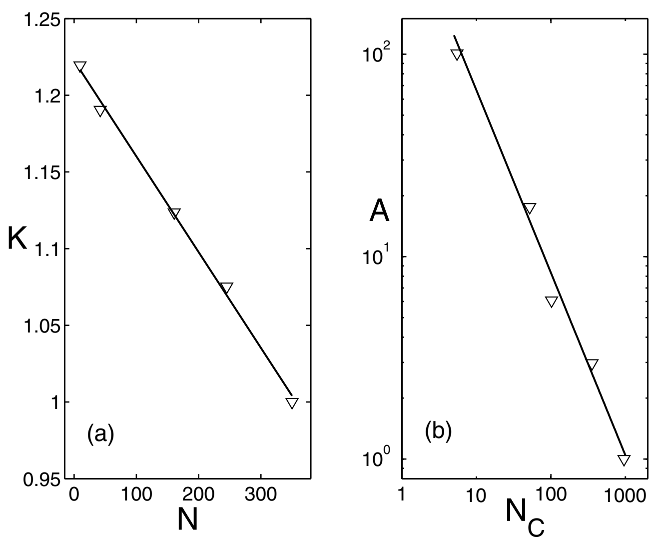


FIG. 2. (a) Scale factor $K(N)$ in units of GeV$/c$. Solid line is a fit by Eq. (3). (b) Power-law behavior of the normalization factor $A(N_c)$. Solid line is a fit by Eq. (4).

$$\Phi(z) = 1200\,(z^2 + 2)^{-4.8}(1 + 25e^{-4.5\,z}), \tag{5}$$

which is shown by the solid line in Fig. 1. We can check its normalization by evaluating the integral,

$$I = \int_0^{10} dz\, z\Phi(z) = 46.2 = \frac{A(N)}{2\pi}\frac{dN_{\pi^0}}{d\eta}. \tag{6}$$

For $N = 200$, for instance, this gives $dN_{\pi^0}/d\eta = 149$, which compares satisfactorily to $dN_{\text{ch}}/d\eta/(0.5N) = 3.2$ at the same $N$ [8]. Since the $\pi^\pm$ data do not extend into the $p_T > 2$ GeV$/c$ region, we do not consider them for centrality analysis here.

The exponential term in Eq. (5) is mainly to fit the low-$z$ data that contain thermodynamical effects. At high $z$, $\Phi(z)$ behaves as a power law that represents the effects of hard collisions and jet quenching. For all $z$, $\Phi(z)$ is a succinct summary of all dynamical effects for all centralities.

In terms of $\Phi(z)$, it is now possible to have an analytic expression of the inclusive distribution of the pions in $p_T$ at midrapidity. For convenience, we shall write it in terms of the momentum fraction $x$:

$$x = p_T/K_0, \tag{7}$$

where $K_0$ is a fixed scale, beyond which no physics of interest need be of concern here. We set $K_0 = 10$ GeV$/c$ for now, although increasing it later, if necessary, is a simple matter. In view of Eq. (1), we thus have

$$z = x\Lambda(N), \qquad \Lambda(N) = K_0/K(N). \tag{8}$$

Converting $(2\pi z)^{-1} dN_\pi/dz$ to the $x$ variable, we define the corresponding pion distribution to be

$$H(x, N) = A^{-1}(N)\,\Phi(x, N), \tag{9}$$

where $A(N) = A[N_c(N)]$. To see the evolution of the pion distribution with increasing $N$, it is more enlightening to study the normalized distribution, defined by

$$P(x, N) = H(x, N) \Big/ \int_0^1 dx\, xH(x, N), \tag{10}$$

where the upper limit of integration is set to 1 on the assumption that the contribution from $p_T > K_0$ is insignificant. Thus, $P(x, N)$ is the probability distribution of producing a $\pi^0$ at $x$, for which the differential phase space is $xdx$ due to the 2D nature of $\vec{p}_T$.

In Fig. 3, we show $P(x, N)$ for four values of $N$. Note how $P(x, N)$ decreases at high $x$ but increases at low $x$, when $N$ is increased. That is the behavior we expect when high-$p_T$ partons are suppressed, giving rise to low-$p_T$ partons. The crossover occurs at around $x = 0.06$, corresponding to $p_T = 0.6$ GeV$/c$.

Such an evolution of the $x$ distribution is reminiscent of the evolution of the parton distribution in $\ln Q^2$ in perturbative QCD. Although no precise relationship between the two has been established, it is known that in the latter

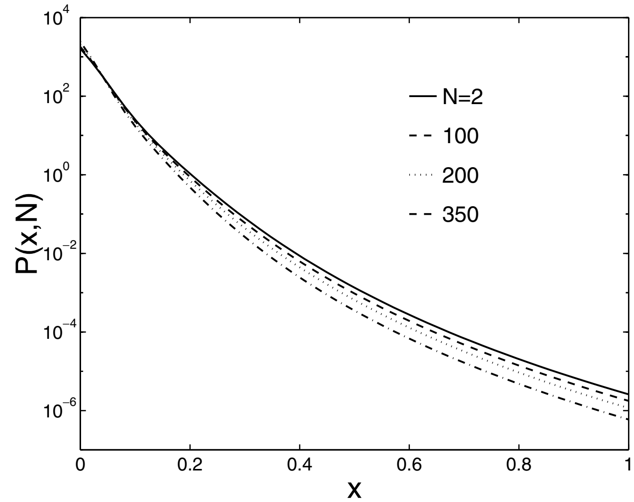


FIG. 3. Probability distribution $P(x, N)$ for four values of $N$.

case the analytical description is simpler in terms of the moments. Thus, let us define the moments

$$P_n(N) = \int_0^1 dx\, x^{n+1} P(x, N). \tag{11}$$

From Eqs. (5), (9), and (10), we can calculate the $N$ dependencies of $P_n(N)$, which are shown in Fig. 4 for $n = 1$ to 5. Evidently, $\ln P_n(N)$ can be well approximated by linear dependence on $N$, i.e.,

$$\ln P_n(N) = a_n - b_n N. \tag{12}$$

The slope parameters $b_n$ are shown in the inset of the same figure. The dependence of $b_n$ on $n$ is also linear. Thus, we may rewrite Eq. (12) as

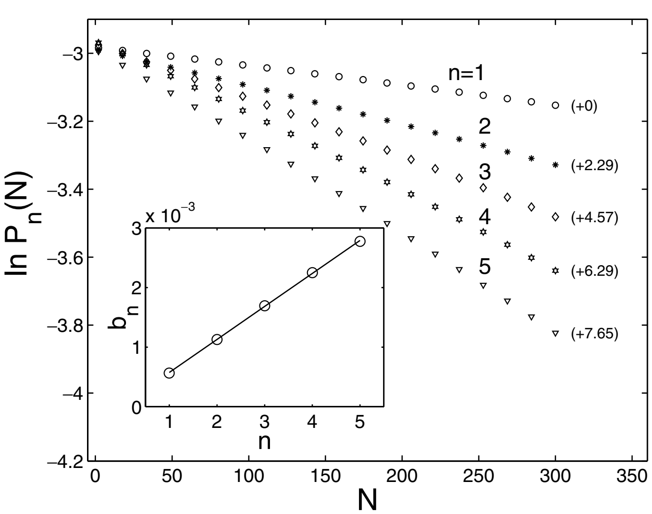


FIG. 4. $N$ dependence of the moments, $P_n(N)$, whose log values are raised by the quantities in the parentheses. The inset shows the slopes $b_n$, the line being a linear fit.

$$\frac{d}{dN} \ln P_n(N) = -\lambda n, \qquad \lambda = 5.542 \times 10^{-4}. \tag{13}$$

This is a very economical way of describing the degradation property of the pion distribution in terms of one basic parameter $\lambda$.

A physical interpretation can readily be given for $\lambda$ when we consider $n = 1$, for which $P_1(N) = \langle x \rangle_N$, the average $x$ at $N$. From Eq. (13), we obtain

$$\langle x \rangle_N = \langle x \rangle_{N_0} \exp[-\lambda (N - N_0)], \tag{14}$$

which exhibits explicitly the exponential decrease of $\langle x \rangle_N$ with increasing $N$, a behavior that solidifies our physical notion of what the dense medium does to $\langle p_T \rangle$. For $N_0 = 2$ and $N = N_{\max}$, we get

$$\langle x \rangle_{N_{\max}} / \langle x \rangle_2 = 0.825, \tag{15}$$

which gives a quantitative measure of the degree of degradation. From Eq. (13) it is easy also to show that

$$\frac{d}{dN} \frac{\langle x^n \rangle_N}{\langle x \rangle_N^n} = 0, \tag{16}$$

where $\langle x^n \rangle_N = P_n(N)$. Hence, the normalized moments of $P(x, N)$ are invariant in $N$. That is a clue to another invariant form of the distribution.

Before we examine the implications of that clue, we note that the properties of $P_n(N)$ displayed in Fig. 4 and described by Eq. (12) cannot be expected to be valid for arbitrarily large $n$, since the definition of $P_n(N)$ in Eq. (11) puts more weight on the high end of $x$ when $n$ is large. Our cutoff at $x = 1$, corresponding to $p_T = K_0 = 10$ GeV$/c$, is based partly on the lack of data at higher $p_T$ and partly on the recognition that the contribution from $p_T > K_0$ is unimportant when $n$ is not too large. To test the validity of our procedure, we have carried out the analysis for $K_0 = 20$ GeV$/c$, using the same $\Phi(z)$, and found that Eq. (13) remains to be an excellent approximation of the $n$ dependence shown in Fig. 4, and that the value of $\lambda$ is larger by just 2%, which is less than the experimental errors. Thus, we claim that our analysis is stable under variations of $K_0$ so long as we consider $K_0 \geq 10$ GeV$/c$ and $n \leq 5$.

The invariance of the normalized moments in Eq. (16) suggests that we should consider yet another scaling variable,

$$u = x / \langle x \rangle_N = p_T / \langle p_T \rangle_N, \tag{17}$$

for any fixed $N$. Let us now define

$$\Psi(u, N) = \langle x \rangle_N^2 P(x, N), \tag{18}$$

whose moments are defined by

$$\Psi_n(N) = \int_0^{\langle x \rangle_N^{-1}} du\, u^{n+1} \Psi(u, N). \tag{19}$$

Transforming this integral to an integration over $x$, we find that

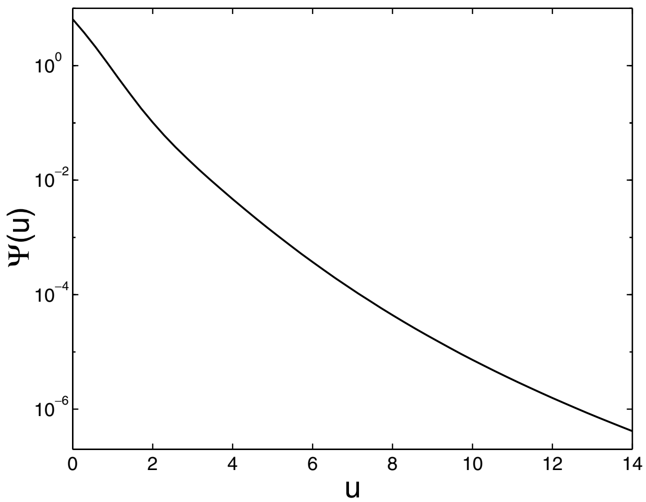


FIG. 5. Scaling distribution $\Psi(u)$.

$$\Psi_n(N) = \langle x\rangle_N^{-n} P_n(N). \tag{20}$$

It then follows from Eq. (16), that

$$d\,\Psi_n(N)/dN = 0. \tag{21}$$

Hence, $\Psi_n(N)$ is independent of $N$ and we have a scaling function $\Psi_n$, which in turn implies that $\Psi(u)$ is also independent of $N$. Indeed, from Eqs. (9) and (10) we see that (18) can be reexpressed as

$$\Psi(u) = \Phi[z(u)] \Big/ \int du\, u\Phi[z(u)], \tag{22}$$

where, by virtue of Eqs. (8) and (17),

$$z(u) = \langle x\rangle_N \Lambda(N) u. \tag{23}$$

Although $\langle x\rangle_N \Lambda(N)$ may appear to depend on $N$, it actually is a constant:

$$\gamma = \langle x\rangle_N \Lambda(N) = \langle z\rangle = \frac{\int dz\, z^2 \Phi(z)}{\int dz\, z\Phi(z)} = 0.414. \tag{24}$$

Thus, using $z = \gamma u$ in Eq. (22), we obtain the scaling function $\Psi(u)$:

$$\Psi(u) = 2.1 \times 10^4 (u^2 + 11.65)^{-4.8} \times [1 + 25 \exp(-1.864u)]. \tag{25}$$

In Fig. 5, we show $\Psi(u)$ whose shape evidently differs from that of $\Phi(z)$ at low $u$ because of the difference in the power-law violating constants. $\Psi(u)$ is a universal form of all the $P(x, N)$ shown in Fig. 3.

The scaling property of $\Psi(u)$ is analogous to the KNO scaling of the multiplicity distributions $P_m(s)$ in hadronic collisions for $\sqrt{s} < 200$ GeV [2]. It was found that in terms of the scaling variable $z = m/\langle m\rangle$, where $m$ is the multiplicity, the KNO function $\psi(z) = \langle m\rangle P_m(s)$ is independent of $s$. Here, we find that $\Psi(u)$, defined in Eq. (18), is independent of centrality when the scaling variable, $u = p_T/\langle p_T\rangle$, is used. As it is with KNO scaling, we have

$$\langle u^n\rangle = \int du\, u^{n+1} \Psi(u) = 1, \qquad \text{for } n = 0, 1. \tag{26}$$

The higher moments are what characterize the scaling function, and perhaps scaling violation at some point. A generalization of the scaling property to include variations in both energy and centrality is considered in Ref. [10].

It should be emphasized that no theoretical models have been used in any part of this investigation. The discovery of a scaling behavior over the whole $p_T$ range that has been measured offers a simple form of the $p_T$ distribution for dynamical models to describe at any centrality and energy. The scaling distribution provides us with not only a simple picture of the complex $p_T$ problem, but also a way of quantifying the degree of degradation of the transverse momentum in the dense medium. More importantly, the mere existence of the scaling behavior presents a phenomenological obstacle to the realization of the theoretical expectation that deconfinement results in an anomalous dependence of the $p_T$ distribution on centrality.

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