Self-Similar Optical Wave Collapse: Observation of the Townes Profile

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Analyses of many different types of nonlinear wave equations indicate that a collapsing wave will transform into a universal blowup profile regardless of its initial shape; that is, the amplitude of the wave increases as the spatial extent decreases in a self-similar fashion. We show experimentally that the spatial profile of a collapsing optical wave evolves to a specific circularly symmetric shape, known as the Townes profile, for elliptically shaped or randomly distorted input beams. These results represent the first experimental confirmation of this universal collapsing behavior and provide deeper insight into the high-power filamentation of femtosecond laser pulses in air.

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Nonlinear wave collapse is an intrinsic feature of many areas of physics, including optics [1,2], hydrodynamics [3], and plasma physics [4]. In optics, propagation of a laser beam through a transparent medium is governed by the two-dimensional (2D) nonlinear Schrödinger equation (NLSE), and wave collapse occurs when nonlinear focusing due to the intensity-dependent refractive index overcomes linear diffraction. This self-focusing process occurs when the power in the beam is above a certain critical power P_{cr} [5], which ultimately provides the upper limit on the power that can be transmitted through the medium without significant changes to the spatial profile of the beam or damage to the material. A remarkable prediction [6–8] of the 2D NLSE, which is shared with other nonlinear equations that exhibit collapse [9–12], is that the spatial profile of the beam evolves into a universal, self-similar, circularly symmetric shape regardless of the shape of the initial profile. For the NLSE, the associated profile is known as the *Townes profile,* as shown in Fig. 1. In this Letter, we present detailed experimental observations of the universal, self-similar nature of optical wave collapse due to Kerr nonlinearity self-focusing for randomly distorted and elliptical beams. This work investigates the propagation dynamics before nonlinearities such as plasma coupling and saturation of the nonlinearity stabilize the field resulting in either defocusing or the formation of spatial solitons [13] and filamentation.

For our studies, we express the NLSE as

$$2ik\frac{\partial A}{\partial z} = -\nabla_T^2 A - \frac{2k^2 n_2}{n_0}|A|^2 A, \qquad (1)$$

where $A = A(x, y, z)$ represents the amplitude of the electric field, $k = 2n_0\pi/\lambda$ is the propagation wave vector, λ is the vacuum wavelength, n_0 is the linear index of refraction, n_2 is the nonlinear index of refraction, $\nabla_T^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the transverse Laplacian, and z is the propagation direction. The first term on the right-hand side represents diffraction, whereas the second term accounts for the intensity-dependent refractive index $n = n_0 + n_2 I$, where $I = (n_0 c/2\pi)|A|^2$ is the intensity and gives rise to self-focusing. The Townes profile corresponds to the unstable solution in which nonlinear focusing and diffraction precisely balance each other, and the beam maintains a constant profile. These waveguide solutions have the form $A(z, r) = (n_0/2k^2 n_2)^{1/2} e^{i\alpha^2 z/2k} \times R_\alpha(r)$, where α is an arbitrary positive constant, $r = (x^2 + y^2)^{1/2}$, $R_\alpha(r) = \alpha R(\alpha r)$, and $R(r)$ satisfies the ordinary differential equation (ODE) [14]

$$\frac{d^2R(r)}{dr^2} + \frac{1}{r}\frac{dR(r)}{dr} - R(r) + R(r)^3 = 0, \qquad (2)$$

subject to the boundary conditions $\frac{dR(r)}{dr}|_{r=0} = 0$ and $\lim_{r\to\infty} R(r) = 0$. Although no analytical expression exists for the Townes solution in terms of elementary functions, the profile can be numerically computed, as shown in Fig. 1(d). These waveguide solutions are unstable to small perturbations from this profile resulting in catastrophic collapse of the beam or in an expanding, diffracting beam. Our numerical simulations have shown that if the input profile is perturbed from the Townes profile such that the beam collapses, the on-axis portion of the beam approaches the profile of a focusing Townes solution given by

$$|A| \sim L(z)^{-1} R[r/L(z)], \qquad (3)$$

where $L(z)$ goes to zero at the blowup point [8]. More generally, even if the beam is not initially circularly symmetric [7,15] or is randomly shaped [16], the collapsing portion of the beam becomes circularly symmetric. Figures 1(a)–1(c) show the numerical simulation for a collapsing beam with an incident power of $3.5P_{cr}$ and an initial elliptical profile $A(x, y) \propto \exp[-(x^2/8w_y^2 + y^2/2w_y^2)]$, where x and y are distances along the two Cartesian axes and w_y is the width of the beam in the y

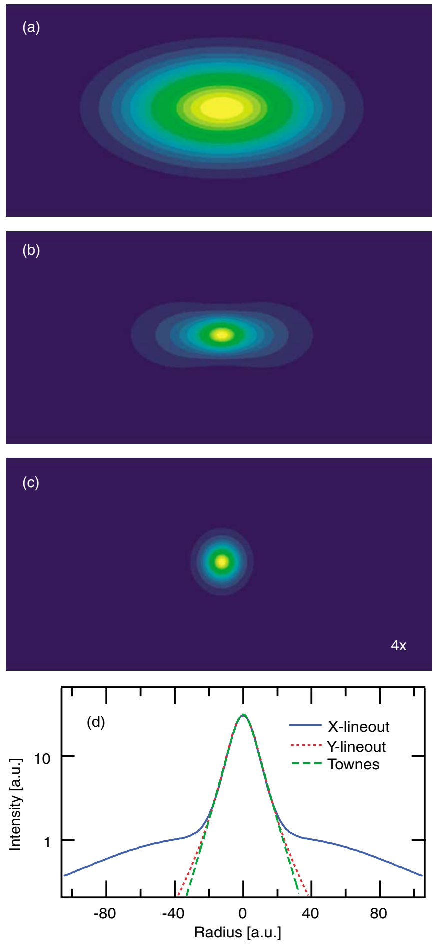

FIG. 1 (color online). The propagation of an elliptically shaped input beam (a) is simulated. As the beam propagates (b), it self-focuses, and a circularly symmetric Townes profile is formed on axis (c) as the beam collapses. Part (c) has been magnified four times to show more detail. Lineouts (d) along both axes through the center of the collapsing beam show that the beam attains the numerically calculated Townes profile and is circularly symmetric on axis.

direction. Although the initial shape deviates substantially from the Townes profile, the central portion of the beam evolves into a collapsing circularly symmetric shape that coincides with the Townes solution. A crucial property of the Townes profile is that the power contained within this collapsing portion of the beam is always precisely equal to the critical power for self-focusing [15,17]. Thus, only a fraction of the beam undergoes collapse, as can be seen in Fig. 1(d) where the wings of the collapsing elliptical beam do not overlap the Townes profile and remain elliptically shaped.

For our experimental studies on the formation of the collapsing Townes profile, we used a beam from an amplified Ti:sapphire laser system which produces 50-fs pulses with a central wavelength of 800 nm. The beam was telescoped to a diameter of ~0.5 mm and allowed to pass through a 30-cm block of BK7 glass as shown in Fig. 2. After the beam was transmitted through the sample, either it impinged on a piece of frosted glass and was imaged by a charge-coupled device (CCD), or it was incident on a thin slit and was imaged by a high dynamic range linear diode array. A computer controlled shutter placed in the beam path and an energy meter sampling a reflection from a glass substrate allowed for the single-shot characterization of the self-focusing dynamics.

Figure 3(a) shows the evolution of the beam profile at the output face of the sample as the energy of the pulse is increased. Although the shape of the beam from the amplifier was approximately Gaussian, substructure was present on the input profile, as indicated by the dotted line in Fig. 3(b) for the case in which the beam was propagated at low power through the sample and imaged with the linear diode array. As the power was increased [solid lines in Figs. 3(b) and 3(c)], a smooth symmetric on-axis component abruptly emerged for powers near the critical power for self-focusing. The Townes profile was numerically calculated by solving the ODE in Eq. (2) and was fitted [dashed lines in Figs. 3(b) and 3(c)] to the experimentally observed peaks by scaling the amplitude and radial width of the solution. The agreement between the intensity of the low- and high-power beams [see Fig. 3(b)] as a function of position in the wings (the low-power lineout has been scaled to account for the change in power) also demonstrates the property of partial-beam collapse inherent to the NLSE where the wave outside the collapsing core undergoes linear diffraction. This observation confirms the theoretical prediction that whole beam collapse is not physically realizable [15,18].

Equation (3) predicts that the peak intensity I_T and the width w_T of the fitted Townes profiles should be related by the power law $w_T \propto I_T^{-0.5}$ if the collapse is self-similar. The inset in Fig. 3(a) shows a plot of the fitted widths versus peak intensity and exhibits an experimentally observed power law of $w_T \propto I_T^{-0.6}$. We believe the slight discrepancy between the experimentally observed power law and the theoretically predicted value of -0.5 is due to the necessity of using a slit. At high intensities, the location and size of the collapsing core were observed to change on a shot-to-shot basis due to power fluctuations in the laser. However, when the beam moves perpendicular to the slit, the peak intensity decreases to a much larger extent than the width, leading to a saturating effect as observed in our experimentally calculated value of -0.6. This movement of the beam also makes it impossible to compare the power in the central portion of the beam to the predicted value of 1.8 MW for BK7 glass [19] since it is not possible to eliminate the noncollapsing wings with a small aperture if the beam is randomly changing location and size. However, since the scaling relation between the width and the peak intensity is obeyed, our results confirm that the power within the collapsing core is constant.

We also considered the case of a randomly distorted circular input beam, as shown in Fig. 4(a), which was

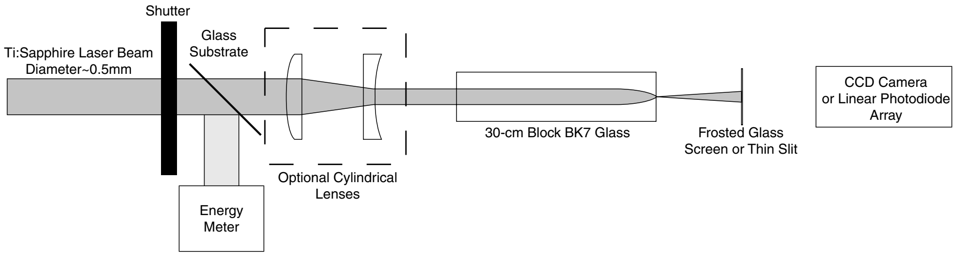

FIG. 2. Experimental setup. An amplified Ti:sapphire laser beam of 50-fs pulses propagates through a BK7 sample and undergoes self-focusing. The power is adjusted such that the beam is collapsing at the output surface. The output beam is imaged by a CCD camera. Optionally, a pair of cylindrical lenses is inserted into the beam to study the collapse dynamics with an elliptically shaped beam. A computer controlled shutter allows for single-shot measurements.

produced by inserting a roughened microscope slide into the path of the input beam. Once again, we observed that as the power was increased from the linear regime until just below the threshold for collapse within the glass sample, the beam radius decreased (an indication of self-focusing), and the profile becomes smooth and nearly perfectly circularly symmetric [see Fig. 4(b)].

We conclude from our numerical simulations that the farther the initial beam shape deviates from the ideal Townes profile, the closer the beam must propagate to the point of complete collapse in order to evolve to the Townes profile. Experimentally, the electric-field amplitude cannot reach arbitrarily large intensities, and higher-order nonlinear effects such as the formation of plasma [20], saturation of the nonlinearity, and in the case of pulsed lasers, temporal effects such as dispersion [21] and self-steepening [22,23] play an increasingly important role, which can result in the arrest of pulse collapse. For the case of an ultrashort laser pulse, an optical "shock" forms at the back edge of the pulse as it undergoes collapse, leading to the production of an extremely broad spectrum of radiation known as super-continuum generation (SCG) [20,24]. As discussed above, we found that for nearly circular input beams, Townes-profile formation could be observed without the occurrence of SCG. However, for the case of elliptical input beams observation of the Towne profile required propagation very near to the collapse point, at which point SCG occurs.

Observation of SCG is thus a signature that the pulse has undergone collapse, and we used this fact to deduce the spatial shape of the beam near the collapse point for the case of an elliptically shaped input beam. Such a beam was produced by using a suitable pair of cylindrical lenses that were inserted before the sample to reduce the beam width in one dimension with respect to the other. We found that regardless of the input-beam

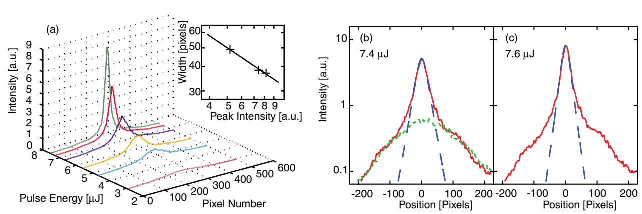

FIG. 3 (color online). Experimental observation of the Townes profile. (a) Lineouts along one axis are taken through the center of the beam at several different input powers. At sufficiently high pulse energies, a strong on-axis component [solid line in (b) and (c)] is observed which matches the Townes profile [dashed line in (b) and (c)] predicted by numerical simulations. The low-power profile [dotted line in (b)] has been scaled by a constant factor to account for different input powers and demonstrates the partial collapse of the beam. The width and peak intensity of the fitted Townes profiles [(a) inset] obey the scaling relation $w_T \propto I_T^{-0.6}$.

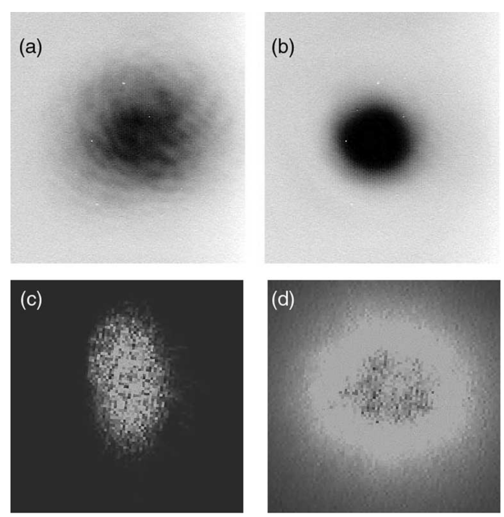

FIG. 4. Images of the transmitted beam profiles for randomly distorted and elliptical input beams. At low powers, (a) a randomly distorted beam and (c) an elliptically shaped input beam with a 3:1 ratio of major to minor axes are passed through the BK7-glass sample. As the power is increased, (b) the distorted beam self-focuses, becoming smaller in diameter as well as smooth and symmetric. For the elliptically shaped beam, (d) circularly symmetric supercontinuum generation is formed. A color-glass filter was used to eliminate the light at the fundamental wavelength and image the yellow component of the SCG. Analysis of the captured image demonstrates that the beam is circular to better than one part in thirty. The speckle in (c) and (d) is a result of the frosted glass used to image the output beam.

ellipticity SCG was always emitted in a circularly symmetric profile. Figures 4(c) and 4(d) display images of the input beam and output SCG for the case in which the diameter of the input beam in the horizontal direction is one-third of the diameter in the vertical direction. Similar behavior was also observed for the highly elliptical case in which a single cylindrical lens was used to focus the beam within the sample.

In various other physical systems, the process of wave collapse has been analytically studied and numerically modeled, but is difficult to study experimentally. For many equations, such as the scalar Zakharov system [9], the generalized Korteweg–de Vries equation [10], the nonlinear heat equation [11], the Davey-Stewartson equations [12], and the Gross-Pitaevskii equation [which governs Bose-Einstein condensates (BEC) in a magnetic trap] [25], the predicted behavior is similar to that shown here with the NLSE in that an initially irregular or asymmetric profile collapses with a smooth and symmetric universal shape. Our experimental results in this nonlinear optical system provide clear evidence that evolution of a wave to the collapse point causes the beam to reshape itself into a characteristic circularly symmetric profile and represent what we believe are the first experimental observations of universal collapse behavior for any physical system.

Our observation that the universal collapse profile is a scaled Townsian, rather than other blowup profiles which have been sometimes used in NLSE analysis (e.g., Gaussian, sech, etc.), is important because the Townes profile is the only one that represents a complete balance between the nonlinearity and diffraction. This implies that the collapse process is highly sensitive to the effects of additional small physical mechanisms (nonparaxiality, vectorial effects, dispersion) which are neglected in the NLSE model since the two dominant terms (diffraction and nonlinearity) precisely cancel each other. In fact, these mechanisms can arrest the collapse even while being small compared with diffraction and the nonlinearity [8]. These observations will also lead to an improved understanding of the propagation of high-power femtosecond laser pulses in air [26–28] in which the beam is observed to propagate over very long distances without undergoing any apparent diffraction. It has been suggested and observed that at small transverse length scales, the spatial profile of the beam breaks up into multiple filaments and goes through repeated focusing/defocusing cycles [29,30]. Since the input power is significantly higher than $P_{\rm cr}$ for air, we conjecture that the shape of each individual filament corresponds to a Townes profile. This is supported by the experimental results presented in Ref. [26] in which the energy contained within each filament was measured to be approximately equal. Lastly, a 3D analog of the Townes solution exists, and thus self-similar 3D collapse also will occur for ultrashort laser pulses in the anomalous-dispersion regime [31] and for BEC's [32].

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