

Dynamical Symmetry Breaking of Extended Gauge Symmetries

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We construct asymptotically free gauge theories exhibiting dynamical breaking of the left-right gauge group $G_{LR} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, and its extension to the Pati-Salam gauge group $G_{422} = SU(4)_{PS} \times SU(2)_L \times SU(2)_R$. The models incorporate technicolor for electroweak breaking, and extended technicolor for the breaking of G_{LR} and G_{422} and the generation of fermion masses. They include a seesaw mechanism for neutrino masses, without a grand unified theory (GUT) scale. These models explain why G_{LR} and G_{422} break to $SU(3)_c \times SU(2)_L \times U(1)_Y$, and why this takes place at a scale ($\sim 10^3$ TeV) large compared to the electroweak scale, but much smaller than a GUT scale.

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The standard-model (SM) gauge group $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$ has provided a successful description of both strong and electroweak interactions. Although the standard model itself predicts zero neutrino masses, its fermion content can be augmented to accommodate the current evidence for neutrino masses and lepton mixing. But the origin of the electroweak symmetry breaking (EWSB) is still not understood. It might occur via the Higgs mechanism, as in the SM. An alternative is dynamical symmetry breaking (DSB) of the electroweak symmetry, driven by a strongly coupled, asymptotically free, vectorial gauge interaction associated with an unbroken gauge symmetry, denoted generically as technicolor (TC) [1–8].

There has also long been interest in models with gauge groups larger than G_{SM} . One such model has the gauge group [9]

$$G_{LR} = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \quad (1)$$

in which the fermions of each generation transform as $(3, 2, 1)_{1/3,L}$, $(3, 1, 2)_{1/3,R}$, $(1, 2, 1)_{-1,L}$, and $(1, 1, 2)_{-1,R}$. The gauge couplings are defined via the covariant derivative $D_\mu = \partial_\mu - ig_3 \mathbf{T}_c \cdot \mathbf{A}_{c,\mu} - ig_{2L} \mathbf{T}_L \cdot \mathbf{A}_{L,\mu} - ig_{2R} \mathbf{T}_R \cdot \mathbf{A}_{R,\mu} - i(g_U/2)(B-L)\mathbf{U}_\mu$. In this model the electric charge is given by the elegant relation $Q = T_{3L} + T_{3R} + (B-L)/2$, where B and L denote baryon and (total) lepton number. G_{LR} would break at a scale Λ_{LR} well above the electroweak scale.

The model based on G_{LR} may be further embedded in a model with gauge group [10]

$$G_{422} = SU(4)_{PS} \times SU(2)_L \times SU(2)_R. \quad (2)$$

This model provides a higher degree of unification since it combines $U(1)_{B-L}$ and $SU(3)_c$ (in a maximal subgroup) in the Pati-Salam group $SU(4)_{PS}$ and hence relates g_U and g_3 . Denoting the generators of $SU(4)_{PS}$ as $T_{PS,i}$, $1 \leq i \leq 15$, with $T_{PS,15} = (2\sqrt{6})^{-1} \text{diag}(1, 1, 1, -3)$ and setting $U_\mu = A_{PS,15,\mu}$, one has $(B-L)/2 = \sqrt{2/3} T_{PS,15}$, and hence $(g_U/g_{PS})^2 = 3/2$ at Λ_{PS} , where Λ_{PS} is the breaking

scale of the G_{422} group. This model also has the appeal that it quantizes electric charge, since $Q = T_{3L} + T_{3R} + \sqrt{2/3} T_{PS,15} = T_{3L} + T_{3R} + (1/6) \text{diag}(1, 1, 1, -3)$.

The conventional approach to the gauge symmetry breaking of these models employs elementary Higgs fields and arranges for a hierarchy of breaking scales by making the vacuum expectation values (VEV's) of the Higgs that break G_{LR} or G_{422} to G_{SM} much larger than the Higgs VEV's that break $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ [9,11]. This hierarchy is necessitated by the experimental lower limits on the masses of a possible W_R or Z' [12]. An interesting question is whether one can construct asymptotically free gauge theories containing the group G_{LR} and/or G_{422} that exhibit dynamical breaking of all the gauge symmetries other than $SU(3)_c$ and $U(1)_{em}$, that naturally explain the hierarchy of breaking scales, and that yield requisite light neutrino masses. In this Letter, we present the first such models.

Technicolor itself cannot provide a mechanism for all the breaking, because it is too weak at the scale Λ_{LR} or Λ_{PS} and because the technifermion condensate $\langle \bar{F}F \rangle = \langle \bar{F}_L F_R \rangle + \langle \bar{F}_R F_L \rangle$ would break both $SU(2)_L$ and $SU(2)_R$ at the same scale [to the diagonal (vector) group $SU(2)_V$]. Of course, to explain quark and lepton mass generation and incorporate the three families, technicolor has to be enlarged to an extended technicolor (ETC) theory [3]. Our models are ETC-type theories, with the breaking of G_{LR} and G_{422} to G_{SM} being driven by the same interactions that break the ETC group and generate quark and lepton masses.

Taking the technicolor gauge group to be $SU(N_{TC})$, the technifermions comprise an additional family, viz., $Q_L = \binom{U}{D}_L$, $L_L = \binom{N}{E}_L$, U_R , D_R , N_R , E_R transforming according to the fundamental representation of $SU(N_{TC})$ and the usual representations of G_{SM} (where color and TC indices are suppressed). Vacuum alignment considerations yield the desired color- and charge-conserving TC condensates [14]. To satisfy constraints from flavor-changing neutral-current processes, the ETC vector bosons that can

mediate generation-changing transitions must have large masses. We envision that these arise from self-breaking of an ETC gauge symmetry, which requires that ETC be a strongly coupled, chiral gauge theory. The self-breaking occurs in stages, for example, at the three stages $\Lambda_1 \sim 10^3$ TeV, $\Lambda_2 \sim 50$ TeV, and $\Lambda_3 \sim 3$ TeV, corresponding to the three standard-model fermion generations. Hence $N_{\text{ETC}} = N_{\text{TC}} + 3$.

A particularly attractive choice for the technicolor group, used in the models studied here, is $\text{SU}(2)_{\text{TC}}$, which thus entails $N_{\text{ETC}} = 5$. With $N_f = 8$ vectorially coupled technifermions in the fundamental representation, studies suggest that this $\text{SU}(2)_{\text{TC}}$ theory could have an (approximate) infrared fixed point (IRFP) in the confining phase with spontaneous chiral symmetry breaking [15,16]. This approximate IRFP produces a slowly running (“walking”) TC gauge coupling, which can yield realistically large quark and charged lepton masses [5]. The choice $N_{\text{TC}} = 2$ and the walking can strongly reduce TC contributions to the S parameter [8,17]. Further ingredients may be needed to account for the top-quark mass.

In Ref. [18], we studied the generation of neutrino masses in an ETC model of this sort and showed that light neutrino masses and lepton mixing can be produced via a seesaw without any superheavy mass scales. Here we extend this model to the groups G_{LR} and G_{422} .

We recall that Λ_{TC} is determined by using the relation $m_W^2 = (g^2/4)(N_c f_Q^2 + f_L^2) \simeq (g^2/4)(N_c + 1)f_F^2$, where for our purposes we take $f_L \simeq f_Q \equiv f_F$. This gives $f_F \simeq 130$ GeV. In QCD, $f_\pi = 93$ MeV and $\Lambda_{\text{QCD}} \sim 170$ MeV, so that $\Lambda_{\text{QCD}}/f_\pi \sim 2$; using this as a guide to technicolor, we infer $\Lambda_{\text{TC}} \sim 260$ GeV. The induced fermion masses in the i th generation are given by $m_{f_i} \sim g_{\text{ETC}}^2 \eta_i N_{\text{TC}} \Lambda_{\text{TC}}^3 / (4\pi^2 M_i^2)$, where $M_i \sim g_{\text{ETC}} \Lambda_i$ is the mass of the ETC gauge bosons that gain mass at scale Λ_i and g_{ETC} is the running ETC gauge coupling evaluated at this scale. The quantity η_i is a possible enhancement factor incorporating walking, for which $\eta_i \sim \Lambda_i/f_F$ [5,19].

We first consider the standard-model extension based on G_{LR} . Our model for the DSB utilizes the gauge group

$$G = \text{SU}(5)_{\text{ETC}} \times \text{SU}(2)_{\text{HC}} \times G_{LR} \quad (3)$$

where HC denotes hypercolor, a second strong gauge interaction which, together with ETC, triggers the requisite sequential breaking pattern. The fermion content of this model is listed below; the numbers indicate the representations under $\text{SU}(5)_{\text{ETC}} \times \text{SU}(2)_{\text{HC}} \times \text{SU}(3)_c \times \text{SU}(2)_L \times \text{SU}(2)_R$ and the subscript gives $B - L$:

$$\begin{aligned} (5, 1, 3, 2, 1)_{1/3,L}, & \quad (5, 1, 3, 1, 2)_{1/3,R}, \\ (5, 1, 1, 2, 1)_{-1,L}, & \quad (5, 1, 1, 1, 2)_{-1,R}, \\ (\bar{5}, 1, 1, 1, 1)_{0,R}, & \quad (\bar{10}, 1, 1, 1, 1)_{0,R}, \\ (10, 2, 1, 1, 1)_{0,R}. & \end{aligned} \quad (4)$$

Thus the fermions include a vectorlike set of quarks and

techniquarks in the representations $(5, 1, 3, 2, 1)_{1/3,L}$, $(5, 1, 3, 1, 2)_{1/3,R}$, and leptons and technileptons in $(5, 1, 1, 2, 1)_{-1,L}$, $(5, 1, 1, 1, 2)_{-1,R}$, together with a set of G_{LR} -singlet fermions in $(\bar{5}, 1, 1, 1, 1)_{0,R}$, $(\bar{10}, 1, 1, 1, 1)_{0,R}$, and $(10, 2, 1, 1, 1)_{0,R}$ [20]. The leptons and technileptons are denoted $L_{\chi}^{i,p}$, where $\chi = L, R$, $1 \leq i \leq 5$, and $p = 1, 2$. The G_{LR} singlets are denoted, respectively, $\mathcal{N}_{i,R}$, $\psi_{ij,R}$, and $\zeta_{ij,R}^{\alpha}$, where $1 \leq i, j \leq 5$ are ETC indices and α, β are $\text{SU}(2)_{\text{HC}}$ indices. The models with G_{LR} and G_{422} share several features with the ETC model in [7].

The $\text{SU}(5)_{\text{ETC}}$ theory is an anomaly-free, chiral gauge theory and, like the TC and HC theories, is asymptotically free. There are no bilinear fermion operators invariant under G , and hence there are no bare fermion mass terms. The $\text{SU}(2)_{\text{HC}}$ and $\text{SU}(2)_{\text{TC}}$ subsectors of $\text{SU}(5)_{\text{ETC}}$ are vectorial.

To analyze the stages of symmetry breaking, we identify plausible preferred condensation channels using a generalized-most-attractive-channel (GMAC) approach that takes account of one or more strong gauge interactions at each breaking scale, as well as the energy cost involved in producing gauge boson masses when gauge symmetries are broken. In this framework, an approximate measure of the attractiveness of a channel $R_1 \times R_2 \rightarrow R_{\text{cond}}$ is $\Delta C_2 = C_2(R_1) + C_2(R_2) - C_2(R_{\text{cond}})$, where R_j denotes the representation under a relevant gauge interaction and $C_2(R)$ is the quadratic Casimir.

As the energy decreases from some high value, the $\text{SU}(5)_{\text{ETC}}$ and $\text{SU}(2)_{\text{HC}}$ couplings increase. We envision that at $E \sim \Lambda_{LR} \gtrsim 10^3$ TeV, α_{ETC} is sufficiently strong [16] to produce condensation in the channel

$$(5, 1, 1, 1, 2)_{-1,R} \times (\bar{5}, 1, 1, 1, 1)_{0,R} \rightarrow (1, 1, 1, 1, 2)_{-1} \quad (5)$$

with $\Delta C_2 = 24/5$, breaking G_{LR} to $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$. The associated condensate is $\langle L_R^{i,pT} C \mathcal{N}_{i,R} \rangle$, where $1 \leq i \leq 5$ is an $\text{SU}(5)_{\text{ETC}}$ index and $p \in \{1, 2\}$ is an $\text{SU}(2)_R$ index. With no loss of generality, we use the initial $\text{SU}(2)_R$ invariance to rotate the condensate to the $p = 1$ component, $L_R^{i,p=1} \equiv n_R^i$, which is electrically neutral and has weak hypercharge $Y = 0$; the condensate is thus $\langle n_R^{iT} C \mathcal{N}_{i,R} \rangle$ so that the n_R^i and $\mathcal{N}_{i,R}$ gain dynamical masses $\sim \Lambda_{LR}$.

There exists a more attractive channel than (5) in a simple MAC analysis: $(\bar{10}, 1, 1, 1, 1)_{0,R} \times (10, 2, 1, 1, 1)_{0,R} \rightarrow (1, 2, 1, 1, 1)_0$, with $\Delta C_2 = 36/5$. But with the coupling g_{HC} also large at Λ_{LR} , a sizable energy price would be incurred in this channel to generate the vector boson masses associated with the breaking of the $\text{SU}(2)_{\text{HC}}$. We assume here that this price is higher than the energy advantage due to the greater attractiveness of the channel $(\bar{10}, 1, 1, 1, 1)_{0,R} \times (10, 2, 1, 1, 1)_{0,R} \rightarrow (1, 2, 1, 1, 1)_0$ [21].

The condensation (5) generates masses

$$m_{W_R} = \frac{g_{2R}}{2} \Lambda_{LR}, \quad m_{Z'} = \frac{g_{2U}}{2} \Lambda_{LR}, \quad (6)$$

where $g_{2u} \equiv \sqrt{g_{2R}^2 + g_U^2}$, for the $W_{R,\mu}^\pm = A_{R,\mu}^\pm$ gauge

bosons and the linear combination

$$Z'_\mu = \frac{g_{2R}A_{3,R,\mu} - g_U U_\mu}{g_{2U}}. \quad (7)$$

This leaves the orthogonal combination

$$B_\mu = \frac{g_U A_{3,R,\mu} + g_{2R} U_\mu}{g_{2U}} \quad (8)$$

as the weak hypercharge $U(1)_Y$ gauge boson, which is massless at this stage. The hypercharge coupling is then

$$g' = \frac{g_{2R}g_U}{g_{2U}}. \quad (9)$$

so that, with $e^{-2} = g_{2L}^{-2} + (g')^{-2} = g_{2L}^{-2} + g_{2R}^{-2} + g_U^{-2}$, the weak mixing angle is given by

$$\sin^2 \theta_W = \left[1 + \left(\frac{g_{2L}}{g_{2R}} \right)^2 + \left(\frac{g_{2L}}{g_U} \right)^2 \right]^{-1} \quad (10)$$

at the scale Λ_{LR} . The experimental value of $\sin^2 \theta_W$ at M_Z can be accommodated naturally, for example, with all couplings in (10) of the same order (even with $g_{2R} = g_{2L}$) and with the modest renormalization group (RG) running from Λ_{LR} to M_Z .

For $E < \Lambda_{LR}$, the fermion content is

$$\begin{aligned} (5, 1, 3, 2)_{1/3,L}, & \quad (5, 1, 3, 1)_{4/3,R}, & \quad (5, 1, 3, 1)_{-2/3,R}, \\ (5, 1, 1, 2)_{-1,L}, & \quad (5, 1, 1, 1)_{-2,R}, & \\ (\bar{10}, 1, 1, 1)_{0,R}, & \quad (10, 2, 1, 1)_{0,R}, \end{aligned} \quad (11)$$

where the entries refer to $SU(5)_{ETC} \times SU(2)_{HC} \times SU(3)_c \times SU(2)_L$ and Y is a subscript. This is precisely the gauge group and fermion content of the ETC model that we analyzed in Ref. [18] with a focus on the formation of neutrino masses. We therefore summarize the subsequent stages of breaking only briefly, drawing on results of [18].

At a value $E \sim \Lambda_1 \sim 10^3$ TeV comparable to Λ_{LR} , a GMAC analysis suggests that there is condensation in the channel

$$(\bar{10}, 1, 1, 1)_{0,R} \times (\bar{10}, 1, 1, 1)_{0,R} \rightarrow (5, 1, 1, 1)_0. \quad (12)$$

Thus, $SU(5)_{ETC}$ self-breaks to $SU(4)_{ETC}$, producing masses $\sim g_{ETC} \Lambda_1$ for the nine gauge bosons in the coset $SU(5)_{ETC}/SU(4)_{ETC}$. As at Λ_{LR} , we assume that a GMAC analysis favors this channel over the $10 \times \bar{10}$ channel in which $SU(2)_{HC}$ -breaking gauge boson masses $\sim g_{HC} \Lambda_1$ would have to be formed. Although the latter channel is more attractive, a very large energy price would have to be paid for the associated vector boson mass generation for sufficiently large $\alpha_{HC} > \alpha_{ETC}$. Also, although (12) has the same ΔC_2 value ($= 24/5$) as (5), it is plausible that $\Lambda_1 \lesssim \Lambda_{LR}$, since an energy price ($\sim g_{ETC} \Lambda_1$) is incurred by the breaking of $SU(5)_{ETC}$.

The $SU(5)_{ETC} \rightarrow SU(4)_{ETC}$ breaking entails the separation of the first generation of quarks and leptons from

the components of $SU(5)_{ETC}$ fermion fields with indices $2 \leq i \leq 5$. The further ETC gauge symmetry breaking occurs in stages, leading eventually to the $SU(2)_{TC}$ subgroup of the original $SU(5)_{ETC}$ group. We have identified two plausible sequences for this breaking [7,18]. Both sequences yield a strongly coupled $SU(2)_{TC}$ gauge interaction that produces a TC condensate, breaking $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ [22].

Dirac mass terms for the neutrinos are formed dynamically, involving the left-handed neutrinos in the $(5, 1, 1, 2, 1)_{-1,L}$, but not their respective right-handed counterparts in the $(5, 1, 1, 1, 2)_{-1,R}$. Instead, the right-handed partners emerge from the $(\bar{10}, 1, 1, 1, 1)_{0,R}$ (as $\psi_{1j,R}$, $j = 2, 3$). Thus there are only two right-handed neutrinos. In a model in which L is not gauged, it is a convention how one assigns the lepton number L to the SM-singlet fields. Here, $L = 0$ for the fields that are singlets under G_{LR} or G_{422} , since they are singlets under $U(1)_{B-L}$ and have $B = 0$. Hence, the neutrino Dirac mass terms violate L by one unit. There are also larger, Majorana masses generated for the $\psi_{ij,R}$ fields themselves; the seesaw mechanism then leads to left-handed $\Delta L = 2$ Majorana neutrino bilinears [23].

We next consider the further extension of the standard-model gauge group to G_{422} . In this case, our full model is based on the gauge group $G = SU(5)_{ETC} \times SU(2)_{HC} \times G_{422}$ with fermion content

$$\begin{aligned} (5, 1, 4, 2, 1)_L, & \quad (5, 1, 4, 1, 2)_R, & \quad (\bar{5}, 1, 1, 1, 1)_R, \\ (\bar{10}, 1, 1, 1, 1)_R, & \quad (10, 2, 1, 1, 1)_R. \end{aligned} \quad (13)$$

Again, as E decreases from high values, the $SU(5)_{ETC}$ and $SU(2)_{HC}$ couplings increase. At a scale Λ_{PS} , the $SU(5)_{ETC}$ coupling will be large enough to produce condensation in the channel

$$(5, 1, 4, 1, 2)_R \times (\bar{5}, 1, 1, 1, 1)_R \rightarrow (1, 1, 4, 1, 2). \quad (14)$$

This breaks $SU(4)_{PS} \times SU(2)_R$ directly to $SU(3)_c \times U(1)_Y$. The value $\Lambda_{PS} \sim 10^3$ TeV satisfies phenomenological constraints, e.g., from the upper limit on $BR(K_L \rightarrow \mu^\pm e^\mp)$. The associated condensate is again $\langle n_R^{iT} \mathcal{N}_{i,R} \rangle$, and the n_R^i and $\mathcal{N}_{i,R}$ gain masses $\sim \Lambda_{PS}$. The results (6)–(10) apply with the condition $(g_U/g_{PS})^2 = 3/2$ at Λ_{PS} .

Further breaking at lower scales proceeds as in the G_{LR} model and as described in Ref. [18]. Dirac mass terms for the neutrinos are formed from the $(5, 1, 4, 2, 1)_L$ and the $(\bar{10}, 1, 1, 1, 1)_R$, leading to the same type of seesaw as in [18] and the G_{LR} model.

The experimental value of $\sin^2 \theta_W$ can again be accommodated by (10), although this now necessarily requires $g_{2R} < g_{2L}$ at Λ_{PS} . To see this, we evolve the SM gauge couplings from $\mu = m_Z$ to the EWSB scale $\Lambda_{EW} = 2^{-3/4} G_F^{-1/2} = 174$ GeV and then from Λ_{EW} up to Λ_{PS} using $d\alpha_j/dt = -b_0 \alpha_j^2/(2\pi) + O(\alpha_j^3) + \dots$ where $t = \ln \mu$, $\alpha_1 \equiv (g')^2/(4\pi)$, and \dots denotes theoretical uncertainties associated with mass thresholds. In the interval

$\Lambda_{EW} \leq \mu \leq \Lambda_{PS}$ we include the contributions from the t quark and relevant technifermions, so that $b_0^{(3)} = 13/3$, $b_0^{(2)} = 2/3$, and $b_0^{(1)} = -10$. The initial values at m_Z are $\alpha_3(m_Z) = 0.118$, $\alpha_{em}(m_Z)^{-1} = 129$, and $(\sin^2\theta_W)_{\overline{MS}}(m_Z) = 0.231$ [13,17]. With $\Lambda_{PS} = 10^6$ GeV and the calculated values $\alpha_3 = 0.064$, $\alpha_{2L} = 0.032$, $\alpha_1 = 0.012$ at Λ_{PS} , we find $\alpha_{2R}(\Lambda_{PS}) \approx 0.013$ so that $g_{2R}/g_{2L} \approx 0.64$ at this scale.

It may be possible to allow $g_{2R} = g_{2L}$ at Λ_{PS} , and still match $(\sin^2\theta_W)_{\text{exp}}$, by further expanding the (4D) gauge theory to one with, e.g., $SU(4)_{PS} \times SU(2)^4$ as in [24] but with DSB; we are currently studying this [25].

To summarize, we have constructed the first asymptotically free models with dynamical symmetry breaking of the extended gauge groups G_{LR} and G_{422} . These models involve higher unification, and G_{422} has the appeal of quantizing electric charge. Our models naturally explain why (i) G_{LR} and G_{422} break to G_{SM} , and (ii) this breaking occurs at the scales $\Lambda_{LR}, \Lambda_{PS} \gg m_{W,Z}$. The models incorporate technicolor for EWSB, and ETC for fermion mass generation including a seesaw mechanism for the generation of realistic neutrino masses. That our seesaw mechanism for dynamically generating light neutrino masses in ETC theories can be extended to more unified strong-EW gauge groups strengthens our demonstration that realistic neutrino masses do not require the presence of a GUT scale.

A different approach appears to be needed to construct a theory with dynamical breaking of the grand unified groups $G_{GUT} = SU(5)$ or $SO(10)$. One reason is that if the ETC group is to commute with G_{GUT} , then, with the standard fermion assignments in these GUT groups, the quarks and charged leptons would not all transform in a vectorial manner under G_{ETC} , so that the usual ETC mechanism for the corresponding fermion mass generation (e.g., for u, c, t) would not apply.

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- [14] S. Weinberg, Phys. Rev. D **13**, 974 (1976); M. Peskin, Nucl. Phys. **B175**, 197 (1980); J. Preskill, *ibid.* **B177**, 21 (1981). The TC theory forms condensates $\langle \bar{F}F \rangle$, where $F = U^a, D^a, E, N$, but not, e.g., $\langle \bar{U}_a E \rangle$, $\langle \bar{D}_a E \rangle$, $\langle \bar{U}_a N \rangle$, $\langle \bar{D}_a N \rangle$, $\langle \bar{U}_a D^a \rangle$, $\langle \bar{E} N \rangle$, or, for $N_{TC} = 2$, $\langle \epsilon_{ij} F_{\chi}^{i\tau} C F_{\chi}^{j\tau} \rangle$, $\chi = L, R$. The excluded condensates would incur an energy price due to gauge boson mass generation when the (weaker) gauge symmetries are broken.
- [15] A vectorial $SU(N)$ theory with N_f massless fermions in the fundamental representation is expected to exist in a confining phase with $S_{\chi}SB$ if $N_f < N_{f, \text{cond}}$, where $N_{f, \text{cond}} \approx (2/5)N(50N^2 - 33)/(5N^2 - 3)$ and in a non-Abelian Coulomb phase if $N_{f, \text{cond}} < N_f < 11N/2$. For $N = 2$, we have $N_{f, \text{cond}} \approx 8$.
- [16] In the approximation of single-gauge-boson exchange, the critical coupling for condensation $R_1 \times R_2 \rightarrow R_{\text{cond}}$ is given by $\frac{3\alpha}{2\pi} \Delta C_2 = 1$, where $\Delta C_2 = [C_2(R_1) + C_2(R_2) - C_2(R_{\text{cond}})]$ and $C_2(R)$ is the quadratic Casimir.
- [17] We note that global electroweak fits yielding S and T are complicated by the NuTeV anomaly reported in G. Zeller *et al.*, Phys. Rev. Lett. **88**, 091802 (2002).
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- [19] Here $\eta_a = \exp[\int_{f_F}^{\Lambda_a} (d\mu/\mu) \gamma(\alpha(\mu))]$, and in walking TC theories the anomalous dimension $\gamma \approx 1$ so $\eta_a \approx \Lambda_a/f_F$.
- [20] We write SM-singlet fields as right-handed.
- [21] This problem is currently under study. The analysis is more challenging than perturbative vacuum alignment [14] since all the relevant couplings are strong.
- [22] For a different approach to DSB of G_{LR} using Nambu Jona-Lasino-type four-fermion couplings, see E. Akhmedov *et al.*, Phys. Lett. B **368**, 270 (1996); Phys. Rev. D **53**, 2752 (1996). See also remarks in B. Holdom, Phys. Rev. D **23**, 1637 (1981).
- [23] In Ref. [18], which did not use a gauged $B - L$ symmetry, we employed a different convention, assigning $L = 1$ to $\psi_{ij,R}$ so that these Dirac mass terms conserve L and the $\Delta L = 2$ violation was manifest in induced $\psi_{i,R}^T C \psi_{j,R}$ operators as well as left-handed Majorana bilinears.
- [24] P. Hung, A. Buras, and J. Bjorken, Phys. Rev. D **25**, 805 (1982).
- [25] For a higher-dimensional approach to $SU(4)_{PS} \times SU(2)^4$, see Z. Chacko, L. Hall, and M. Perelstein, hep-ph/0210149.