Quiet-Time Statistics of Electrostatic Turbulent Fluxes from the JET Tokamak and the W7-AS and TJ-II Stellarators

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The statistics of the quiet times between successive turbulent flux bursts measured at the edge of the JET tokamak and the W7-AS and TJ-II stellarators are analyzed in search for evidence of self-organized critical behavior. The results obtained are consistent with what would be expected in the situation where the underlying plasma is indeed in a near critical state.

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Recently, it has been recently pointed out [1,2] that some intriguing experimental findings regarding the outward energy transport in magnetically confined plasmas might be understood if examined from the perspective of self-organized criticality (SOC) [3]. SOC generally appears in externally driven systems in which a large disparity exists between the time scales associated with the external drive and the system response. This is the case when an instability threshold exists. The resulting steady state exhibits many of the properties of critical points in phase transitions [4]: namely, self-similarity and criticality, although restricted to a limited range of scales, known as the *self-similar range*, because of the finite system size. In magnetically confined plasmas, instability thresholds are indeed common. The possible relevance of the SOC hypothesis is thus based on this fact. Its experimental validation is, however, complicated because a conclusive test for SOC is still lacking. However, some necessary features have been detected in electrostatic fluctuations from the edge of several devices; in particular, long-term temporal correlations [5] and self-similarity [6]. In addition, multifractal analysis has suggested the existence of a self-similar range for at least time scales longer than a few times the turbulence decorrelation time [7].

The purpose of this Letter is to report on new experimental evidence, found in one tokamak (JET [8]) and two stellarators (W7-AS [9] and TJ-II [10]), that is consistent with the SOC hypothesis. It involves the statistics of quiescent times (from now on, *quiet times*) between successive bursts of turbulent flux measured just within their edge, which are found to be in agreement with what would be expected in a randomly driven SOC system. Regarding the quiet-time statistics in SOC systems, two main points were raised in Ref. [11]. First, it is important to measure quiet times instead of waiting times [12,13] (see Fig. 1) if we intend to separate the correlations induced by the drive and the ones induced by the dynamics. Second, the distribution of the quiet times between all transport events in the system essentially reflects the statistics of the drive [14]. In the case of a randomly driven sandpile, the probability density function (pdf) of the quiet time for all events must thus follow a Poisson or exponential law. However, SOC scale invariance requires sufficiently long (large) events to be strongly correlated to each other. How long (or large) they need to be is determined by the beginning of the self-similar range. For this reason, this correlation can be made apparent by constructing the pdf of the quiet times between transport events selected according to the following criterium [11]: that their duration (or size) must lie within this self-similar range. The

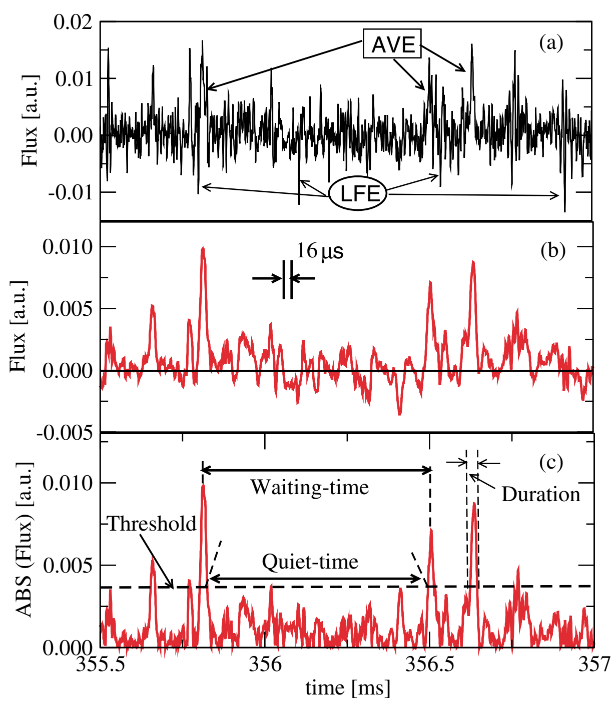

FIG. 1 (color online). (a) Detail of raw W7-AS flux signal; (b) same signal averaged with $m = 32$; (c) absolute value of averaged signal together with a sketch of relevant definitions.

pdf suffers, then, a strong distortion that, due to the self-similar nature of its cause, takes the form of a power law. This behavior is again characteristic of SOC dynamics, which further reinforces the soundness of the plasma SOC paradigm.

We have examined several records of electrostatic turbulent flux data obtained by means of a moving triple Langmuir probe [15]: density fluctuations are measured at one point, $\tilde{n} = \tilde{n}(r_1, \theta_1, t)$, and the plasma potential at two nearby positions, $\phi_1 = \phi_1(r_1, \theta_1 - \delta, t)$ and $\phi_2 = \phi_2(r_1, \theta_1 + \delta, t)$; the instantaneous turbulence-induced particle flux at r_1 is then estimated as $\Gamma_t \simeq \tilde{n}(\tilde{\phi}_2 - \tilde{\phi}_1)/2r_1\delta B$. The part of the records of interest corresponds to the temporal window in which the signal remains stationary and the probe tips stay *within* the plasma. The number of data points are usually scarce, so that survival functions [16] have been used instead of pdfs. The survival function $S^q(s)$ of any quantity of interest q gives the probability that q exceeds s. In our case, q will be either quiet times or burst durations. Therefore, $q \geq 0$ and $S^q(0) = 1$. From the experimental data, $S^q(s)$ is easily constructed by sorting all recorded values of q in decreasing order $T \equiv \{q_j, j = 1, 2, \ldots, N\}$; then a *rank* number is assigned to each distinct value in T, q_k^*, by using $r_k = r_{k-1} + n_k$, where n_k is the number of appearances of q_k^* and $r_0 = 0$. The survival function is then given by: $S^q(q_k^*) = r_k/N$, and is related to the pdf by $p^q(s) = -dS^q(s)/ds$. Notice that $S^q(s)$ carries the same information that $p^q(s)$, exhibiting exponential or power-law behavior only if the pdf does. In order to detect power laws, all survival functions are fitted to

$$S^{q,\mathrm{fit}}(s) = \frac{e^{-s/s_1}}{1 + (s/s_2)^k}, \tag{1}$$

where $k > 0$ and $s_1 > s_2 > 0$. Thus, $S^q(s) \sim s^{-k}$ for scales $s_2 \ll s \ll s_1$. The number of decades of power-law behavior is given by $D \equiv \log[s_1/s_2]$. To claim power-law behavior, $D \sim 1$ will be at least required. Furthermore, to ensure that the power law is not an artifact of the type of fit chosen, all fits are redone after relaxing the condition $s_1 > s_2$. If the best fit tends then to a pure exponential (s_2 increases then beyond the largest q_k^* and s_1 gives an average avalanche-crossing rate) and it is similarly good, the power-law fit is discarded. As goodness-of-the-fit, we use the usual chi-square merit function [17]:

$$\chi^2 = \sum_{k=1}^{N^*} [S^q(q_k^*) - S^{q,\mathrm{fit}}(q_k^*)]^2, \tag{2}$$

where N^* is the number of distinct q values.

A last technical issue that must be solved to compute meaningful quiet times is the identification of the events that are related to avalanches crossing the probe location (from now on, AVEs), which will be convoluted and mixed with other events associated with faster local fluctuations (from now on, LFEs). One approach is to apply an amplitude threshold, but there is no *a priori* reason for expecting AVEs to be more intense than LFEs (only the local flux is measured, not the integrated avalanche flux). It seems more natural to assume that AVEs are longer than LFEs, since they imply several successive relaxations near the probe. (Further evidence of this has been provided by the multifractal analysis of similar fluctuations, that associated LFEs to scales shorter than a few microseconds [7].) Therefore, the faster LFE scales can be largely eliminated by convoluting the signal with a m-point smoothing window. The value of m to be used must exceed the minimum above which the resulting smoothed signals become self-similar [18]. For instance, for the W7-AS data, it turns out that $m \geq 16$. In Fig. 1, the raw and smoothed signals are compared. Clearly, in the latter, most LFEs have merged into a rather continuous band on top of which clearly separated longer events prevail. This band can now be eliminated by amplitude thresholding. Before proceeding, it must be clarified that the AVEs discussed here are found within the last closed magnetic surface (LCS), in contrast to the so-called "blobs" found in the scrape-off layer (SOL) [19]. While the connection, if it exists, between them is not clear, the physics governing their time scales may be very different, since SOL magnetic field lines are open.

Results from W7-AS will be presented first (discharge No. 35427 [5]). The signal has been sampled at $\nu_s = 2$ MHz and has 200 000 usable points. The probe tips are located 0–2 cm within the LCS. Examples of quiet-time survival functions obtained with the $m = 32$ smoothed signal are shown in Fig. 2. Clearly, the survival function obtained without selecting the bursts according

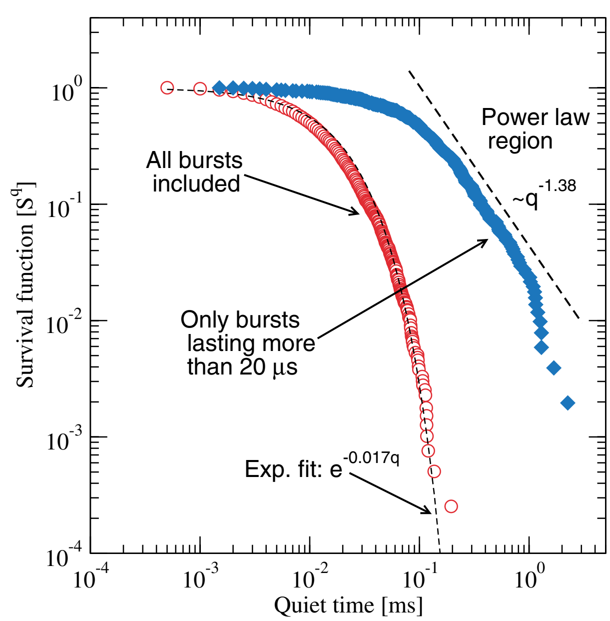

FIG. 2 (color online). Examples of quiet-time survival functions for W7-AS shot No. 35427.

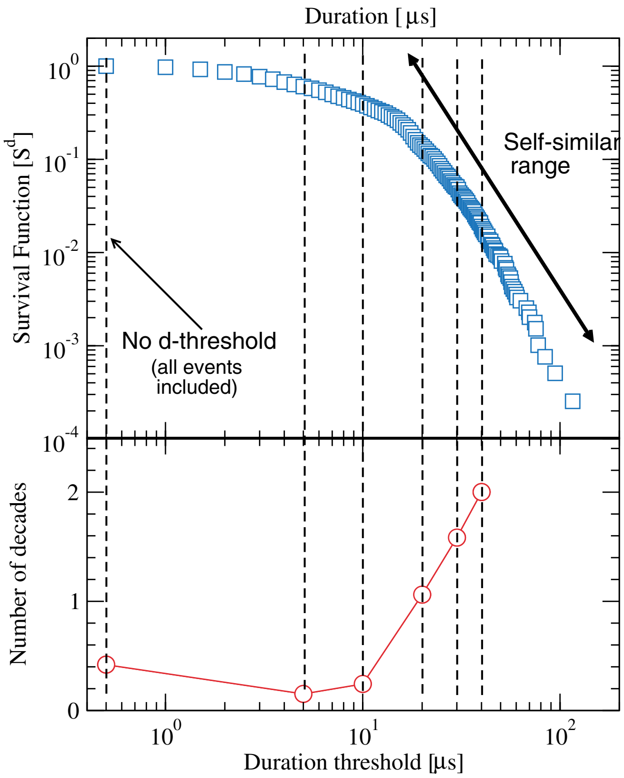

FIG. 3 (color online). Above: burst-duration survival function for W7-AS data. Below: decades of extension of power-law behavior vs duration threshold d. Situation of d relative to self-similar duration range shown by means of dashed vertical lines.

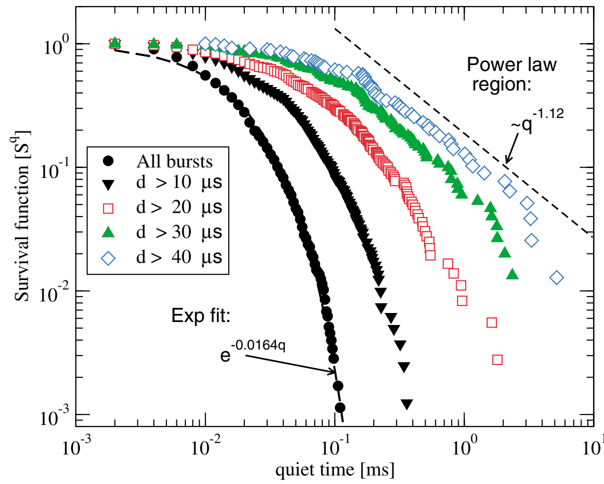

FIG. 4 (color online). Quiet-time survival functions for JET shot No. 54278.

to their duration can be fit well by an exponential law. This suggests that avalanches crossing by the probe location do so randomly, at a slower pace than the signal decorrelation time ($\tau_{\text{dec}} \sim 3$–$5\ \mu$s while the average time between crossings is $t_{\text{cros}} \sim 17\ \mu$s as shown in Fig. 2). Next, we study how the survival function changes when only flux bursts are considered, that exceed a minimum duration such that they lie in the self-similar temporal range. The lower boundary of this range is estimated to start near $d \sim 15$–$20\ \mu$s, from the burst duration survival function (Fig. 3). The correlation between the emergence of power laws in the quiet-time survival function (see Fig. 2, for an example) and the choice for the duration threshold (d) is clear. A sudden increase in D is coincident with d entering the self-similar range (Fig. 3). The significance of these power laws is supported by the comparison of the χ^2 values with those obtained when fitting the same survival functions to an exponential law (see Table I).

Data from the JET tokamak (discharge No. 54278 [20]) were analyzed next. Here, the usable points are about 40 000 ($\nu_s = 1$ MHz), and the probe is 0–2 cm inside. After determining $m \geq 8$, the quiet-time survival function of the smoothed signal was computed for increasing duration thresholds (see Fig. 4). Prior to thresholding the data the survival function is again fitted well by an exponential. This suggests the existence of a random drive, with the average time between two successive avalanches crossing the probe being $t_{\text{cros}} \sim 16\ \mu$s ($t_{\text{dec}} \sim 4$–$5\ \mu$s). The self-similar range for the burst duration of these data starts now in the range $d \sim 15$–$25\ \mu$s, which can be correlated again with the duration threshold, $d \sim 20\ \mu$s, above which quiet-time survival functions exhibits extended power laws (see Table II and Fig. 5).

Finally, edge turbulent flux data from the TJ-II heliac (discharge No. 5639 [21]) were also examined. The signal had 25 000 usable points ($\nu_s = 0.5$ MHz), and was averaged using $m = 8$. The probe was 3–4 cm inside. Prior to thresholding the data, the quiet-time survival function

TABLE I. Results of the fits to Eq. (1) of the $S^q(s)$ obtained by d thresholding the W7-AS data. First line corresponds to the no thresholded case. χ^2_{exp} gives χ^2 for the exponential fit.

d [μs]	s_1 [ms]	s_2 [ms]	k	D	χ^2	χ^2_{exp}
⋯	0.038	0.011	1.34	0.53	0.023	0.031
5	0.052	0.034	1.05	0.18	0.12	0.040
10	0.088	0.047	1.00	0.27	0.031	0.092
20	1.25	0.10	1.38	1.09	0.051	0.13
30	8.49	0.22	1.17	1.59	0.043	0.32
40	49.97	0.50	1.00	2.00	0.084	0.26

TABLE II. Same as Table I for JET data.

d [μs]	s_1 [ms]	s_2 [ms]	k	D	χ^2	χ^2_{exp}
⋯	0.035	0.021	1.77	0.22	0.018	0.026
10	0.15	0.036	1.36	0.62	0.014	0.035
20	0.75	0.054	1.11	1.14	0.012	0.24
30	13.05	0.10	1.16	2.11	0.014	0.20
40	59.87	0.16	1.12	2.57	0.031	0.21

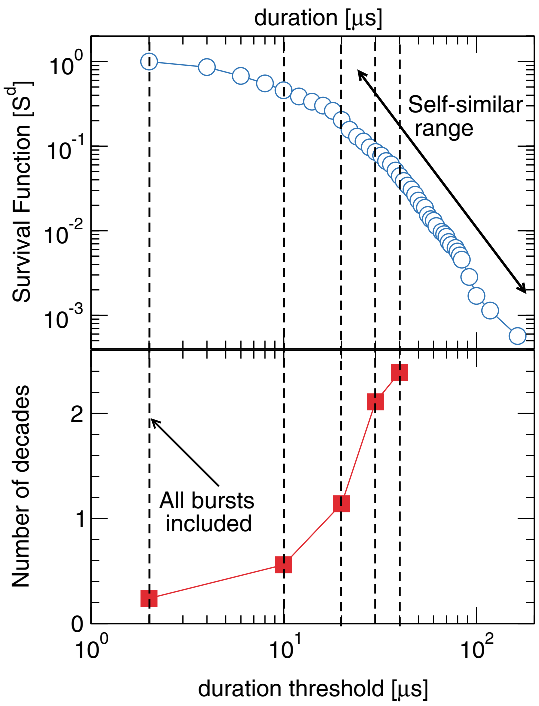

FIG. 5 (color online). Idem as Fig. 3 but for JET data.

was fitted equally well by an exponential law, yielding an average time between avalanche crossings of $t_{\rm cros} \sim 31$ μs. Similar to the previous cases, the duration thresholded cases developed once again a power law above $d \simeq 20$ μs (see Table III), which can be related to the start of the burst duration self-similar range ($d \simeq 15$–30 μs).

It is interesting to compare these new results with those reported in Ref. [13], obtained at the edge of the RFX [22] reverse-field-pinch (RFP). There, power laws with $D > 2$ were found without previously smoothing the data and using the same amplitude threshold prescription. These results were then interpreted as the first experimental contradiction with the SOC paradigm, at least for RFPs. A naive comparison with our results might lead one to conclude that RFPs behave very differently. However, in the light of Ref. [11], the extended power laws might also be taken to be an indication that the drive of the RFP edge is not random, but correlated and self-similar. This is a reasonable hypothesis in view of the fact that their edge is driven by a powerful turbulent flux originated inside the reversal surface, where transport is dominated by magnetic fluctuations [23,24]. This flux can be estimated as $\Gamma_{\rm mag} \simeq -(\tilde{J}_{\|,e} \cdot \tilde{B}_r)/eB$, where $\tilde{J}_{\|,e}$ and $\tilde{B}_r$ are the fluctuating parts of the parallel electron current and the radial magnetic field. And in MST [25], another RFP, the power spectra of both quantities have been found to decay as f^{-1} and $f^{-3/2}$ (see Fig. 3 of Ref. [23]).

In conclusion, we have found new evidence supporting that, just inside the LCS of tokamaks and stellarators, turbulent transport behaves as would be expected if SOC would indeed dominate the dynamics. The technique presented might also prove useful in the study of other physical systems.

TABLE III. Same as Table I for TJ-II data.

d [µs]	s_1 [ms]	s_2 [ms]	k	D	χ^2	$\chi^2_{\rm exp}$
...	0.089	0.032	2.20	0.44	0.041	0.058
10	0.090	0.041	1.93	0.34	0.024	0.051
20	0.20	0.048	1.51	0.62	0.007	0.032
30	1.50	0.078	1.30	1.28	0.013	0.061
40	9.03	0.12	1.34	1.88	0.012	0.044

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