Magnetization Transport and Quantized Spin Conductance

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We analyze transport of magnetization in insulating systems described by a spin Hamiltonian. The magnetization current through a quasi-one-dimensional magnetic wire of finite length suspended between two bulk magnets is determined by the spin conductance which remains finite in the ballistic limit due to contact resistance. For ferromagnetic systems, magnetization transport can be viewed as transmission of magnons, and the spin conductance depends on the temperature T. For antiferromagnetic isotropic spin-1/2 chains, the spin conductance is quantized in units of order $(g\mu_B)^2/h$ at $T = 0$. Magnetization currents produce an electric field and, hence, can be measured directly. For magnetization transport in electric fields, phenomena analogous to the Hall effect emerge.

Transport of magnetization in various magnetic systems has received considerable attention both theoretically and experimentally [1–4]. A spatially varying magnetic field gives rise to a current of magnetic dipoles [3,4], similar to the transport of electric charge driven by an electric gradient. Here we consider insulating magnets described by a spin Hamiltonian, where magnetization can be transported by excitations such as magnons and spinons without transport of charge. Theoretical work on such systems has been focused on the long-wavelength limit for magnets with translational invariance [3,5].

In contrast, we propose to investigate magnetization transport in systems with broken translational invariance. In particular, we consider a quasi-one-dimensional system of finite length, e.g., a spin chain sandwiched between two bulk magnets which act as reservoirs for magnetization, where the magnetic field gradient is nonzero only over the system. Then, the magnetization current is determined by the spin *conductance G* which remains finite in the ballistic limit due to the contact resistance between the reservoirs and the system, in analogy to electronic transport in mesoscopic systems [6]. This is in stark contrast to the spin conductivity which diverges in the ballistic limit due to translational invariance [3,5]. Here, we derive the spin conductance G for both ferromagnetic (FM) and antiferromagnetic (AF) systems. We find that, for FM systems, magnetization transport can be viewed as transmission of magnons and the conductance is temperature dependent. For the AF spin-1/2 chain, the conductance has a value of order $(g\mu_B)^2/h$, where g is the gyromagnetic ratio and μ_B the Bohr magneton. Further, spin currents produce an electric field which allows one to measure G. We discuss magnetization transport in an external electric field and show that phenomena analogous to the Hall effect exist.

Ferromagnetic systems.—We first discuss a system with isotropic FM exchange interaction in a magnetic field $\mathbf{B}(\mathbf{x}_i) = B_i\mathbf{e}_z$. The spins occupy the sites $\mathbf{x}_i$ of a simple d-dimensional lattice with lattice constant a,

$$\hat{H} = J \sum_{\langle ij \rangle} \hat{\mathbf{s}}_i \cdot \hat{\mathbf{s}}_j + g\mu_B \sum_i B_i \hat{s}_{i,z}, \tag{1}$$

with $J < 0$. Here, $\hat{\mathbf{s}}_i$ is the spin operator of the spin with spin quantum number S at $\mathbf{x}_i$, and $\langle ij \rangle$ denotes nearest neighbor sites. For spatially constant $B_i = B > 0$, the elementary excitations of the system are magnons with dispersion [7]

$$\epsilon_{\mathbf{k}} \simeq g\mu_B B + |J|Sa^2k^2, \tag{2}$$

which carry a magnetic moment $-g\mu_B\mathbf{e}_z$. Here, $\mathbf{k}$ is the magnon wave vector. For temperature $T \ll g\mu_B B/k_B$, the magnon density is small and the noninteracting-magnon theory is valid for all d.

We now consider a setup for a magnetization transport measurement as sketched in Fig. 1(a). A spin chain extends from $x = -L/2$ to $L/2$ and is suspended between two large three-dimensional (3D) reservoirs, R1 and R2.
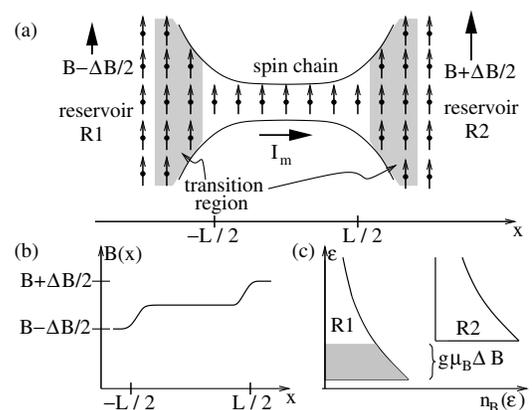

FIG. 1. (a) Proposed experimental setup for the measurement of a magnetization current I_m. (b) A magnetic field difference ΔB between the two bulk systems gives rise to $I_m = G\Delta B$. (c) ΔB shifts the Bose functions $n_B(\epsilon)$ in the reservoirs R1 and R2. Magnons with energies ϵ within the shaded region in R1 are not transmitted to R2.

$L \gg a$ is sufficiently small that magnons propagate ballistically through the chain. The reservoirs narrow adiabatically towards the chain ["transition region" in Fig. 1(a)]. The system is still described by Eq. (1), with the sites $\mathbf{x}_i$ occupying a bounded region in space [Fig. 1(a)]. A small spatially varying magnetic field $\delta B(x)\mathbf{e}_z$ with $\delta B(x) = -\Delta B/2$ $(\Delta B/2)$ for $x < -L/2$ $(x > L/2)$ is superimposed on the offset field $B\mathbf{e}_z$ for $t > 0$ [Fig. 1(b)]. For $|x| < L/2$, $\delta B(x)$ interpolates smoothly between the values $\pm \Delta B/2$ in the reservoirs. The field gradient results in a magnetization current I_m from R1 to R2. In linear response theory, I_m can be expressed in terms of the spin conductivity $\sigma(x, x', \omega)$,

$$I_m(x, \omega) = \int dx' \, \sigma(x, x', \omega) \partial_{x'} \, \delta B(x', \omega). \tag{3}$$

To calculate $I_m(x, \omega)$, knowledge of σ for $x, x' \in [-L/2, L/2]$ is sufficient because $\partial_{x'} \, \delta B(x', \omega) = 0$ inside the reservoirs. For a quasi-one-dimensional system, due to the continuity equation, σ is related to the susceptibility χ by $\sigma(q, \omega) = -i\omega \chi(q, \omega)/q^2$ [8]. In the noninteracting-magnon approximation,

$$\chi(q, \omega) = -\frac{(g\mu_B)^2}{\hbar} \int_{-\infty}^{\infty} \frac{dk}{2\pi} \frac{n_B(\epsilon_{k+q}) - n_B(\epsilon_k)}{(\epsilon_{k+q} - \epsilon_k)/\hbar + \omega + i0}. \tag{4}$$

Here, $n_B(\epsilon) = 1/[\exp(\beta\epsilon) - 1]$ is the Bose distribution function and $\beta = 1/k_B T$. In the limit $\omega \to 0$ of a dc field, from Eq. (4) we find that $\lim_{\omega \to 0} \sigma(x, x', \omega) = (g\mu_B)^2 n_B(g\mu_B B)/h$ is independent of x and x'. Integrating over x' in Eq. (3), we find that

$$I_m(x) = \frac{(g\mu_B)^2}{h} n_B(g\mu_B B)\Delta B = G\Delta B \tag{5}$$

is constant and depends only on the difference of magnetic fields in the reservoirs, ΔB. Although magnetization is transported ballistically, the spin *conductance G* remains finite due to the contact resistance for magnons between reservoirs and the system, similar to the related phenomenon in charge transport [6].

In FM systems, the magnetization current is carried by magnons. This allows us to reproduce Eq. (5) from the Landauer-Büttiker approach [6]. The field difference ΔB switched on at $t = 0$ results in a shift of the magnon energies ϵ in the reservoirs [Eq. (2)] and of the magnon distribution functions $n_B(\epsilon)$ [Fig. 1(c)]. Hence, a nonequilibrium situation is established. The magnetization in the reservoirs relaxes towards the new equilibrium values by magnetization transport from R1 to R2, i.e., the magnetization current I_m. All magnons incident on the spin chain from R2 are transmitted into R1 [9]. In contrast, magnons with $\epsilon \in [g\mu_B(B - \Delta B/2), g\mu_B(B + \Delta B/2)]$ are not transmitted from R1 to R2. This results in a net magnetization transport current

$$I_m = g\mu_B \int_0^{g\mu_B \Delta B} d\epsilon \, v(\epsilon)\rho(\epsilon)n_B(\epsilon + g\mu_B B)$$
$$\simeq \frac{(g\mu_B)^2}{h} n_B(g\mu_B B)\Delta B = G\Delta B, \tag{6}$$

where $v(\epsilon) = \partial_{k_x} \epsilon_{k_x}/\hbar$ is the magnon velocity and $\rho(\epsilon) = 1/hv(\epsilon)$ is the magnon density of states in the spin chain.

If the system connecting R1 and R2 consists of several chains with finite interchain exchange J', $G = (g\mu_B)^2 \sum_{k_\perp} n_B(g\mu_B B + \epsilon_{k_\perp})/h$, where $\epsilon_{k_\perp}$ is the energy of the transverse magnon mode. At $T = 0$, $G = 0$ because the system and the reservoirs are in the spin-polarized ground state.

Antiferromagnetic systems.—As we show next, magnetization transport in antiferromagnets is significantly different from ferromagnets but similar to charge transport in Fermi liquids. In an AF chain with half-integer spin, the elementary excitations are massless, and we will show that $G \neq 0$ even at $T = 0$. The spin-1/2 chain is believed to capture the essential features [10–13]. Thus, we now consider a spin-1/2 chain with isotropic AF exchange interaction $J > 0$ in Eq. (1) suspended between two AF reservoirs [14]. For $t > 0$, a magnetic field $B(x)$ is applied along $\mathbf{e}_z$ such that $B(x) = -\Delta B/2$ $(\Delta B/2)$ for $x < -L/2$ $(x > L/2)$. By a Jordan-Wigner transformation and subsequent bosonization, the spin chain can be mapped onto a Luttinger liquid (spinless fermions). Then, at $T = 0$, the Euclidean Lagrangian flows into a massless free theory under renormalization group [10,11,13],

$$\mathcal{L}_E = \int dx \frac{K}{2} \left[\frac{1}{v}(\partial_\tau \phi)^2 + v(\partial_x \phi)^2 \right], \tag{7}$$

where $K = 2$, $v = (\pi/2)Ja/\hbar$, and the homogeneous part of $\hat{s}_z$ is identified with $\partial_x \phi/\sqrt{\pi}$. The imaginary-time spin conductivity is $\sigma(q, \omega_n) = (g\mu_B)^2(v/\pi\hbar K)\omega_n/(\omega_n^2 + v^2 q^2)$ [15]. However, in order to calculate G, it is not sufficient to evaluate the dc limit $\omega \to 0$ of $\sigma(q, \omega)$ because the elementary excitations change on propagation from the reservoirs (magnons) through the chain (spinons). Following the related analysis for charge transport through a Luttinger liquid coupled to Fermi leads [16], we model the transition from a 3D ordered AF state to the spin chain by spatially varying $K(x)$ and $v(x)$ in the Lagrangian Eq. (7). For simplicity, we assume that $K(x)$ and $v(x)$ change discontinuously from the values of the spin chain to the ones of a bulk antiferromagnet at $x = \pm L/2$ [Fig. 2(a)]. The values K_b and v_b in the bulk region are chosen such that Eq. (7) correctly reproduces the dynamic susceptibility of a 3D AF ordered state. From the nonlinear sigma model description [17], we estimate $v_b \simeq \sqrt{3}Ja/\hbar$ and $K_b \simeq 4\sqrt{3}/\pi$. The spin conductance then follows from $G = [(g\mu_B)^2/\pi\hbar]\lim_{\omega_n \to 0} \omega_n G_{\phi\phi}(x, x', \omega_n)$, where the time ordered Green's function

$$G_{\phi\phi}(x, x', \omega_n) = \int_0^{\infty} d\tau \, e^{-i\omega_n \tau} \langle T_\tau \phi(x, \tau)\phi(x', 0) \rangle, \tag{8}$$
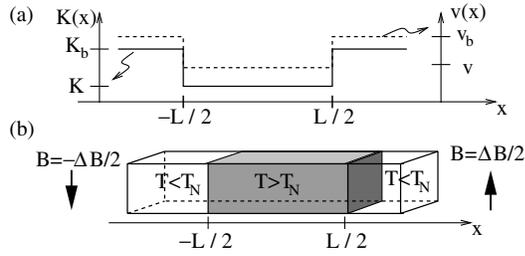
FIG. 2. (a) The transition from the 3D AF ordered bulk to the spin-1/2 chain is modeled by spatially varying $K(x)$ (solid line) and $v(x)$ (dashed line) in Eq. (7). (b) Setup for a transport measurement in an AF system.

must be evaluated for the *inhomogeneous* system including the transition regions [16]. For given $x' \in [-L/2, L/2]$, $G_{\phi\phi}(x, x', \omega_n)$ is obtained from the ansatz $G_{\phi\phi}(x, x', \omega_n) = a\exp[\omega_n x/v(x)] + b\exp[-\omega_n x/v(x)]$ for the four regions $x < -L/2$, $-L/2 < x < x'$, $x' < x < L/2$, and $L/2 < x$. The boundary conditions for the spin current are automatically satisfied by evaluating Eq. (8). We find that $\lim_{\omega_n \to 0} \omega_n G_{\phi\phi}(x, x', \omega_n) = 1/2K_b$ is independent of x, x' and of the parameters K and v of the spin chain. The spin conductance at $T = 0$,

$$G = \frac{(g\mu_B)^2}{hK_b},\qquad (9)$$

depends only on the parameter K_b of the bulk system.

We next argue that Eq. (9) remains valid also for a finite temperature and an offset magnetic field. Both analytical [18] and recent numerical [5] analysis indicate that, even at finite $T \ll J/k_B$, Eq. (7) describes the low energy behavior of the spin-1/2 chain correctly. Similarly, a finite offset magnetic field, $g\mu_B B \ll J$, suppresses quantum fluctuations of $\hat{s}_z$ in the bulk, leading only to a slight decrease of K_b. Our result Eq. (9) is, hence, robust both for finite temperatures and magnetic fields. In summary, an AF spin-1/2 chain suspended between AF reservoirs acts as a transport channel for magnetization with a spin conductance of order $(g\mu_B)^2/h$. For N parallel spin chains with vanishing interchain exchange interaction, each chain acts as an independent transmission channel and G increases by a factor N. The spin conductance is quantized in units of order $(g\mu_B)^2/h$.

Preparation of a sample as shown in Fig. 1(a) is challenging. A promising strategy is the use of a bulk material with an intrachain exchange J much stronger than the interchain exchange, such as Sr_2CuO_3. If heated to temperatures T much larger than the Néel ordering temperature T_N, the spin chains decouple and magnetization is transported predominantly along the spin chains. Hence, an AF wire heated to $T > T_N$ in its central part, but cooled to $T \ll T_N$ at its ends [Fig. 2(b)], provides a realization of the system in Fig. 1(a). Recent experiments [19] provide strong evidence that elementary excitations

in various quasi-one-dimensional systems have mean-free paths of several hundred nanometers at temperatures up to 50 K. The mean-free path is limited by the defect concentration in the samples. For $L < 1 \mu m$, transport through the system shown in Fig. 2(b) then is indeed ballistic as assumed above [Eq. (9)].

Detection of spin currents.—A current of magnetic dipoles produces an electric dipole field. The electric field is most easily calculated by decomposing the magnetization current into contributions propagating at a certain velocity v, $I_m = g\mu_B \sum_v n(v)v$, where $n(v)$ is the line density of magnetic dipoles with velocity v. For each v, the electric field in the laboratory frame is obtained by a Lorentz transform of the magnetic dipole field in the comoving frame. Summing over v, we find that the total electric dipole field [Fig. 3(a)]

$$\mathbf{E}_m(\mathbf{x}) = \frac{\mu_0}{2\pi} \frac{I_m}{r^2} (0, \cos 2\phi, -\sin 2\phi) \qquad (10)$$

depends only on I_m. Here, $\sin\phi = y/r$, $\cos\phi = z/r$, and $r = \sqrt{y^2 + z^2}$. For a numerical estimate, we now consider N parallel uncoupled AF spin-1/2 chains connecting two AF reservoirs. With Eqs. (9) and (10),

$$|\mathbf{E}_m(\mathbf{x})| \sim N\frac{\mu_0}{2\pi}\frac{(g\mu_B)^2}{h}\frac{\Delta B}{r^2} = N\frac{g^2}{4} \times 10^{-19} \frac{\Delta B[T]}{r[m]^2}\frac{V}{m}. \qquad (11)$$

Even for moderate $\Delta B = 10^{-3}$ T and large $r = 10^{-5}$ m, the magnetization current transported by $N \simeq 10^4$ parallel spin chains leads to an electric field $E_m \sim 10^{-8}$ V/m. The voltage drop between the two points $(0, r, 0)$ and $(0, 0, r)$ indicated in Fig. 3(a) is then $V_m = E_m r \simeq 10^{-13}$ V, which is within experimental reach [20,21].

Spin currents in electric fields.—A moving magnetic dipole moment also interacts with an external electric field $\mathbf{E}(\mathbf{x})$, leading to phenomena analogous to the Hall effect. A magnetic dipole $-g\mu_B \mathbf{e}_z$ moving in an electric field acquires an Aharonov-Casher phase [22] and the
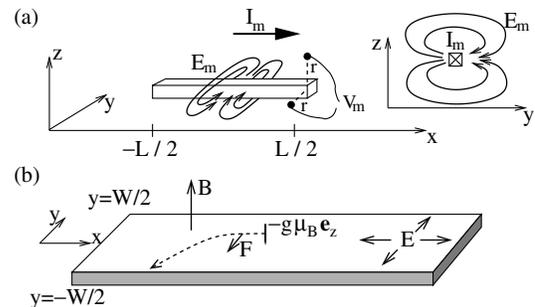

FIG. 3. (a) A current of magnetic dipole moment I_m produces an electric dipole field leading to a measurable voltage V_m. (b) Magnetic dipoles $-g\mu_B \mathbf{e}_z$ driven by a magnetic field gradient ∇B in an inhomogeneous electric field $\mathbf{E}(\mathbf{x})$ experience a force $\mathbf{F}$ analogous to the Lorentz force.

spin Hamiltonian is modified to

$$\hat{H} = \frac{J}{2} \sum_{\langle ij \rangle} [\hat{s}_i^+ \hat{s}_j^- e^{-i\theta_{ij}} + \hat{s}_i^- \hat{s}_j^+ e^{i\theta_{ij}} + 2\hat{s}_{i,z}\hat{s}_{j,z}]$$
$$+ g\mu_B \sum_i B_i \hat{s}_{i,z}, \tag{12}$$

where $\hat{s}_j^\pm = \hat{s}_{j,x} \pm i\hat{s}_{j,y}$ and $\theta_{ij} = g\mu_B \int_{\mathbf{x}_i}^{\mathbf{x}_j} d\mathbf{x} \cdot (\mathbf{E} \times \mathbf{e}_z)/\hbar c^2$. Introducing magnon creation and annihilation operators, Eq. (12) can be rewritten in terms of magnons with single-magnon Hamiltonian $\hat{h}$. From Eq. (12),

$$\hat{h} = \frac{|J|Sa^2}{\hbar^2}(\hat{\mathbf{p}} - g\mu_B \mathbf{E} \times \mathbf{e}_z/c^2)^2 + g\mu_B B. \tag{13}$$

Here, we discuss only the classical motion of magnons propagating with velocity $\mathbf{v} = -v_x \mathbf{e}_x$ in a 2D system of finite width W in the y direction [Fig. 3(b)], where $I_m = g\mu_B n v_x W$, and n is the magnon density. From the equation of motion implied by Eq. (13), one obtains the force acting on a magnon, $\mathbf{F} = -g\mu_B \nabla[B - (\mathbf{v} \times \mathbf{E}) \cdot \mathbf{e}_z/c^2]$. The second term accounts for the interaction with the electric field. We now focus on $\mathbf{E} = E'(x, y, -2z)$ with $E' =$ const. Then, the equation of motion of the magnons is formally identical to that of electrons in a constant magnetic field. Magnons are deflected into the $\mathbf{e}_y$ direction perpendicular to the transport direction $\mathbf{e}_x$. Stationarity is reached when the magnon repulsion equals the driving force along $\mathbf{e}_y$ due to the electric field. Taking into account only dipolar forces between the magnons, in the stationary state $B - v_x E' y/c^2$ is constant as function of y. The difference in magnetic fields $\Delta B = B(y = W/2) - B(y = -W/2)$ is related to the magnetization current density by the spin Hall conductance G_H,

$$\frac{I_m}{W} = -G_H \frac{\Delta B}{W} = -\frac{g\mu_B n c^2}{E'} \frac{\Delta B}{W}. \tag{14}$$

In the hydrodynamic regime, the drift velocity v_x is determined by the magnon scattering time τ. At low temperatures, τ is limited by impurities in the sample. For τ on the order of 10^2–10^3 ns, as measured for yttrium iron garnet (YIG) at 1–4 K (Ref. [23]), $\partial_x B = 10^5$ T/m, $J = 200$ K k_B, $S = 1$, and $a = 1$ Å, the drift velocity is $v_x = 10^3$–10^4 m/s. A variation of electric field $\Delta E = E(y = W/2) - E(y = -W/2) = 10^7$ V/m across the magnetic system then would lead to $\Delta B = 10^{-3}$–10^{-2} G resulting from the spin Hall effect. Thus, the spin Hall conductance G_H is within experimental reach.

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