Circuit Theory of Unconventional Superconductor Junctions

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(Received 1 August 2002; published 24 April 2003)

We extend the circuit theory of superconductivity to cover transport and proximity effect in mesoscopic systems that contain unconventional superconductor junctions. The approach fully accounts for zero-energy Andreev bound states forming at the surface of unconventional superconductors. As a simple application, we investigate the transport properties of a diffusive normal metal in series with a d-wave superconductor junction. We reveal the competition between the formation of Andreev bound states and proximity effect that depends on the crystal orientation of the junction interface.

DOI: 10.1103/PhysRevLett.90.167003 PACS numbers: 74.50.+r, 74.70.Kn

In the last decade the mesoscopic superconducting systems have been the subject of intensive experimental and theoretical research. The transport in these systems is essentially contributed by the so-called Andreev reflection [1], a unique process specific for normal metal/superconductor interface. The phase coherence between incoming electrons and Andreev reflected holes persists in the normal metal at mesoscopic length scale [2,3]. This results in strong interference effects on the Andreev reflection rate [4]. The transport properties of mesoscopic N/S junctions have been theoretically investigated with various approaches [4–7].

One of the authors has proposed a generic circuit theory of nonequilibrium superconductivity which accounts for the effects mentioned above [8,9]. The mesoscopic system is presented as a network of nodes and connectors. A connector is characterized by a set of transmission coefficients and can present anything from ballistic point contact to tunnel junction. The circuit theory is based on conservation laws for so-called spectral currents. These additional conservation laws present interference of electrons and holes. The spectral currents through each connector are functions of spectral vectors in the nodes. There is one-to-one correspondence between spectral vectors and currents and Keldysh Green functions in the underlying microscopic approach [5]. Kirchhoff-type equations determine spectral currents and vectors in each node and connector, and electric current in the circuit.

Unconventional superconductors bring about very unusual interface physics. The transport through the interface is influenced by formation of Andreev bound states (ABS) at this interface [10–12]. Those result from the interference of injected and reflected quasiparticles. The ABS manifest themself as a zero-bias peak in tunneling conductance (ZBCP) [11,12]. Indeed, ZBCP has been reported in various superconductors that have anisotropic pairing symmetry [12]. The proper theory of transport in the presence of ABS has been formulated [11,12] for conditions of ballistic transport only. This theory has to be revisited to account for diffusive transport in the normal metal. The point is that the diffusive transport provides an Andreev reflection mechanism for ZBCP which does not involve any unconventional superconductivity. This mechanism may compete with the formation of ABS. The anomalous size dependence of transport in yttrium barium copper oxide (YBCO) junctions reported in recent experiment [13] seems to arise from this competition.

All this has motivated us to extend the circuit theory to the systems containing unconventional superconductor junctions. We stress that this extension is by no means straightforward. The circuit theory cannot be directly applied to an unconventional superconductor since it requires the isotropization. The latter is just incompatible with the mere existence of unconventional superconductivity. Fortunately, there is a way around. We concentrate on the matrix currents via the unconventional superconductor junction to or from diffusive parts of the system. If one knows the relation between these currents and the spectral vectors (isotropic Green functions) in the diffusive part, one is able to use Kirchhoff rules to complete the evaluation of the matrix currents everywhere in the system.

This relation shall be derived from microscopic theory and presents the main result of this work. We stress that applicability of this relation is not restricted to circuit theory. One can regard our result as a boundary condition for the traditional Keldysh-Usadel equations of nonequilibrium superconductivity [5]. As an immediate application, we study a d-wave superconductor junction in series with normal metal. The resistance of the system appears to depend strongly on the angle α between the normal to the interface and the robe direction of d-wave superconductor (misorientation angle). This reveals the competition between the effect of ABS and proximity-induced reflectionless tunneling.

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To derive the relation between matrix current and Green functions, we make use of the method proposed in [9]. The method puts the older ideas [3] to the framework of Landauer-Büttiker scattering formalism. One expresses the matrix current in a constriction in terms of one-dimensional Green functions $\check{g}_{n,\sigma;n',\sigma'}(\epsilon; x, x')$, where n,n' and σ, $\sigma' = \pm 1$ denote the indices of transport channels and the direction of motion along the x axis, respectively. The "check" represents the Keldysh-Nambu structure. These Green functions are to be expressed in terms of the transfer matrix that incorporates all information about the scattering, and asymptotic Green functions $\check{G}_{1,2}$ presenting boundary conditions deep in each side of the constriction. The isotropization assumption requires that these $\check{G}$ do not depend on the channel number. Under this assumption, the current is universal depending on transmission eigenvalues only. Although the isotropization assumption is good for conventional superconductors and normal metals, it fails to grasp the physics of an unconventional superconductor where the Green function essentially depends on the direction of motion and thus on the channel number. To avoid this difficulty, we restrict the discussion to a conventional model of *smooth interface,* assuming momentum conservation in the plane of the interface. Within the model, the channel number eventually numbers possible values of this in-plane momentum and the transfer matrix becomes block diagonal in the channel index. We thus solve Green functions $\check{g}_{n,\sigma;n',\sigma'}(\epsilon, x, x')$ separately for each channel. The asymptotic Green function in the unconventional superconductor *does* depend on the direction of motion σ,

$$\check{G}_{2}; n, \sigma; n, \sigma = \check{G}^{(n)}_{2+}\frac{1-\sigma}{2} + \check{G}^{(n)}_{2-}\frac{1+\sigma}{2}, \qquad (1)$$

reflecting different asymptotic conditions for incoming ($\check{G}^{(n)}_{2+}$) and outgoing ($\check{G}^{(n)}_{2-}$) waves in each channel. The asymptotic Green function $\check{G}_1$ in normal metal is the same for both waves and all channels (see Fig. 1). All these matrices satisfy unitary relation $(\check{G}^{(n)}_{2\pm})^2 = \check{G}_1^2 = 1$.

After some algebra we obtain the matrix current in the following form:

$$\begin{aligned}\check{I} &= \frac{4e^2}{\mathrm{h}}\sum_m [\check{G}_1, \check{B}_m],\\ \check{B}_m &= \{-\Xi_m[\check{G}_1, \check{H}^{(m)-1}_{-}] + \check{H}^{(m)-1}_{-}\check{H}^{(m)}_{+}\\ &\quad - \Xi_m^2 \check{G}_1 \check{H}^{(m)-1}_{-}\check{H}^{(m)}_{+}\check{G}_1\}^{-1}\\ &\quad \times [\Xi_m(1-\check{H}^{(m)-1}_{-}) + \Xi_m^2\check{G}_1\check{H}^{(m)-1}_{-}\check{H}^{(m)}_{+}], \end{aligned} \qquad (2)$$

with $\check{H}^{(m)}_{\pm} = (\check{G}^{(m)}_{2+} \pm \check{G}^{(m)}_{2-})/2$.

Here $\Xi_m \equiv T_m/(1+\sqrt{1-T_m})^2$ is related to the transmission coefficient T_m in a given channel m. The above relation reduces to the isotropic result of Ref. [9] provided $\check{G}^{(n)}_{2+} = \check{G}^{(n)}_{2-} = \check{G}_2$. The above 4×4 matrix relation is the main result of the present work. It incorporates the most general situation and allows for many applications that involve unconventional superconductors. Below we provide a simple but extensive application example that both illustrates circuit theory method and demonstrates an interesting interplay of ABS and proximity effect.

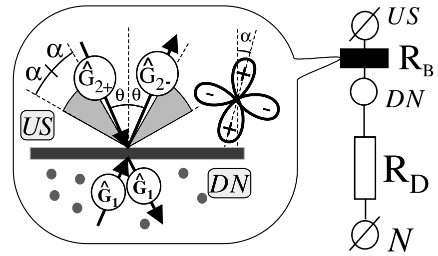

FIG. 1. The unconventional superconductor junctions (solid box) can be incorporated into circuit theory by means of the matrix current relation (2). This relation accounts for anisotropic features of the US, as sketched for a d-wave superconductor.

The circuit is the one given in Fig. 1: diffusive conductor of resistance R_D in series with unconventional superconductor junction. We disregard decoherence between electrons and holes in the diffusive conductor ("leakage" current in terms of Ref. [9]), and this is justified at energies not exceeding Thouless energy of this piece of normal metal. We restrict our attention to the d-wave superconductor, being the most practical example of the singlet unconventional superconductor that preserves time reversal symmetry. For simplicity, we have in mind a "two-dimensional" superconductor made from the layers stacked in the z direction. The z axis lies in the plane of the interface and is normal to the plane of Fig. 1. The interface normal (x axis) makes an angle α with the main crystal axis. The propagation directions of the waves are thus in the xy plane and are parametrized by the angle θ with the x axis. The angular dependence of the superconducting order parameter is thus given by $\Delta(\theta) = \Delta_0\cos(2(\theta - \alpha))$. A scattering channel consists of an incoming wave in direction $\pi - \theta$ and an outgoing wave in the direction θ. The sums over channels can be reduced to integrals over θ:

$$\sum_m \propto \int_{-\pi/2}^{\pi/2} d\theta \cos\theta. \qquad (3)$$

The Green functions are fixed in the "US" terminal and in the "N" terminal, and the voltage V is applied to N terminal. The Green function in the node "DN," $\check{G}_1$, is not fixed and shall be determined from the balance of the matrix currents. There is a natural separation of balance equations for 2×2 spectral currents that set advanced or retarded parts of $\check{G}_1$ and for a particle current at a given energy that sets the distribution function in the node DN [8].

We address the balance of the spectral currents first. The advanced 2×2 Green functions are fixed in the N and US terminals and read $\hat{G}_N = \tau_z$, $\hat{G}_{2\pm} = (\Delta_\pm \tau_x + i\epsilon\tau_z)/\sqrt{\Delta_\pm^2 - \epsilon^2}$, $\boldsymbol{\tau}$ being Pauli matrices, $\Delta_\pm = \Delta_0 \cos[2(\theta \pm \alpha)]$ being superconducting order parameters that correspond to the direction of the incoming (outgoing) wave. This suggests that the corresponding Green function in the DN node assumes a form $\sin\gamma \cdot \tau_x + \cos\gamma \cdot \tau_z$ where γ is yet to be determined. γ is the measure of proximity effect. All spectral currents are proportional to τ_y. The spectral current $\boldsymbol{i}_D^{(s)}$ through the diffusive conductor is proportional to the spectral angle drop [8], and the spectral current $\boldsymbol{i}_B^{(s)}$ via the interface is obtained from Eq. (2). The balance equation thus reads

$$\boldsymbol{i}_B^{(s)} + \boldsymbol{i}_D^{(s)} = 0, \qquad \boldsymbol{i}_B^{(s)} = -\frac{2e^2}{\mathrm{h}} \sum_m F(\gamma, \epsilon, T_m), \qquad \boldsymbol{i}_D^{(s)} = \gamma / R_D. \tag{4}$$

Under the conditions considered, the transport is determined by the energy-symmetric distribution function, which is conventionally called f_t [5]. The balance of particle currents at each energy determines this distribution function in the DN node. We will assume that the temperature $1/\beta$ is much smaller than the typical value of the superconducting energy gap, so we can disregard quasiparticle excitations in the superconductor. The particle current through the diffusive conductor is given by the drop of the distribution function at its ends, and the particle current via the interface is given by the corresponding block of Eq. (2). This yields

$$\boldsymbol{i}_B^{(p)} + \boldsymbol{i}_D^{(p)} = 0, \qquad \boldsymbol{i}_B^{(p)} = f_t \frac{2e^2}{\mathrm{h}} \sum_m T^*(\gamma, \epsilon, T_m), \qquad \boldsymbol{i}_D^{(p)} = (f_t - f_0)/R_D, \tag{5}$$

f_0 being the symmetrized distribution function in the normal reservoir, $f_0 = \frac{1}{2}\{\tanh[\beta(\epsilon + eV)/2] - \tanh[\beta(\epsilon - eV)/2]\}$. The above relation becomes especially transparent if one regards T^*'s as effective transmission coefficients in each channel. It just shows that the full (energy-dependent) resistance of the system is the sum of the resistance of diffusive metal and the interface resistance, the latter being influenced by the proximity effect. The degree of the proximity effect is determined from Eq. (4). If we define the average over the angle as

$$\langle A(\theta) \rangle = \int_{-\pi/2}^{\pi/2} d\theta \cos\theta A(\theta) \Big/ \int_{-\pi/2}^{\pi/2} d\theta T(\theta) \cos\theta$$

with $T(\theta) = T_m$, both balance equations can be rewritten in a compact form.

$$R = R_D + R_B / \langle T^*(\gamma, \epsilon, T_m) \rangle, \tag{6}$$

$$\gamma = \langle F(\gamma, \epsilon, T_m) \rangle R_D / R_B. \tag{7}$$

Here R_B is the interface resistance in normal state, R is the full resistance. It may depend on energy, so the full electric current is given by $eI_{el} = \int d\epsilon f_0(\epsilon)/R$.

To reveal the underlying physics, we present the concrete expressions for $F = F(\gamma, \epsilon, T_m)$ and $T^* = T^*(\gamma, \epsilon, T_m)$ assuming $|\epsilon| \ll |\Delta_\pm|$. It turns out that these expressions are essentially different for $\Delta_+ \Delta_- \gtrless 0$, this manifesting the formation of ABS in the latter case. For $\Delta_+ \Delta_- < 0$ (ABS channels) we have

$$F = \frac{-2T_m \sin\gamma}{T_m \cos\gamma + i(2 - T_m)\epsilon/\tilde{\Delta}} = \xrightarrow{\epsilon \to 0} -2\tan\gamma, \tag{8}$$

$$T^* = \frac{T_m^2 (1 + |\cos\gamma|^2 + |\sin\gamma|^2)}{T_m^2 \cos^2\gamma + (2 - T_m)^2 (\epsilon/\tilde{\Delta})^2} = \xrightarrow{\epsilon \to 0} \frac{2}{\cos^2\gamma} \tag{9}$$

with $\tilde{\Delta} = (2|\Delta_+||\Delta_-|)/(|\Delta_+| + |\Delta_-|)$. It is somewhat counterintuitive that the zero-energy limit does not depend on the actual transmission, giving finite currents even for insulating interfaces. This is the signature of the resonance forming precisely at zero energy [11]. If the transmission is low, the resonance feature persists in a narrow energy interval $\simeq T_m \Delta_\pm$ only. The spectral current F eventually suppresses the proximity effect. The explanation is that ABS form a reservoir of *normal* electrons within the unconventional superconductor, and F can be viewed as a connection to this normal reservoir. The effective transmission coefficient T^* at resonance is always bigger than 2, and is enhanced by the proximity effect. One can understand this as a *multiple* Andreev reflection induced by the corresponding ABS.

In the case of $\Delta_+ \Delta_- > 0$ ("conventional" channels) the resonance feature is absent and energy dependence can be safely disregarded. The expressions are identical to those of *conventional* superconductor

$$F = \frac{2T_m s \cos\gamma}{2 - T_m + T_m s \sin\gamma}, \tag{10}$$

$$T^* = \frac{2T_m [T_m + (2 - T_m) s \sin\gamma]}{|2 - T_m + T_m s \sin\gamma|^2}. \tag{11}$$

Here $s \equiv \mathrm{sgn}(\Delta_+) = \mathrm{sgn}(\Delta_-)$. The spectral current F thus induces the proximity effect of the corresponding sign s. The effective transmission T^* does not exceed 2 (which is the limiting case of the ideal Andreev reflection). Being compared with the transmission in the normal state, the effective transmission is suppressed (enhanced) at $T_m < (>)\ 2/3$. The fully developed proximity effect ($\gamma = s\pi/2$) restores the normal transmission.

To summarize, the proximity effect originates from the conventional channels and is suppressed by ABS channels. While the proximity effect is present, it enhances transmission via ABS channels. It restores the effective transmission of conventional channels to that in the normal state. The full resistance of the structure is determined by competition of all these effects. It is essential

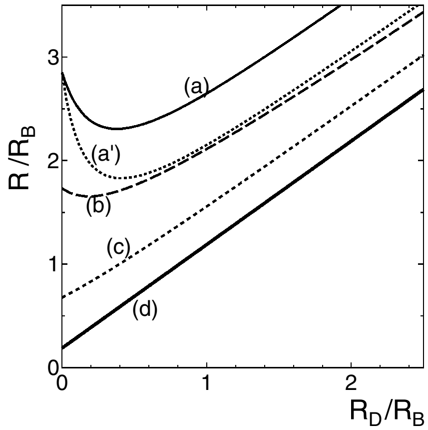

FIG. 2. Full resistance of the circuit versus R_D for various α. (a) $\alpha = 0$, (b) $\alpha = 0.01\pi$, (c) $\alpha = 0.05\pi$, and (d) $\alpha = 0.25\pi$. $Z = 1$. The curve (a') presents the same dependence for a conventional superconductor. Similar results for $\alpha = 0$ and $\alpha = \pi/4$ were obtained in [14] by numerical simulations.

that one can tune the relative number of conventional and ABS channels by changing the misorientation angle α. As one can see from Fig. 1, the ABS channels are there in the angle interval $\pi/4 - |\alpha| < |\theta| < \pi/4 + |\alpha|$. If $\alpha = 0$, there are no such channels. If $\alpha = \pi/4$, there are no conventional channels. This gives no chance to the proximity effect.

To illustrate this further, we calculate with Eqs. (6) and (7) the zero-voltage resistance ($\epsilon \to 0$) at different values of α as a function of R_B/R_D. The angular dependence of the transmission coefficient was assumed to be $T(\theta) = \cos^2\theta/(\cos^2\theta + Z)$ with barrier parameter Z. The results are presented in Figs. 2 and 3. At $R_D = 0$ there is no proximity effect in DN and the resistances are given by the quasiballistic formulas of Ref. [11]. The proximity effect may develop with increasing R_D and decreases the interface resistance. This gets the curves down. The curves 2(a) and 2(a') correspond to the d-wave junction at $\alpha = 0$ and conventional superconductor junction, respectively. One sees that the proximity effect is weaker in the d-wave system. This is due to competition of the conventional channels having different signs of Δ_+. The ABS channels appear with increasing α. The curve 2(b) demonstrates interface conductance reduced slightly below its normal state value. This manifests the enhanced transmission in ABS channels. The ABS channels quench the proximity effect very efficiently at $\alpha > 0.02\pi$. The total resistance can be approximated by $R_D + R_{R_D=0}$, although this becomes exact only at $\alpha = \pi/4$. Following Ref. [15], we regard the counterintuitive negative sign of $(dR/dR_D)_{R_D=0}$ as a signal of importance of the proximity effect [or reflectionless tunneling (RLT)]. We evaluate this sign at different Z and α (Fig. 3). The sign of dR/dR_D is negative for junctions of low transmissivity in a relatively narrow range of α.

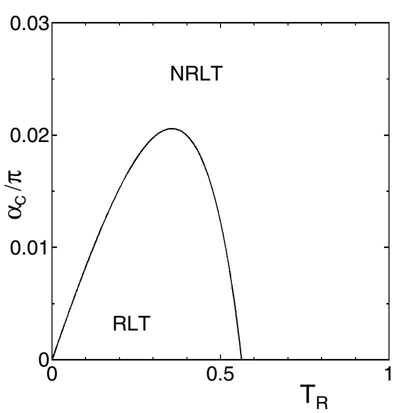

FIG. 3. "RLT" ("NRLT") marks the region where $dR/dR_D < 0$ ($dR/dR_D > 0$) at $R_D = 0$. T_R is the average transmissivity of the junction in the normal state, $T_R \equiv 1 - Z/(2\sqrt{Z+1})\ln[(\sqrt{Z+1}+1)/(\sqrt{Z+1}-1)]$ for the model in use.

In conclusion, we have extended the circuit theory of superconductivity to include unconventional superconductor junctions. We have derived a general relation for a matrix current to or from an unconventional superconductor. An elaborate example demonstrates the interplay of ABS and the proximity effect in a d-wave junction. The theory presented will facilitate the analysis of more complicated mesoscopic systems that include unconventional superconductors.

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