Electron-Phonon Scattering in Quantum Point Contacts

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We study the negative correction to the quantized value $2e^2/h$ of the conductance of a quantum point contact due to the backscattering of electrons by acoustic phonons. The correction shows activated temperature dependence and also gives rise to a zero-bias anomaly in conductance. Our results are in qualitative agreement with recent experiments studying the 0.7 feature in the conductance of quantum point contacts.

The quantization of conductance in units of $G_0 = 2e^2/h$ observed [1] in narrow one-dimensional constrictions—the quantum point contacts (QPC)—is well understood within a simple model of noninteracting electrons [2]. In this approach the electrons are backscattered by the effective one-dimensional potential created by the walls of the constriction, and the conductance follows the energy dependence of the transmission coefficient. In the last few years a number of experiments [3–8] studied the deviations of the conductance from this picture that appear as a shoulderlike feature at a conductance near $0.7G_0$. Several experiments have demonstrated that when a strong in-plane magnetic field is applied, this 0.7 anomaly evolves into the spin-polarized conductance plateau at $0.5G_0$ [3–5]. This observation led to the proposal that the anomaly originated from a spin-dependent mechanism [3]. Theoretical attempts to understand the 0.7 anomaly mostly followed this conjecture [9–17].

Here we concentrate on another feature of the 0.7 anomaly, namely, its strong temperature dependence. The experiments [3,5,6] showed that the anomalous shoulder is weak at the lowest available temperatures but grows when the temperature is increased. A detailed study [6] revealed that the correction to the conductance follows an Arrhenius law

$$\delta G \propto e^{-T_A/T} \tag{1}$$

with the activation temperature T_A of the order of 1 K. The activated temperature dependence implies that a certain backscattering mechanism turns on at temperature $T \approx T_A$ and leads to partial suppression of the conductance. The phenomenological proposals for this mechanism include scattering off a plasmon [18] or a spin wave [16] localized in the QPC, a low-lying spin-split subband [12], as well as spin-flip scattering by a Kondo impurity [5,17].

In this Letter we explore another backscattering mechanism, which results in the activated temperature dependence (1), namely, the scattering of electrons by acoustic phonons. This mechanism was discussed in the cases of scattering by surface acoustic waves [19] and by equilibrium phonons at relatively high temperatures [20].

At low temperatures the effect of electron-phonon scattering on transport in quantum wires is strongly suppressed. Indeed, in order to backscatter an electron at the Fermi level the phonon must have a wave vector $q = 2k_F$ in the direction along the wire. Thus the minimum energy of such a phonon is

$$T_A = 2\hbar s k_F = \sqrt{E_s E_F}, \tag{2}$$

where s is the speed of sound, E_F is the Fermi energy of electrons in the wire, $E_s = 8$ ms^2, and m is the effective mass of electrons. At temperatures $T \ll T_A$ the occupation numbers of such phonons and their contribution to the conductance are exponentially suppressed [Eq. (1)]. For typical GaAs quantum wires one can estimate $E_s \approx 0.3$ K and $E_F \sim 100$ K, resulting in $T_A \sim 5$ K. Thus the electron-phonon scattering is negligible in typical low-temperature experiments. On the other hand, the electron density in a QPC tuned into the vicinity of the first conductance step is very low. Estimating the Fermi energy at the center of a QPC to be about 2 K, one obtains a value of $T_A \approx 0.8$ K, in reasonable agreement with experiment [6].

To study transport in a QPC near a conductance step as a function of the gate voltage, one has to account for the effect of the confining potential. We follow the idea of Refs. [2,21] and model the QPC by a one-dimensional electron gas in an external potential approximated as $U(x) = -\frac{1}{2}m\Omega^2 x^2$. This approximation is valid in the vicinity of the conductance step, provided that the potential is sufficiently smooth. The transmission coefficient of such a barrier is well known [22],

$$T_0(E) = \frac{1}{1 + e^{-2\pi E/\hbar\Omega}}. \tag{3}$$

In this Letter we concentrate on the case when the Fermi energy is well above the top of the barrier, $E_F \gg \hbar\Omega/2\pi$. In this regime the transmission coefficient (3) at the Fermi level equals unity up to an exponentially small correction, and even a weak backscattering by acoustic phonons gives a significant correction (1) to the quantized conductance G_0.

The amplitude of electron backscattering by a phonon with wave vector q_x is proportional to the matrix element

$$I(q_x) = \langle \psi_R | e^{iq_x x} | \psi_L \rangle, \tag{4}$$

where $\psi_{R,L}(x)$ are the wave functions of the right- and left-moving electrons. Because of the fact that the speed of sound is small, the typical phonon energy $\hbar s q$ is much smaller than $\hbar\Omega$. Therefore, in evaluating the matrix element (4), the right- and left-moving electrons can be assumed to have the same energy E_F. At $E_F \gg \hbar\Omega$ the calculation of the matrix element (4) can be further simplified by using semiclassical expressions for the wave functions:

$$\psi_{R,L}(x) = \sqrt{\frac{m}{2\pi\hbar p_F(x)}} \exp\left[\pm \frac{i}{\hbar} \int_0^x dx' p_F(x') \right]. \tag{5}$$

Here the quasiclassical Fermi momentum is defined as $p_F(x) = \sqrt{2m[E_F - U(x)]}$.

We begin by studying the regime of small temperature and bias, $T, eV \ll T_A$, when the dominant contribution to δG is due to long-wavelength phonons, $q_x \ll 2k_F$. (Here $k_F = \sqrt{2mE_F}/\hbar$ is the electron wave vector at the center of the QPC.) In this regime the integral (4) is easily evaluated by the saddle-point method, and we find

$$I(q_x) = \left(\frac{k_F^2}{2\pi\hbar\Omega E_F q_x \sqrt{4k_F^2 - q_x^2}} \right)^{1/2} e^{-F_1(q_x/2k_F)},$$
$$F_1(u) = \frac{2E_F}{\hbar\Omega}(\arccos u - u\sqrt{1 - u^2}). \tag{6}$$

Since E_F is large compared to $\hbar\Omega$, this result is valid even when q_x approaches $2k_F$, as long as $F_1 \gg 1$.

Here λ labels the three possible polarizations of the acoustic phonons, $\omega_{\mathbf{q}\lambda} \propto q$ is the phonon frequency, ρ is the mass density of the material, $f_L(E')$ is the Fermi function of the left-moving electrons in the contact, and $N(\omega_{\mathbf{q}\lambda})$ is the equilibrium occupation number of a phonon of wave vector $\mathbf{q}$ and polarization λ.

The exact form of the matrix element $M_\lambda(\mathbf{q})$ depends on the nature of the electron-phonon coupling. At relatively high temperatures [20] the deformation potential coupling dominates, and $M_\lambda(\mathbf{q}) \propto q$. However, at low temperatures in GaAs the leading contribution is due to piezoelectric coupling, for which $M_\lambda(\mathbf{q})$ depends on the direction of $\mathbf{q}$, but not on its length q.

The backscattering of electrons by phonons reduces the total current carried by the right-moving electrons of energy E in the contact. One can therefore find the correction to the total current $I = G_0 V$ by integrating the backscattering rate (8) with the occupation numbers of the respective states:

At zero temperature only the processes of emission of phonons are allowed. The energy of the phonon cannot exceed applied voltage, and thus the maximum possible q_x in such a process is $eV/\hbar s$. Therefore, at $eV < T_A$ the correction to the differential conductance is exponentially small as $\delta G \propto \exp[-2F_1(eV/T_A)]$.

At nonzero temperature, both emission and absorption of phonons are possible. In the most interesting limit of zero bias, both processes are exponentially suppressed at $T \ll \hbar s q$. Thus the total backscattering rate is small as $\exp[-2F_1(q_x/2k_F) - \hbar s q_x/T]$. One can easily see that this rate assumes its maximum value at $q_x = 2k_F\sqrt{1 - (\hbar\Omega T_A/8E_F T)^2}$. Thus, the temperature dependence of the correction to the quantized conductance has the following exponential behavior:

$$\delta G = -G^* \exp\left[-\frac{T_A}{2T}\left(\frac{\arcsin v}{v} + \sqrt{1 - v^2} \right) \right], \tag{7}$$

where $v = \hbar\Omega T_A/8E_F T$. In a broad range of temperatures $\frac{\hbar\Omega}{E_F} T_A \ll T \ll T_A$ this correction shows activated behavior (1). The deviation from the Arrhenius law (1) at low T is due to the presence of the potential barrier $-\frac{1}{2}m\Omega^2 x^2$ for the one-dimensional electrons in the QPC.

In order to find the preexponential factor G^* in Eq. (7) and to evaluate δG in the regime when either temperature or voltage exceeds T_A, one has to perform a more formal calculation of the electron backscattering rate. Using the golden rule approach, one can find the following expression for the scattering rate of a right-moving electron of energy E to all the available left-moving states:

$$\tau_R^{-1}(E) = 2\pi \sum_\lambda \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{|M_\lambda(\mathbf{q})|^2}{2\rho\omega_{\mathbf{q}\lambda}} \int dE' |I(q_x)|^2 [1 - f_L(E')]$$
$$\times \{ N(\omega_{\mathbf{q}\lambda})\delta(E - E' + \hbar\omega_{\mathbf{q}\lambda}) + [N(\omega_{\mathbf{q}\lambda}) + 1]\delta(E - E' - \hbar\omega_{\mathbf{q}\lambda}) \}. \tag{8}$$

$$\delta I = -2e \int_{-\infty}^{\infty} [\tau_R^{-1}(E)f_R(E) - \tau_L^{-1}(E)f_L(E)]dE. \tag{9}$$

Here the factor of 2 accounts for the electron spins; the scattering rate τ_L^{-1} for left-moving electrons is obtained from Eq. (8) by replacing the subscripts $R \leftrightarrow L$.

The correction δI depends on the voltage V across the contact through the difference of the chemical potentials $\mu_R - \mu_L = eV$ entering the Fermi functions $f_R(E)$ and $f_L(E)$. Thus, the correction to the conductance G_0 of the contact can be found as $\delta G = d\delta I/dV$. The resulting expression for δG is simplified greatly if one makes the following approximations. First, we again neglect the dependence of the matrix element (4) on the energies E and E' and assume $E = E' = E_F$. Second, following the standard procedure [23], we replace $|M_\lambda(\mathbf{q})|^2$ and the sound velocities with their values $|M_\lambda|^2$ and s_λ averaged over the directions of $\mathbf{q}$. Then the integrals with respect

to the energies E and E' as well as the transverse components of q can be done analytically, and we find

$$\delta G = -G_0 T \sum_\lambda \frac{|M_\lambda|^2}{2\rho s_\lambda^2} \int_{-\infty}^{\infty} dq_x |I(q_x)|^2 K_\lambda(q_x). \quad (10)$$

Here the function $K_\lambda(q_x)$ is given by

$$K_\lambda(q_x) = 2 \ln \frac{1}{1 - e^{-\hbar s_\lambda |q_x|/T}} - \frac{\hbar s_\lambda |q_x|}{T}$$

$$+ \frac{eV - \hbar s_\lambda |q_x|}{2T} \coth \frac{eV - \hbar s_\lambda |q_x|}{2T}$$

$$+ \frac{eV + \hbar s_\lambda |q_x|}{2T} \coth \frac{eV + \hbar s_\lambda |q_x|}{2T}. \quad (11)$$

Unlike our previous results, the expression (10) for the correction δG is not limited to the regime $T, eV \ll T_A$. In addition, when the temperature and voltage are small compared to T_A, Eq. (10) enables one to find the preexponential factors, such as G^* in Eq. (7).

To find G^*, we first notice that at $T \ll T_A$ the longitudinal phonon mode can be ignored. Indeed, the sound velocity s_t of the transverse modes is lower than that of the longitudinal mode, $s_t < s_l$. Accordingly, the activation temperature (2) is lower for the transverse modes, i.e., the longitudinal mode gives a negligible contribution to δG at low temperatures. We therefore assume $s = s_t$ in the definition (2) of the activation temperature.

At zero bias and $T \ll \hbar s_t |q_x|$, we find $K(q_x) = (2\hbar s_t |q_x|/T) e^{-\hbar s_t |q_x|/T}$. The integral in Eq. (10) can then be evaluated by the saddle-point method. As a result we reproduce the exponential temperature dependence (7) with the prefactor

$$G^* = \gamma G_0 \sqrt{\frac{2T}{\pi T_A}} \frac{1}{\sqrt[4]{1 - \nu^2}}, \qquad \gamma = \frac{2|M_t|^2}{\rho s_t} \frac{m}{\hbar^2 \Omega}. \quad (12)$$

The dimensionless parameter γ determines the magnitude of the phonon backscattering effect on conductance at $T = T_A$. The numerical value of γ can be estimated from the data available in the literature [23]. We write the coupling parameter for the transverse phonons as $|M_t|^2 = \frac{8}{35}(ee_{14}/\epsilon)^2$, where for GaAs the permittivity $\epsilon = 13.2\epsilon_0$, and $e_{14} = 0.16$ C/m^2. Substituting $\rho = 5.36$ g/cm^3, $s = s_t = 3 \times 10^3$ m/s, and $m = 0.067 m_e$, we find $\gamma = 0.0052$ meV/$\hbar\Omega$.

We now turn to the evaluation of the correction (10) to the conductance of the QPC in the regime when the temperature and/or bias is large compared to $T_A = 2\hbar s_t k_F$. In this case the typical wave vector q of the phonons emitted or absorbed by electrons is large, $q \gg k_F$. To find δG we notice that the matrix element $I(q_x)$ has a peak near $q_x = 2k_F \ll q$. Thus, the electron backscattering in this regime is dominated by phonons propagating in the direction normal to the channel. One can then substitute the asymptotic expression for K_λ at $q_x \to 0$ into Eq. (10) and find δG in the form

$$\delta G = -\tilde{\gamma} G_0 \left(\frac{T}{T_A} \ln \frac{T}{T_A} + \frac{eV}{2T_A} \coth \frac{eV}{2T} \right). \quad (13)$$

Here the dimensionless parameter $\tilde{\gamma}$ is defined as

$$\tilde{\gamma} = \sum_\lambda \frac{|M_\lambda|^2 s_t}{\rho s_\lambda^2} \frac{m}{\hbar^2 \Omega}.$$

Because of the contribution of the longitudinal phonon mode, $\tilde{\gamma} > \gamma$. To estimate $\tilde{\gamma}$ we write the average matrix element as $|M_l|^2 = \frac{12}{35}(ee_{14}/\epsilon)^2$, Ref. [23]. Then using the value $s_l = 5.12 \times 10^3$ m/s of the velocity of longitudinal sound in GaAs, we find $\tilde{\gamma} = 0.0065$ meV/$\hbar\Omega$.

It is interesting to note that the negative correction (13) to the quantized conductance G_0 grows not only with temperature, but also with bias. When V is increased, more left-going states become available for the right-moving electrons to scatter into, and the conductance decreases. Thus, the electron-phonon scattering gives rise to a zero-bias anomaly similar to the one observed in experiments [5,6]. The linear shape of the zero-bias peak at $eV \gg T$ is consistent with the one observed in Ref. [5]. The height of the peak is determined by the limits of applicability of Eq. (13) at high bias. The most important limitation of our derivation was the assumption that the electrons are purely one dimensional. At sufficiently high bias the typical wave vector $q \sim eV/\hbar s$ of the phonons becomes comparable to the width w of the one-dimensional channel. Since the backscattering is mostly due to the phonons propagating in the transverse direction, their coupling to the electrons at $q > 1/w$ becomes weak, and the linear dependence $\delta G(V)$ given by Eq. (13) saturates. This saturation occurs at $eV \sim T_A \sqrt{\Delta/E_F}$, where $\Delta \sim \hbar^2/mw^2$ is the subband spacing in the QPC. Thus the height of the zero-bias peak in conductance is expected to be of the order $\tilde{\gamma} G_0 \sqrt{\Delta/E_F}$.

The zero-bias peak observed in experiment [5] had a height of about $0.15 G_0$. To compare this result with our estimate $\tilde{\gamma} G_0 \sqrt{\Delta/E_F}$, we use the device parameters $\hbar\Omega \approx 0.3$ meV and $\Delta \approx 0.9$ meV reported for similar samples [24]. To estimate E_F we assume the transmission coefficient $T_0(E_F) \approx 0.9$ and from Eq. (3) find $E_F \approx \frac{1}{3}\hbar\Omega$. This results in the peak height $\tilde{\gamma} G_0 \sqrt{\Delta/E_F} \approx 0.07 G_0$, which is reasonably close to the experimentally observed value $0.15 G_0$.

Unlike the bias dependence of δG, the temperature dependence $\delta G = -\tilde{\gamma} G_0 (T/T_A) \ln(T/T_A)$ obtained from Eq. (13) at $V = 0$ does not saturate at $T \sim T_A \sqrt{\Delta/E_F}$. The suppression of coupling to phonons with $q > 1/w$ does cut off the factor $\ln(T/T_A)$. However, the main linear in T dependence of δG originates from the phonon occupation numbers $N(\omega_q) = T/\hbar\omega_q$ at $T \gg \hbar\omega_q$ and remains even at $T \gg T_A \sqrt{\Delta/E_F}$ [25]. The experiment [5] does show a stronger suppression of conductance $\delta G \sim -0.3 G_0$ at high temperature than $\delta G \sim -0.15 G_0$ at high bias. It is also worth noting that a device with a higher value of

$\hbar\Omega = 2.6$ meV and, consequently, lower $\tilde{\gamma}$ shows a weaker temperature dependence of δG [26].

The one-dimensional electron gas at the center of a QPC at gate voltage corresponding to the first quantized step of conductance is very dilute and, consequently, Coulomb interactions are strong. We expect the interactions to enhance the correction δG by a large factor of order E_F/T_A, in analogy with the enhancement of electron backscattering by an impurity in a Luttinger liquid [27].

The phonon-induced backscattering effect discussed in this Letter is not limited to the first conductance step. Although most experiments observe the anomalous shoulder in the conductance at the first step, several observations of similar behavior at the second [4,7,26] and even higher steps [26] have been reported.

By applying an in-plane magnetic field B one can polarize the electron spins and observe a conductance step of height $0.5G_0$. The phonon-induced backscattering should then result in a negative correction to conductance similar to the 0.7 anomaly at $B = 0$. However, the experiments [3–5,28,29] do not show an anomalous plateau at $0.7 \times (0.5G_0)$. The likely reason for the apparent absence of the phonon backscattering effects is that at temperatures $T \gtrsim T_A$ the electrons in the channel are no longer completely spin-polarized. Indeed, in the experiments [28,29] the spin-split conductance plateau at $G = 0.5G_0$ *rises* to values about $0.6G_0$ when the temperature is increased from 60 mK to 1.5 K, indicating that the second spin-split subband contributes to the conductance. This conclusion is supported by the estimate of the spin-splitting $g\mu_B B \approx 3$ K in a typical field $B = 10$ T. Thus, at a temperature of order $T_A \sim 1$ K the second spin-split subband gives a significant positive contribution to the conductance that compensates for the decrease in conductance due to the phonons. To observe the phonon-induced backscattering features in conductance, magnetic fields significantly higher than 10 T are required.

In conclusion, we have studied the effect of backscattering of electrons in quantum point contacts by acoustic phonons. We found a significant negative correction to the quantized conductance. The correction grows exponentially as a function of temperature or voltage at T, $eV \ll T_A$. Above the activation temperature T_A, the correction grows roughly linearly with T and V [Eq. (13)]. Our results are consistent with the experimentally observed features of the conductance near $0.7(2e^2/h)$.

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