Beliaev Damping and Kelvin Mode Spectroscopy of a Bose-Einstein Condensate in the Presence of a Vortex Line

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(Received 18 November 2002; published 7 May 2003)

It is demonstrated theoretically that the counter-rotating quadrupole mode in a vortex of Bose-Einstein condensates can decay into a pair of Kelvin modes via the Beliaev process. We calculate the spectral weight of a density-response function within the Bogoliubov framework, taking account of both Beliaev and Landau processes. Good agreement with experiment on ^{87}Rb by Bretin *et al.* [Phys. Rev. Lett. **90**, 100403 (2003)] allows us to unambigiously identify the decayed mode as the Kelvin wave propagating along a vortex line.

DOI: 10.1103/PhysRevLett.90.180401 PACS numbers: 03.75.Lm, 05.30.Jp, 67.40.Vs

Much attention has been focused on Bose-Einstein condensation (BEC) realized in alkali-atom gases [1]. Quantized vortices which are a hallmark of superfluidity [2] are created in various methods, such as phase imprinting [3], mechanical rotation by optical spoon [4], and topological Berry phase engineering [5]. Hundreds of vortices are trapped in a BEC system [6]. The creation and decay processes of vortices are investigated experimentally and theoretically, giving rise to a general consensus [2] that quantized vortices in a scalar, i.e., one-component BEC are well described by the Bogoliubov framework regarding to the static properties, such as the density profile or core radius, etc.

In contrast, regarding the dynamical aspects, the study of low-lying collective modes is rather scarce in theory and particularly in experiment. Needless to say, the low-lying Fermionic excitations in a vortex have played a fundamental role in charged or neutral Fermion systems, that is, the mixed state in a superconductor [7] and superfluid ^{3}He. Here we have a unique opportunity to investigate Bosonic excitations associated with a vortex, which was difficult in superconductivity. In particular, the so-called Kelvin mode [8] propagating along the vortex line, which is studied in classical normal fluids and superfluid ^{4}He, is interesting to identify and characterize in the present dilute Bose gases. This is an unexplored region.

Note that Bosonic excitations with lower energy in a vortex-free BEC are thoroughly studied; the breathing or monopole mode with the azimuthal angular momentum $q_\theta = 0$, the dipole Kohn mode $q_\theta = 1$, and the quadrupole mode $q_\theta = 2$ for an axis-symmetric system [1].

Recently, Bretin *et al.* [9] performed an experiment to examine the quadrupole modes with $q_\theta = \pm 2$ for a long cigar-shaped BEC with a vortex line, observing the decay process. Their results are summarized as follows: (i) The one of the splitted quadrupole mode $q_\theta = -2$, which rotates opposite to the vortex winding, decays two times faster than the other corotating quadrupole mode with $q_\theta = +2$. (ii) After the decay of $q_\theta = -2$ mode, there remains a density oscillation pattern along the long axis (z axis) whose nodes are 7 or 8 within the length of the condensate. The oscillation pattern is localized near the vortex core, seen in the radial direction profile.

Here we investigate the physical implication of these interesting observations, by calculating the density-density response function based on the wave functions and eigenvalues of the Bogoliubov–de Gennes equation for describing the collective modes of Bosonic excitations.

Before going into detailed computation of a cylindrical system with a vortex, we first give a clear physical picture for these phenomena which is fully justified microscopically later: At low temperatures, among the two possible decay channels, the Beliaev process dominates over the remaining Landau process [10]. The counter-rotating quadrupole mode with the energy ω_{-2} and the angular momentum $q_\theta = -2$ (in units of the radial harmonic frequency ω_r and $\hbar = 1$, respectively) can decay into a pair of the dipole mode with ω_{-1}, conserving the energy $\omega_{-2} \to 2\omega_{-1}$ and the angular momentum $(q_\theta = -2) \to (q_\theta = -1) + (q_\theta = -1)$. However, as seen from Fig. 1 where a Beliaev process is depicted as a function of the wave number q_z along the z axis, the pair created dipole modes [see Fig. 1(b)] should have a finite $\tilde{q}_z$, namely, $\omega_{-2}(q_z = 0) \to \omega_{-1}(\tilde{q}_z) + \omega_{-1}(-\tilde{q}_z)$. Note that in the absence of a vortex shown in Fig. 1(a) there is no

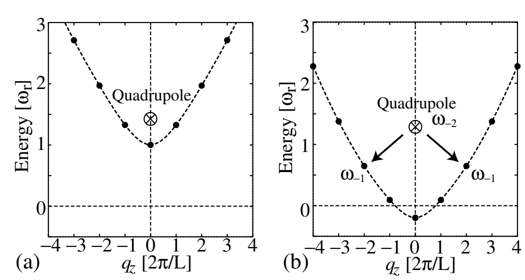

FIG. 1. Solid circles show the dispersion relations $\omega_{-1}(q_z)$ of $(q_\theta, q_r) = (-1, 0)$ along q_z in the case of the vortex-free state (a) and a single-vortex state (b). ⊗ denotes ω_{-2}.

Beliaev process because $2\omega_1(q_z) > \omega_2$, which is well known, leaving only Landau decay active [11]. Likewise as for the other comoving quadrupole mode with $q_\theta = +2$ there is no Beliaev process because $2\omega_{+1} > \omega_2$. Only the Landau process, in which the quadrupole mode annihilates with the thermally already excited modes into other modes, is responsible for the decay.

Here it is central that the so-called anomalous mode ω_{-1} at $q_z = 0$ has a negative value [12–14] relative to the condensate at the zero energy and that their wave function is radially localized at the core region whose wave length is an order of the coherence length. Along the z direction it oscillates sinusoidally with $\tilde{q}_z$. This explains the above experimental facts (i) and (ii) simultaneously. In addition the decay time of ω_{+2} should be the same as in the vortex-free case, which is indeed the case [9].

Let us now consider this novel phenomenon of the single-vortex condensate from the microscopic viewpoint. The Hamiltonian in a rotating frame with the angular frequency $\mathbf{\Omega}_{\rm rot}$ is given by $\hat{\mathcal{H}} = \int d\mathbf{r}\hat{\Psi}^\dagger(\mathbf{r})\{h(\mathbf{r}) + \frac{g}{2}\hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}(\mathbf{r})\}\hat{\Psi}(\mathbf{r})$. The creation and annihilation operators of the Bose particle are $\hat{\Psi}^\dagger$ and $\hat{\Psi}$, which are decomposed into $\hat{\Psi}(\mathbf{r}) = \phi(\mathbf{r}) + \hat{\psi}(\mathbf{r})$. The single-particle Hamiltonian is given as $h(\mathbf{r}) = -(\hbar^2\nabla^2/2m) - \mu + V(\mathbf{r}) - \mathbf{\Omega}_{\rm rot}\cdot(\mathbf{r}\times\mathbf{p})$ with the confining potential $V(\mathbf{r})$ and the chemical potential μ. The last term in $\hat{\mathcal{H}}$ describes the interaction between the particles, which is classified into the resulting eight terms according to the noncondensate part $\hat{\psi}$, by using the decomposition of the field operator obtained above [15].

The quadratic Hamiltonian may be diagonalized by the usual Bogoliubov transformation,

$$\begin{pmatrix}\hat{\psi}\\ \hat{\psi}^\dagger\end{pmatrix} = \sum_{\mathbf{q}}\begin{pmatrix}u_{\mathbf{q}} & -v_{\mathbf{q}}^*\\ -v_{\mathbf{q}} & u_{\mathbf{q}}^*\end{pmatrix}\begin{pmatrix}\eta_{\mathbf{q}}\\ \eta_{\mathbf{q}}^\dagger\end{pmatrix} \equiv \sum_{\mathbf{q}}\hat{U}_{\mathbf{q}}^\dagger\begin{pmatrix}\eta_{\mathbf{q}}\\ \eta_{\mathbf{q}}^\dagger\end{pmatrix}. \quad (1)$$

This diagonalization leads to the following conditions. First we impose the condition on the condensate wave function $\phi(\mathbf{r})$ as $[h(\mathbf{r}) + g|\phi(\mathbf{r})|^2]\phi(\mathbf{r}) = 0$, which is the so-called Gross-Pitaevskii (GP) equation. The Bogoliubov–de Gennes (BdG) equation for the quasiparticle is given in terms of the eigenfunctions $u_{\mathbf{q}}$ and $v_{\mathbf{q}}$

$$\begin{aligned}&\{h(\mathbf{r}) + 2g|\phi(\mathbf{r})|^2\}u_{\mathbf{q}}(\mathbf{r}) - g\phi^2(\mathbf{r})v_{\mathbf{q}}(\mathbf{r}) = \varepsilon_{\mathbf{q}}u_{\mathbf{q}}(\mathbf{r}),\\ &\{h(\mathbf{r}) + 2g|\phi(\mathbf{r})|^2\}v_{\mathbf{q}}(\mathbf{r}) - g\phi^{*2}(\mathbf{r})u_{\mathbf{q}}(\mathbf{r}) = -\varepsilon_{\mathbf{q}}v_{\mathbf{q}}(\mathbf{r}),\end{aligned} \quad (2)$$

where $\mathbf{q}$ is the quantum number for the eigenstate. It is well known that the eigenvalues $\varepsilon_{\mathbf{q}}$ correspond to the normal modes of the condensate and the values are good agreement with experiment for systems with negligible thermal cloud [1].

We consider the collective mode by the linear response theory [15]. In the presence of an external field coupled to the density $\langle\hat{\Psi}^\dagger\hat{\Psi}\rangle$, the linear response is characterized by a retarded correlation function $D^R(\mathbf{rr}', t-t') = -i\langle[\hat{\Psi}^\dagger(\mathbf{r}t)\hat{\Psi}(\mathbf{r}t), \hat{\Psi}^\dagger(\mathbf{r}'t')\hat{\Psi}(\mathbf{r}'t')]\rangle\theta(t-t')$, where $\langle\cdots\rangle$ denotes the thermal average. In order to calculate D^R directly, it is convenient to introduce a Matsubara correlation function $\mathcal{D}(\mathbf{rr}', \tau-\tau') = -\langle[\hat{\Psi}^\dagger(\mathbf{r}\tau)\hat{\Psi}(\mathbf{r}\tau)\times\hat{\Psi}^\dagger(\mathbf{r}'\tau')\hat{\Psi}(\mathbf{r}'\tau')]\rangle$. The Fourier coefficient $\mathcal{D}(\mathbf{rr}', i\Omega_n)$ can be related to the retarded correlation function, $D^R(\mathbf{rr}', \omega)$, by using the analytic continuation $i\Omega_n \to \omega + i\eta$. Here η is a positive infinitesimal constant and we use $\eta = 0.005\omega_r$ in our calculation. In the dilute Bose system, the Matsubara correlation function is characterized by a matrix

$$\mathcal{D}(\mathbf{rr}', i\Omega_n) \simeq -[\phi^*(\mathbf{r}), \phi(\mathbf{r})]\hat{\mathcal{G}}(\mathbf{rr}', i\Omega_n)[\phi(\mathbf{r}')\phi^*(\mathbf{r}')], \quad (3)$$

where $\hat{\mathcal{G}}$ is a 2×2 matrix renormalized Green's function defined as $\hat{\mathcal{G}}(\mathbf{r}\tau, \mathbf{r}'\tau') = -\langle T_\tau[\hat{\mathcal{A}}(\mathbf{r}\tau)\hat{\mathcal{A}}^\dagger(\mathbf{r}'\tau')]\rangle$ with the matrix operator $\hat{\mathcal{A}}^\dagger = (\hat{\psi}^\dagger, \hat{\psi})$. Using the Beliaev-Dyson equation, we have $\hat{\mathcal{G}} = (\hat{\mathcal{G}}_0^{-1} - \hat{\Sigma})^{-1}$ [10,15] with the bare Green's function $\hat{\mathcal{G}}_0$ which is constructed from quasiparticles and the canonical transformation Eqs. (1) and (2).

The self-energy is expressed, by the second order perturbation theory on g, as $\hat{\Sigma} = \hat{\Sigma}^{(1)} + \hat{\Sigma}^{(2)}$. The first order term $\hat{\Sigma}^{(1)}$, which contributes to the energy shift, is

$$\hat{\Sigma}^{(1)} = \int d\mathbf{r}\hat{U}_{\mathbf{q}_1}^\dagger(\mathbf{r})\begin{pmatrix}2g\rho - \delta\mu & g\kappa\\ g\kappa^* & 2g\rho - \delta\mu\end{pmatrix}\hat{U}_{\mathbf{q}_1}(\mathbf{r}), \quad (4)$$

where the noncondensate density $\rho(\mathbf{r}) = \sum_{\mathbf{q}}[|u_{\mathbf{q}}(\mathbf{r})|^2 f(\varepsilon_{\mathbf{q}}) + |v_{\mathbf{q}}(\mathbf{r})|^2\{f(\varepsilon_{\mathbf{q}}) + 1\}]$, the anomalous average $\kappa(\mathbf{r}) = -\sum_{\mathbf{q}}u_{\mathbf{q}}(\mathbf{r})v_{\mathbf{q}}^*(\mathbf{r})\{2f(\varepsilon_{\mathbf{q}}) + 1\}$, and the Bose function $f(\varepsilon_{\mathbf{q}}) = [\exp(\beta\varepsilon_{\mathbf{q}}) - 1]^{-1}$ with $\beta = 1/k_BT$. $\delta\mu$ corresponds to the change in the chemical potential, arising from the inclusion of mean field interactions in the GP equation, while the change of the condensate shape is small in low temperatures and may be ignored [16]. The second order term $\hat{\Sigma}^{(2)}$ is given by

$$\begin{aligned}\hat{\Sigma}^{(2)}(\mathbf{q}_1\mathbf{q}_1', i\Omega_n) = {}& g^2\sum_{\mathbf{q},\mathbf{q}'}\begin{bmatrix}A_{1,\mathbf{qq}'}(\mathbf{q}_1)\\ A_{2,\mathbf{qq}'}(\mathbf{q}_1)\end{bmatrix}[A_{1,\mathbf{qq}'}^*(\mathbf{q}_1')A_{2,\mathbf{qq}'}^*(\mathbf{q}_1')]\frac{f(\varepsilon_{\mathbf{q}'}) - f(\varepsilon_{\mathbf{q}})}{i\Omega_n - (\varepsilon_{\mathbf{q}} - \varepsilon_{\mathbf{q}'})}\\ &+ \frac{g^2}{2}\sum_{\mathbf{q},\mathbf{q}'}\begin{bmatrix}B_{1,\mathbf{qq}'}^a(\mathbf{q}_1)\\ B_{2,\mathbf{qq}'}^a(\mathbf{q}_1)\end{bmatrix}[B_{1,\mathbf{qq}'}^{a*}(\mathbf{q}_1')B_{2,\mathbf{qq}'}^{a*}(\mathbf{q}_1')]\frac{1 + f(\varepsilon_{\mathbf{q}}) + f(\varepsilon_{\mathbf{q}'})}{-i\Omega_n - (\varepsilon_{\mathbf{q}} + \varepsilon_{\mathbf{q}'})}\\ &+ \frac{g^2}{2}\sum_{\mathbf{q},\mathbf{q}'}\begin{bmatrix}B_{1,\mathbf{qq}'}^b(\mathbf{q}_1)\\ B_{2,\mathbf{qq}'}^b(\mathbf{q}_1)\end{bmatrix}[B_{1,\mathbf{qq}'}^{b*}(\mathbf{q}_1')B_{2,\mathbf{qq}'}^{b*}(\mathbf{q}_1')]\frac{1 + f(\varepsilon_{\mathbf{q}}) + f(\varepsilon_{\mathbf{q}'})}{i\Omega_n - (\varepsilon_{\mathbf{q}} + \varepsilon_{\mathbf{q}'})},\end{aligned} \quad (5)$$

which determines the excitation and its damping. In Eq. (5), the first term describes Landau processes: $\mathbf{q}_1' + \mathbf{q}' \rightarrow \mathbf{q} \rightarrow \mathbf{q}_1 + \mathbf{q}'$, while the second and third terms correspond to Beliaev processes: $\mathbf{q}_1' \rightarrow \mathbf{q} + \mathbf{q}' \rightarrow \mathbf{q}_1$. The matrix elements A and B are the overlap integrals of the condensate and the excitation amplitudes. The ultraviolet divergence in κ and $\Sigma^{(2)}$ is removed because we use the contact potential approximation for the T matrix rather than the bare potential [16]. The frequency spectrum of the collective modes is given as

$$S(\omega) = -\int d\mathbf{r} \int d\mathbf{r}' F^*_{Q_\theta}(\mathbf{r}) \mathrm{Im} D^R(\mathbf{r}\mathbf{r}', \omega) F_{Q_\theta}(\mathbf{r}'). \quad (6)$$

In the case of the surface mode ($Q_r = 0$), the general excitation operator is given as $F_{Q_\theta} = r^{|Q_\theta|} e^{iQ_\theta \theta}$ [17].

We take up the quadrupole excitation experiment on ^{87}Rb atoms by Bretin *et al.* [9]. Assuming their long-cigar system as a cylinder, we introduce a cylindrical coordinate: $\mathbf{r} = (r, \theta, z)$. The quantum number $\mathbf{q} = (q_\theta, q_z, q_r)$ may take the following values: $q_\theta = 0, \pm 1, \pm 2, \ldots$, $q_z = 0, \pm 2\pi/L, \pm 4\pi/L, \ldots$, and $q_r = 0, 1, 2, \ldots$, where L is the period of the length along the z axis [12] and we take $L = 15$ μm. The linear density along the z axis can be estimated as $n_z = 6 \times 10^9$/m with the radial trap frequency $\omega_r/2\pi = 97.3$ Hz ($T_c \sim 200$ nK). At finite temperatures, we use the chemical potential μ to fix the total number.

In Fig. 2(a) we show $S(\omega)$ in the frequency region near the quadrupole excitations $\omega_{\pm 2}$. The two resonance peaks are seen from it. The peak around $\omega = 1.55\omega_r$ corresponds to the main resonance of the quadrupole mode $q_\theta = +2$, while the other mode $q_\theta = -2$ has a peak around $\omega = 1.3\omega_r$. These resonance frequencies correspond to the observations ($\omega_{+2}/2\pi = 159.5 \pm 1.0$ Hz $\simeq 1.6\omega_r/2\pi$, $\omega_{-2}/2\pi = 116.8$ Hz $\simeq 1.2\omega_r/2\pi$). The damping rates of two quadrupole modes $\Gamma_{\pm 2}(\omega)$ can be estimated by the shape of the spectrum $S(\omega)$. We see that at $T \sim 15$ nK the value of two damping constants $\Gamma_{\pm 2}$ at each resonance is $\Gamma_{-2} > \Gamma_{+2}$, which is evidence that the counter-rotating mode decays faster than the other corotating mode and qualitatively agrees with the observation [9].

The width of $S(\omega)$ comes from the imaginary part of the self-energy $\hat{\Sigma}^{(2)}$ in Eq. (5) and the dominant matrix element $\Sigma^{(2)}_{11}$ for $q_\theta = -2$ and $q_\theta = +2$ is shown in Figs. 2(b) and 2(c). For the quadrupole mode $q_\theta = +2$, only the Landau process $(+2, 0, 0) + (q_\theta, q_z, q_r) \rightarrow (q_\theta + 2, q_z, q_r')$ is active as in the vortex-free case. Thus the damping constant for $q_\theta = +2$ is nearly the same for both single-vortex and vortex-free cases. This is exactly seen by Bretin *et al.* [9]. On the other hand, the Beliaev process exclusively dominates the damping of the $q_\theta = -2$ mode because, as mentioned before, $(-2, 0, 0)$ can decay into the two dipole modes ω_{-1} by conserving the angular momentum and the total energy. As is seen from Fig. 2(b), for $q_\theta = -2$, there is a single peak due to this

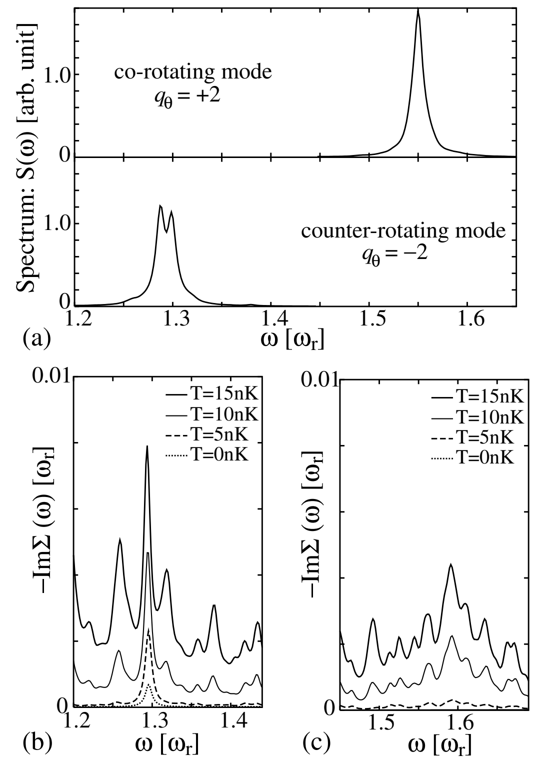

FIG. 2. Spectrum of two quadrupole modes $q_\theta = \pm 2$ at $T = 15$ nK for $\eta = 0.005\omega_r$ (a). The imaginary part of the self-energy for the quadrupole mode $q_\theta = -2$ (b) and $q_\theta = +2$ (c) at $T = 0$, 5, 10, and 15 nK. The peak around $\omega = 1.3\omega_r$ corresponds to the Beliaev process $(q_\theta = -2, q_z = 0) \rightarrow (-1, +2) + (-1, -2)$.

particular Beliaev process at $\omega \sim 1.3\omega_r$ while for $q_\theta = +2$ there is no prominent peak around the resonance $\omega \sim 1.55\omega_r$ Fig. 2(c). As T increases, the Landau processes becomes effective, giving rise to the satellite peaks in addition to the main peak as shown in Fig. 2(b). On the other hand, as T decreases, the Landau process quickly vanishes, leaving only the particular Beliaev process active. This should be checked experimentally.

The created modes via the Beliaev process of the counter-rotating quadrupole mode $q_\theta = -2$ can be identified to the so-called anomalous mode [12,13], or Kelvin mode characterized by the quantum number $q_\theta = -1$, $q_r = 0$, and $\pm\tilde{q}_z = 4\pi/L$ as shown in Fig. 1(b). This dipole mode is counter-rotating to the vortex flow direction. Since this mode has the zero-relative angular momentum to the condensate, the wave function $u(r, z) = u_{q_\theta=-1}(r)e^{iq_z z}$ does not vanish at the center $r = 0$ and localized at the core while all other wave functions with $q_\theta \neq -1$ vanish at $r = 0$. In Fig. 3 we display the condensate $|\phi(r, z)|$ and the wave function of the Kelvin mode with $\tilde{q}_z = \pm 4\pi/L$, which is created by decay of the counter-rotating quadrupole mode. One can see that this anomalous mode with a negative eigenvalue at $q_z = 0$, which is first identified theoretically by Isoshima and Machida [12] and Dodd *et al.* [13], is localized within

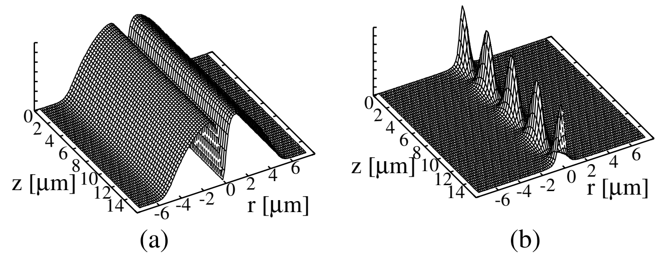

FIG. 3. Spatial profiles of the condensate $|\phi(r,z)|$ (a) and the Kelvin mode $|u(r,z)| = |u_{q_\theta=-1}(r)\cos(\tilde{q}_z z)|$ with $\tilde{q}_z = \pm 4\pi/L$ (b).

the core region along the radial direction whose scale is the coherence length. The characteristic wave number $\tilde{q}_z = 4\pi/L$ approximately corresponds to the observation [9]. These particular features, the localization around the core in the radial direction and propagation along the vortex line direction, are exactly what Bretin *et al.* [9] have detected. Since there is no core localized mode other than this $q_\theta = -1$ mode [12], we conclude that a pair of these anomalous modes with $\tilde{q}_z$ and $-\tilde{q}_z$ are created. Thus the Kelvin wave, or the wave motion of the vortex line propagating along z axis, is now identified and imaged by their experiment (see Fig. 3 in Ref. [9]).

In the following we consider several situations to help identifying the Kelvin mode: Let us first consider the external rotation effect on the decay process. Under the external rotation frequency $\Omega_{\rm rot}$, $\omega_{\pm 2}(\Omega_{\rm rot}) = \omega_{\pm 2} \mp 2\Omega_{\rm rot}$ as is seen from Eq. (2) [2]. Accordingly, the spectral response function $S(\omega)$ shown in Fig. 4 exhibits; (i) The two resonances switch their positions at around $\Omega_{\rm rot} \simeq 0.05\omega_r$, and (ii) the resonance widths become comparable as $\Omega_{\rm rot}$ increases. It is due to the suppression of the Beliaev decay; Under the rotation the energy of the relevant modes with $q_\theta = -1$ and -2 increases according to the rule $\omega_{q_\theta}(\Omega_{\rm rot}) = \omega_{q_\theta}(0) - q_\theta \Omega_{\rm rot}$. Because of the Bose factor the population of these modes decreases.

The finite temperature affects both Beliaev and Landau processes. As increasing T, $\omega_{-1}(T)$ is known to be larger while $\omega_{-2}(T)$ is relatively independent of T except the region near T_c [18]. Thus $\tilde{q}_z(T)$ becomes small with T and

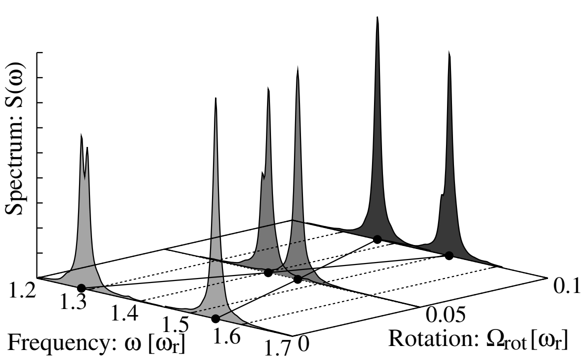

FIG. 4. The external rotation effect on spectrum of the co- and counter-rotating quadrupole modes at 15 nK.

above some critical temperature the Beliaev process $\omega_{-2} \to \omega_{-1}(\tilde{q}_z) + \omega_{-1}(-\tilde{q}_z)$ ceases to exist, unabling to create the Kelvin modes as the decay channel of ω_{-2}. Generally the Landau process for both $\omega_{\pm 2}$ modes becomes important as T increases because thermally excited modes become more available.

In Bretin *et al.* experiment [9] the counter-rotating ω_{-2} mode is used to excite the Kelvin mode via the Beliaev process. It is also possible to use the monopole mode ω_0 to create the Kelvin mode, namely $\omega_0 \to \omega_{+1}(\tilde{q}_z) + \omega_{-1}(-\tilde{q}_z)$. This provides yet another spectroscopic method to analyze the Kelvin mode.

In conclusion, we have demonstrated that the Kelvin mode as a propagation wave along a vortex line can be excited via Beliaev decay processes for the counter-rotating quadrupole mode. It enables us to understand the experiment by Bretin *et al.* [9] successfully, namely, their resonance position and width, and to predict the external rotation and temperature effects. We have also shown that utilizing these decay channels provides a novel spectroscopic method for low-lying Bosonic excitations in a vortex, in particular, the unexplored Kelvin mode.

The authors thank P. Rosenbusch for communicating their results prior to publication.

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