Experimental Verification of Anticipated and Retarded Synchronization in Chaotic Semiconductor Lasers

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Experimental observation of both anticipated and retarded synchronization is demonstrated using unidirectionally coupled semiconductor lasers with delayed optoelectronic feedback. Depending on the difference between the transmission time and the feedback delay time, the lasers fall into either the anticipated or the retarded synchronization regime, where the driven receiver laser leads or lags behind the driving transmitter laser. The two regimes are observed to have the same stability of chaos synchronization in the presence of small perturbations by noise and parameter mismatches. In both regimes the observed time shift between the synchronized chaotic waveforms is found to be equal to the difference between the transmission time and the feedback delay time.

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Chaos synchronization has been intensively investigated in various nonlinear dynamical systems [1–3] for its potential applications in chaotic communications [4,5]. Recently, a new regime of anticipated synchronization was discovered by Voss [6]. In this regime, a driven receiver system synchronizes with the future state of a driving transmitter system. Thus the receiver can anticipate the chaotic dynamics of the transmitter in real time. In Ref. [6], Voss gave analytical and numerical evidence of the occurrence of anticipated synchronization in a system described by unidirectionally coupled delay differential equations. Later on, Masoller [7] numerically investigated the rate equations of two unidirectionally coupled semiconductor lasers with optical feedback and further identified the existence of a regime of anticipated synchronization.

As both theoretical and numerical investigations have identified anticipated synchronization in nonlinear dynamical systems with delayed feedback [6–9], experimental evidence which can prove the theory of anticipated synchronization becomes very important. In Ref. [10], the authors state that they have observed anticipated synchronization in two semiconductor lasers with delayed optical feedback and bidirectional optical coupling. However, in bidirectionally coupled systems, the different roles of driving and responding of the coupled systems are not clearly defined because the outputs of the dynamical systems are mutually coupled. Heil *et al.* [11] have investigated two semiconductor lasers with bidirectional optical coupling and have demonstrated that the two lasers can take different roles of leader and laggard depending on the frequency detuning between the two lasers. Heil *et al.* [11] have further demonstrated that in such a mutually coupled laser system it is the leading laser that synchronizes its lagging counterpart, whereas the synchronized lagging laser drives the coupling-induced instabilities. Therefore, whether the observed phenomenon in Ref. [10] is anticipated synchronization or not still needs to be further investigated. Furthermore, in Ref. [10], the observed anticipation time is found not to depend on the feedback delay time, which is in disagreement with the theoretical expectation indicated by the work of Voss and Masoller [6–9]. In Ref. [12], chaos synchronization is demonstrated in two semiconductor lasers with delayed optical feedback and unidirectional optical coupling. However, in this reference, the driven receiver laser actually lags behind the driving transmitter laser. Therefore, the receiver laser is synchronized to the transmitter laser in a retarded synchronization regime.

To the best of our knowledge, experimental demonstration of anticipated synchronization which can verify the theoretical results of Voss and Masoller has not been reported in the literature. In this Letter, we report the first experimental evidence of anticipated synchronization, using unidirectionally coupled semiconductor lasers with delayed optoelectronic feedback. The experimental results are in full agreement with the theoretical expectation by Voss and Masoller [6–9].

The schematic of two unidirectionally coupled semiconductor lasers with delayed optoelectronic feedback is shown in Fig. 1. The transmitter laser has an optoelectronic feedback loop with a delay time τ. Part of the

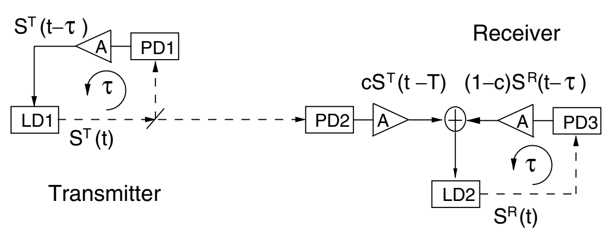

FIG. 1. Schematic of the experiment of anticipated and retarded synchronization in semiconductor lasers with delayed optoelectronic feedback. LD's: laser diodes; PD's: photodetectors; A's: amplifiers.

optical output from the transmitter laser is converted into an electronic signal by a photodetector PD1. After amplification, the electronic signal is fed back to drive the transmitter laser again. Driven by the delayed feedback signal $S^T(t - \tau)$, the transmitter laser output $S^T(t)$ becomes chaotic. Part of the transmitter laser output is then unidirectionally coupled to the receiver laser through an optical channel, a photodetector PD2, and an amplifier. In general, the receiver laser can also have its own optoelectronic feedback loop through a photodetector PD3 and an amplifier. The total driving signal to the receiver laser is $cS^T(t - T) + (1 - c)S^R(t - \tau)$, where T is the transmission time and τ is the delay time of the feedback loop in the receiver, which is the same as that in the transmitter. The factor c can be varied from 0 to 1. When $c = 1$, the receiver has an open loop. When $c < 1$, the receiver has a closed feedback loop. Under chaos synchronization, the receiver laser is forced to follow the transmitter laser as $S^R(t - \tau) = S^T(t - T)$, which is equivalent to $S^R(t) = S^T(t - T + \tau)$. Therefore, for true chaos synchronization, there is a time shift between the outputs of the transmitter and receiver lasers. When $T > \tau$, the receiver is synchronized to the transmitter with a retardation time $T - \tau$ in a retarded synchronization regime. When $T < \tau$, the receiver is synchronized to the transmitter with an anticipation time $\tau - T$ in an anticipated synchronization regime.

In the experiment, the lasers are identical InGaAsP/InP single-mode distributed-feedback lasers at 1.299 μm wavelength and are both stabilized at 21.00 °C. The photodetectors are InGaAs photodetectors (6 GHz bandwidth). The amplifiers are Avantek SSF86 amplifiers (0.4–3 GHz bandpass). The optical outputs detected by the photodetectors are observed with a Tektronix TDS 694C real-time digitizing oscilloscope with a 3 GHz bandwidth and a sampling rate up to 1×10^{10} samples/sec. Chaos synchronization is observed in both the open-loop and the closed-loop configurations of the receiver. The results obtained in the closed-loop configuration with $c = 0.8$ are demonstrated as the typical results of this system.

In the first experiment, the time difference is set to be $T - \tau = 0.0$ ns by adjusting the transmission path and the feedback loops. Figure 2(a) shows the waveforms of the chaotic outputs of the transmitter laser (upper trace) and the receiver laser (lower trace), respectively. As can be clearly seen, the two waveforms are almost identical chaotic pulsing waveforms and the time shift between them is zero, which indicates that the receiver laser is synchronized to the transmitter laser with no retardation because $T = \tau$. In the second experiment the time difference is set to be $T - \tau = +4.0$ ns by prolonging T. Figure 2(b) shows the waveforms of the transmitter and the receiver lasers, respectively, in this situation. It is clear that the receiver laser output lags behind the transmitter laser output with a retardation time of 4.0 ns. The two

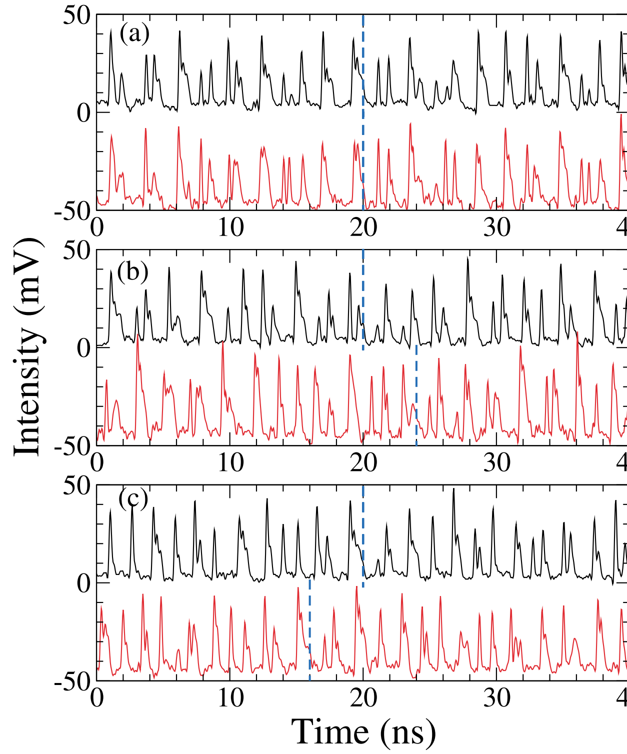

FIG. 2 (color online). Time series of the synchronized chaotic outputs from the transmitter laser (upper trace) and the receiver laser (lower trace) at $c = 0.8$. (a) Synchronization with no retardation with $T - \tau = 0.0$ ns. (b) Retarded synchronization with $T - \tau = +4.0$ ns. (c) Anticipated synchronization with $T - \tau = -4.0$ ns.

lasers are now synchronized in the retarded synchronization regime. In the third experiment the time difference is set to be $T - \tau = -4.0$ ns by shortening T. Figure 2(c) shows the synchronization traces obtained in this situation. We can clearly see that the receiver laser output now leads the transmitter laser output by an anticipation time of 4.0 ns. Thus anticipated synchronization is observed between the transmitter laser and the receiver laser.

The quality of chaos synchronization and the time shift between the chaotic outputs of the synchronized semiconductor lasers can be quantified by a shifted correlation coefficient $\rho(\Delta t)$, which is obtained by calculating the correlation coefficient between the outputs of the transmitter and the receiver when the output of the receiver is continously shifted with respect to the output of the transmitter. Figures 3(a)–3(c) show $\rho(\Delta t)$ obtained from the corresponding traces in Figs. 2(a)–2(c). In Fig. 3(a), $\rho(\Delta t)$ is shown to peak at $\Delta t_1 = 0.0$ ns, which indicates that there is no time retardation between the synchronized transmitter laser and receiver laser outputs. In Fig. 3(b), we can see that $\rho(\Delta t)$ has a sharp peak at $\Delta t_1 = +4.0$ ns, which means the receiver laser is synchronized to the transmitter laser in a retarded manner. In Fig. 3(c), the sharp peak of $\rho(\Delta t)$ appears at $\Delta t_1 = -4.0$ ns, which proves that the receiver laser is

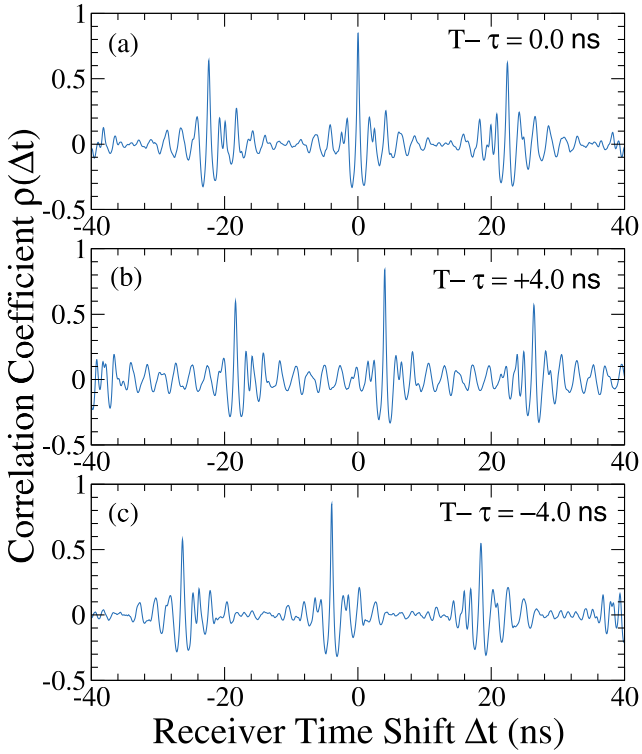

FIG. 3 (color online). Shifted correlation coefficient $\rho(\Delta t)$ calculated for the corresponding traces in Figs. 2(a)–2(c).

synchronized to the transmitter laser in an anticipated manner. In all three cases, $\rho(\Delta t)$ is found to peak at $\Delta t_1 = T - \tau$, corresponding to the change in the time difference $T - \tau$. Comparing the peak values of the shifted correlation coefficient in Figs. 3(a)–3(c), we can see that the semiconductor lasers have the same quality of chaos synchronization in the two regimes of anticipated and retarded synchronization, in spite of the influences of intrinsic noise of the semiconductor lasers and noise from the amplifiers. Further tuning the operating conditions of the receiver laser shows that the two regimes are equally stable in the presence of small parameter mismatches. With an increasing amount of parameter mismatch, the quality of chaos synchronization drops precipitately in both regimes. In Figs. 3(a)–3(c), we also see that secondary peaks appear at $\Delta t = \Delta t_1 \pm n\tau$, with n being integers. These secondary peaks reflect the fact that the chaotic waveform from the transmitter laser has some self-correlation at time intervals of $n\tau$. This self-correlation is a general characteristic of the chaotic waveforms of delay-feedback systems.

It is of great importance to investigate whether the transmitter and the receiver lasers are in separate chaotic states before they are coupled. The dynamics of the two lasers are investigated when the coupling between them is disconnected. With closed optoelectronic feedback loops in both the transmitter and the receiver lasers, respectively, the two lasers are observed to get into separate chaotic pulsing states through a similar route of quasiperiodic pulsing. Figures 4(a) and 4(c) show the chaotic pulsing traces of the transmitter and the receiver, respectively, under the same operating conditions as in Figs. 2 and 3 but with the coupling from the transmitter to the receiver disconnected. As we can see, the outputs from the transmitter and the receiver, though also measured simultaneously, are both chaotic waveforms but are not synchronized in time, because there is no coupling between them. The output from the receiver has less intensity because the strength of the feedback to the receiver is only 20% of that to the transmitter with $1 - c = 0.2$ at $c = 0.8$. Phase portraits of peak series of the chaotic pulsing states in Figs. 4(a) and 4(c) are plotted in Figs. 4(b) and 4(d), respectively. A phase portrait of peak series is obtained by extracting a peak intensity sequence $P(n)$ at the local maxima of a chaotic pulsing waveform and then plotting $P(n + 1)$ versus $P(n)$. As we can see, in both Figs. 4(b) and 4(d) the distribution of the points spreads out over a wide area, which is a typical characteristic of the phase portrait of a chaotic pulsing state [13]. Therefore, it is confirmed that the transmitter and the receiver are in separate chaotic pulsing states before coupling.

The shifted correlation coefficient between the two traces in Figs. 4(a) and 4(c) is also calculated. No correlation peak is observed, indicating that the two lasers are indeed totally unsynchronized when they are not coupled. Nevertheless, high quality of chaos synchronization is observed, as shown in Figs. 2 and 3, for the same two semiconductor lasers when the coupling between them is connected. Indeed, the receiver is found to synchronize to the transmitter, with a time shift of $T - \tau$, after sufficient coupling is applied from the transmitter to the receiver

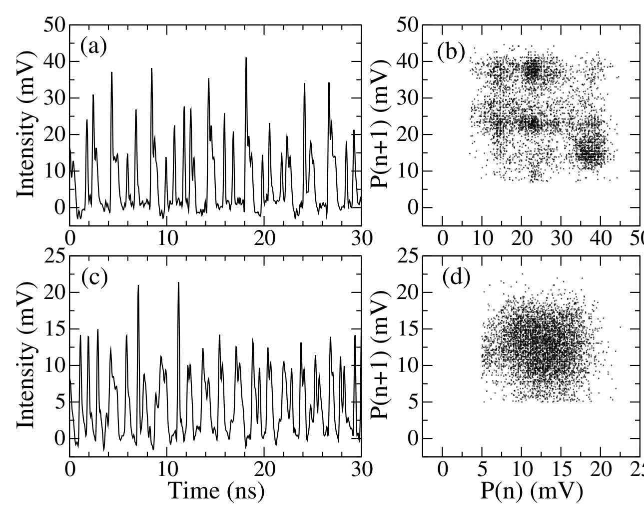

FIG. 4. The time series and phase portraits of the transmitter and the receiver, respectively, with the coupling being disconnected. (a),(b) For the transmitter; (c),(d) for the receiver.

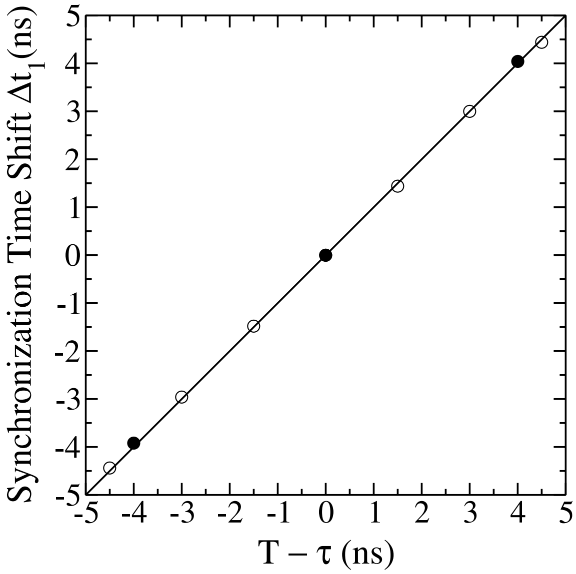

FIG. 5. The time shift of chaos synchronization Δt_1 vs the time difference of $T - \tau$. The open circles are from the open-loop configuration, and the solid circles are from the closed-loop configuration.

even though the two lasers are operated in separate chaotic states before they are coupled.

The experiments have been conducted in both the closed-loop configuration with different c factors and the open-loop configuration with $c = 1$. The phenomena of anticipated and retarded synchronization are observed in all the cases though the quality of chaos synchronization drops dramatically as the value of the c factor drops [14]. The experimentally measured relationship between the time shift of chaos synchronization Δt_1 and the time difference of $T - \tau$ is summarized in Fig. 5. The open circles are obtained from the open-loop configuration, and the solid circles are obtained from the closed-loop configuration. It is clear that all the data points fall within one straight line which has a slope of 1.0. Therefore, it is further proved that the time shift of chaos synchronization is exactly $\Delta t_1 = T - \tau$ in both regimes, where $T > \tau$ falls in the retarded synchronization regime and $T < \tau$ falls in the anticipated synchronization regime. The time shift of $T - \tau$ between the synchronized chaotic waveforms remains the same in both the open-loop and the closed-loop configurations. Therefore, the existence of two regimes of anticipated and retarded synchronization is a general phenomenon in the optoelectronic feedback system with unidirectional coupling.

In conclusion, we have experimentally demonstrated both anticipated and retarded synchronization using semiconductor lasers with delayed optoelectronic feedback. Depending on the difference between T and τ, the two lasers fall into either the anticipated or the retarded synchronization regime. The two regimes have the same stability of chaos synchronization in the presence of small perturbations of noise and parameter mismatches. The time shift of chaos synchronization is demonstrated to be $\Delta t_1 = T - \tau$ in both regimes, which matches the theoretical expectation in Refs. [6–9,15]. The time shift of $T - \tau$ is also proof of true chaos synchronization that is different from other phenomena such as modulation, amplification, injection locking, and driven oscillation, all of which have a different time shift related to only T [16,17]. Therefore, we have experimentally demonstrated that anticipated and retarded synchronization are unified phenomena under the general concept of chaos synchronization with time shift in nonlinear dynamical systems with delayed feedback.

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