

**$e^+e^-$  Annihilation into  $J/\psi + J/\psi$** 

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Recent measurements by the Belle Collaboration of the exclusive production of two charmonia in  $e^+e^-$  annihilation differ substantially from theoretical predictions. We suggest that a significant part of the discrepancy can be explained by the process  $e^+e^- \rightarrow J/\psi + J/\psi$ . Because the  $J/\psi + J/\psi$  production process can proceed through fragmentation of two virtual photons into two  $c\bar{c}$  pairs, its cross section may be larger than that for  $J/\psi + \eta_c$  by about a factor of 3.7, in spite of a suppression factor  $\alpha^2/\alpha_s^2$  that is associated with the QED and QCD coupling constants.

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The charmonium states have long been a key testing ground for our understanding of quantum chromodynamics (QCD), both because of their clear experimental signatures and because of the theoretical simplifications that arise from their nonrelativistic nature. The huge data sample of continuum  $e^+e^-$ -annihilation events now being produced at the SLAC and KEK  $B$  factories makes it possible to investigate charmonium-production processes that have very small cross sections. The exclusive production of double-charmonium states is a particularly interesting example because the theoretical predictions do not involve any unknown nonperturbative parameters. However, the first such measurements by the Belle Collaboration [1] are in substantial disagreement with existing predictions. In this Letter, we argue that a significant part of the discrepancy between experiment and theory may be attributable to the process  $e^+e^-$  annihilation into  $J/\psi + J/\psi$ , where  $J/\psi$  is the lowest spin-triplet charmonium state. This process had been overlooked because it is suppressed by  $\alpha^2/\alpha_s^2$ , where  $\alpha$  is the quantum-electrodynamic (QED) coupling and  $\alpha_s$  is the QCD coupling. However, as we show, this process is enhanced by a kinematic factor that is associated with the fragmentation of photons into  $c\bar{c}$  pairs.

The Belle Collaboration has observed  $e^+e^-$  annihilation into two charmonium states at a center-of-mass energy  $\sqrt{s} = 10.6$  GeV by studying the recoil-momentum spectrum of the  $J/\psi$  [1]. The collaboration measured the production cross section for  $J/\psi + \eta_c$  and also found evidence for  $J/\psi + \chi_{c0}$  and  $J/\psi + \eta_c(2S)$  final states. Recent calculations of the production cross section for  $J/\psi + \eta_c$  have given results that are about an order of magnitude smaller than the Belle measurement [2,3]. This presents a challenge to our current understanding of charmonium production based on nonrelativistic QCD (NRQCD).

States consisting of two charmonia with opposite charge conjugation, such as  $J/\psi + \eta_c$ , can be produced

at order  $\alpha^2\alpha_s^2$  through processes  $e^+e^- \rightarrow c\bar{c} + c\bar{c}$  that involve only a single virtual photon (see Refs. [2,3] for examples). States consisting of two charmonia with the same charge conjugation, such as  $J/\psi + J/\psi$ , can be produced at order  $\alpha^4$  through the processes shown in the diagrams in Fig. 1, which involve two virtual photons. We find that, in spite of its being suppressed by a factor of  $\alpha^2/\alpha_s^2$ , the  $J/\psi + J/\psi$  production cross section may be larger than that for  $J/\psi + \eta_c$ . We suggest that some of the events in Belle's  $J/\psi + \eta_c$  signal may actually be  $J/\psi + J/\psi$  events. By taking this effect into account, one would decrease the discrepancy between the Belle measurements and the predictions of NRQCD.

The effective field theory NRQCD can be used to write a quarkonium production cross section as a sum of products of short-distance coefficients and NRQCD matrix elements [4]. The short-distance coefficients can be calculated in QCD perturbation theory, but the matrix elements are nonperturbative in nature. In the nonrelativistic limit, there is a single independent matrix element for each spin multiplet of charmonium states. The matrix

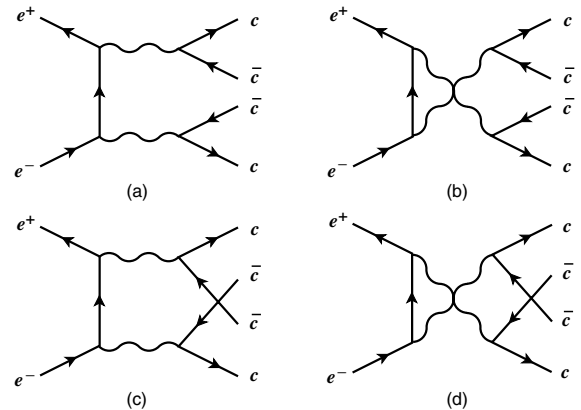


FIG. 1. QED diagrams for  $e^+e^- \rightarrow \gamma^*\gamma^* \rightarrow c\bar{c} + c\bar{c}$ . The upper and lower  $c\bar{c}$  pairs evolve into  $H_1$  and  $H_2$ , respectively.

elements can be determined phenomenologically from electromagnetic annihilation decay rates. Thus, the cross sections for double-charmonium production can be predicted up to relativistic corrections without any unknown nonperturbative factors.

It is convenient to express the cross sections for double-charmonium states in terms of the ratio  $R$ , which is defined by

$$R[H_1 + H_2] = \frac{\sigma[e^+e^- \rightarrow H_1 + H_2]}{\sigma[e^+e^- \rightarrow \mu^+\mu^-]}, \quad (1)$$

where  $\sigma[e^+e^- \rightarrow \mu^+\mu^-] = \pi\alpha^2/(3E_{\text{beam}}^2)$ , and  $E_{\text{beam}} = \sqrt{s}/2$ . We define the angular variable  $x = \cos\theta$ , where  $\theta$

is the angle between the  $J/\psi$  and the beam in the center-of-mass frame. We also introduce a dimensionless kinematic variable  $r = 2m_c/E_{\text{beam}}$ , with  $m_c$  the charm-quark mass. In the nonrelativistic limit, cross sections for the  $1S$  states  $J/\psi$  and  $\eta_c$  have a common NRQCD matrix element  $\langle O_1 \rangle_{1S}$  that is defined in Ref. [2].

The cross section for  $J/\psi + J/\psi$  receives contributions from the diagrams in Fig. 1. The angular distribution is

$$\frac{dR}{dx}[J/\psi + J/\psi] = \frac{16\pi^2\alpha^2}{243m_c^6} \langle O_1 \rangle_{1S}^2 \times \frac{(1-r^2)^{1/2}F(x)}{[4(1-r^2)(1-x^2) + r^4]^2}, \quad (2)$$

where

$$F(x) = r^4x^2(1-x^2)[10 - 3r^4 + r^6 - 4x^2(1-r^4)]^2 + 2(1+x^2)(1-x^2)[6 + r^2 - 4r^4 + r^6 - 4x^2r^2(1-r^2)]^2 + 2r^4x^2(1-x^2)[3 + 4r^2 - 4r^4 + r^6 - 4x^2r^2(1-r^2)]^2 + r^2[f^2(x) + f^2(-x)], \quad (3)$$

$$f(x) = (1+x)[(1-2x)(1+r^2)(2-r^2)(3-r^2) + 6x(1-r^2) - 4x^2(1-2x)r^2(1-r^2)]. \quad (4)$$

The ratio  $R$  is obtained by integrating  $x$  only from 0 to 1, in order to avoid double counting of identical final-state particles. The angular distribution  $dR/dx$  for  $J/\psi + \psi(2S)$  is given by an expression identical to that in Eq. (2), except that one of the factors of  $\langle O_1 \rangle_{1S}$  is replaced by  $\langle O_1 \rangle_{2S}$ , and the range of  $x$  is from  $-1$  to  $1$ .

The cross section for  $\eta_c + \eta_c$  receives contributions only from the two diagrams in Figs. 1(c) and 1(d). The angular distribution is

$$\frac{dR}{dx}[\eta_c + \eta_c] = \frac{16\pi^2\alpha^2}{243m_c^6} \langle O_1 \rangle_{1S}^2 \times r^4(1-r^2)^{5/2}x^2(1-x^2). \quad (5)$$

The ratio  $R$  is obtained by integrating  $x$  from 0 to 1. The angular distribution  $dR/dx$  for  $\eta_c + \eta_c(2S)$  is identical to that in Eq. (5), except that one of the factors of  $\langle O_1 \rangle_{1S}$  is replaced by  $\langle O_1 \rangle_{2S}$ , and the range of  $x$  is  $-1$  to  $1$ .

The production of  $J/\psi + \eta_c$  proceeds through  $e^+e^-$  annihilation into a single virtual photon, which creates a  $c\bar{c}$  pair. The second  $c\bar{c}$  pair is then created either by a virtual gluon or by a virtual photon radiated from the first  $c\bar{c}$  pair. The cross section was recently calculated by Braaten and Lee [2]. The differential ratio is

$$\frac{dR}{dx}[J/\psi + \eta_c] = \frac{4\pi^2}{2187m_c^6} \langle O_1 \rangle_{1S}^2 \times [3\alpha_s r^2 - \alpha(3-r^2)]^2 \times r^2(1-r^2)^{3/2}(1+x^2). \quad (6)$$

The ratio  $R$  is obtained by integrating over  $x$  from  $-1$

to 1. The  $\alpha_s^2$  term was also calculated recently by Liu, He, and Chao [3]. It had been calculated previously, but the analytic expression was not given [5]. The QED terms decrease the cross section by about 20% for  $E_{\text{beam}} = 5.3$  GeV.

When  $E_{\text{beam}}$  is much greater than  $m_c$ , the relative sizes of the double-charmonium cross sections are governed not only by the powers of the coupling constants, but also by the number of kinematic suppression factors  $r$ . The power-counting rules of perturbative QCD [6] require  $dR/dx$  to behave as  $r^4$  in the limit  $r \rightarrow 0$ , with a further suppression factor of  $r^{2|\lambda_1+\lambda_2|}$  if the sum of the charmonium helicities,  $\lambda_1$  and  $\lambda_2$ , does not vanish. In the limit  $r \rightarrow 0$ ,  $dR/dx$  for  $\eta_c + \eta_c$  scales as  $\alpha^2 r^4$ , while  $dR/dx$  for  $J/\psi + \eta_c$  scales as  $\alpha_s^2 r^6$ , with the extra factor of  $r^2$  arising from helicity suppression. As  $r \rightarrow 0$  with  $x$  fixed,  $dR/dx$  for  $J/\psi + J/\psi$  approaches a constant. The helicity selection rules of perturbative QCD are evaded because this process has a contribution, corresponding to the diagrams of Figs. 1(a) and 1(b), in which two virtual photons, with virtuality of order  $m_c$ , fragment into two  $c\bar{c}$  pairs. This contribution is enhanced because the virtual-photon propagators are of order  $1/m_c^2$  instead of order  $1/E_{\text{beam}}^2$ . In the amplitude, there are also two numerator factors of  $m_c$  instead of  $E_{\text{beam}}$ , which arise from the  $c\bar{c}$  electromagnetic currents. Hence, the net enhancement of the squared amplitude is  $(E_{\text{beam}}/m_c)^4$ . The integrated ratio is further enhanced by a factor  $\ln(1/r)$  because the potential logarithmic divergence in the integral of Eq. (2) as  $x \rightarrow 1$  is cut off at  $1-x^2 \sim r^4$ .

Chang, Qiao, and Wang have pointed out that the cross section for  $e^+e^- \rightarrow J/\psi + \gamma$  is enhanced if  $E_{\text{beam}}$  is much greater than  $m_c$  [7]. The reason for the enhancement is that this process also involves a contribution in which a virtual photon fragments into a  $c\bar{c}$  pair.

We use the results given above to predict the cross sections for producing two  $S$ -wave charmonium states at the  $B$  factories, with  $E_{\text{beam}} = 5.3$  GeV. The ratios  $R$  depend on the coupling constants  $\alpha$  and  $\alpha_s$ , the charm-quark mass  $m_c$ , and the NRQCD matrix elements. We take the QCD coupling constant to be  $\alpha_s = 0.21$ , which corresponds to the renormalization scale 5.3 GeV. For  $m_c$ , we use the next-to-leading order pole mass, which we take to be  $m_c = 1.4 \pm 0.2$  GeV. The NRQCD matrix elements can be determined from the electromagnetic annihilation decay rates of  $J/\psi$  and  $\psi(2S)$ . We use the values from the analysis of Ref. [2]. For  $m_c = 1.4$  GeV, they are  $\langle O_1 \rangle_{1S} = 0.335 \pm 0.024$  GeV<sup>3</sup> and  $\langle O_1 \rangle_{2S} = 0.139 \pm 0.010$  GeV<sup>3</sup>, where the error bars are those associated with the experimental uncertainties only. For other values of  $m_c$ , the NRQCD matrix elements should be multiplied by  $(m_c/1.4 \text{ GeV})^2$ .

Our predictions for the double-charmonium cross sections for  $S$ -wave states are given in Table I. The error bars are those associated with the uncertainty in the pole mass  $m_c$  only. The small error bars for  $\eta_c + \eta_c$  in Table I are a consequence of the value of  $m_c$  being fortuitously close to a zero in the derivative of the cross section with respect to  $m_c$ . The cross section for  $\eta_c + \eta_c$  is about 3 orders of magnitude smaller than that for  $J/\psi + \eta_c$ . This suppression comes primarily from the coupling-constant factor  $\alpha^2/\alpha_s^2$ . The cross section for  $J/\psi + J/\psi$  is larger than that for  $J/\psi + \eta_c$ . The suppression factor of  $\alpha^2/\alpha_s^2$  is more than compensated by the kinematic enhancement factor that scales as  $r^{-6}$ . The cross section for  $J/\psi + J/\psi$  is dominated by the photon-fragmentation diagrams in Figs. 1(a) and 1(b). For  $m_c = 1.4$  GeV, they contribute 87.5% of the cross section. In Ref. [2], it was pointed out that there may be large perturbative-QCD and relativistic corrections to the production cross section for  $J/\psi + \eta_c$ . There may also be large perturbative-QCD

TABLE I. Cross sections in fb for  $e^+e^-$  annihilation into two  $S$ -wave charmonium states  $H_1 + H_2$  at  $E_{\text{beam}} = 5.3$  GeV. The errors are only those from variations in the pole mass  $m_c = 1.4 \pm 0.2$  GeV. There are additional large errors associated with perturbative-QCD and relativistic corrections, as described in the text.

$H_1 + H_2$	$\sigma$ (fb)
$J/\psi + J/\psi$	$8.70 \pm 2.94$
$J/\psi + \psi(2S)$	$7.22 \pm 2.44$
$\psi(2S) + \psi(2S)$	$1.50 \pm 0.51$
$J/\psi + \eta_c$	$2.31 \pm 1.09$
$J/\psi + \eta_c(2S)$	$0.96 \pm 0.45$
$\psi(2S) + \eta_c$	$0.96 \pm 0.45$
$\psi(2S) + \eta_c(2S)$	$0.40 \pm 0.19$
$\eta_c + \eta_c$	$(1.83 \pm 0.10) \times 10^{-3}$
$\eta_c + \eta_c(2S)$	$(1.52 \pm 0.08) \times 10^{-3}$
$\eta_c(2S) + \eta_c(2S)$	$(0.31 \pm 0.02) \times 10^{-3}$

and relativistic corrections to the cross section for  $J/\psi + J/\psi$ . These corrections would affect not only the absolute cross sections in Table I, but also the ratios of cross sections. For the  $J/\psi + J/\psi$ ,  $J/\psi + \psi(2S)$ , and  $\psi(2S) + \psi(2S)$  cross sections, the dominant photon-fragmentation contributions receive perturbative-QCD correction factors of about 0.39 and relativistic correction factors of about 0.78, 0.62, and 0.49, respectively [8].

The angular distributions  $d\sigma/d|x|$  for  $m_c = 1.4$  GeV are shown in Fig. 2. At  $x = 0$ , the differential cross section for  $J/\psi + J/\psi$  is larger than that for  $J/\psi + \eta_c$  by about a factor of 2. However, the differential cross section for  $J/\psi + J/\psi$  is strongly peaked near the beam direction at  $x = 0.994$ , where it is larger than that for  $J/\psi + \eta_c$  by about a factor of 16. The reason for the sharp peak is evident from the denominator in Eq. (2).

The Belle Collaboration has recently measured the cross section for  $J/\psi + \eta_c$  by observing a peak in the momentum spectrum of the  $J/\psi$  that corresponds to the two-body final state  $J/\psi + \eta_c$  [1]. The measured cross section is

$$\sigma[J/\psi + \eta_c] \times B[\geq 4] = (33_{-6}^{+7} \pm 9) \text{ fb}, \quad (7)$$

where  $B[\geq 4]$  is the branching fraction for the  $\eta_c$  to decay into at least four charged particles. Since  $B[\geq 4] < 1$ , the right side of Eq. (7) is a lower bound on the cross section to produce  $J/\psi + \eta_c$ . This lower bound is about an order of magnitude larger than the predictions of NRQCD in the nonrelativistic limit [2,3]. Large relativistic corrections may account for part of the discrepancy [2]. There may also be large nonperturbative corrections associated with the double-charmonium cross section being a significant fraction of the total  $c\bar{c} + c\bar{c}$

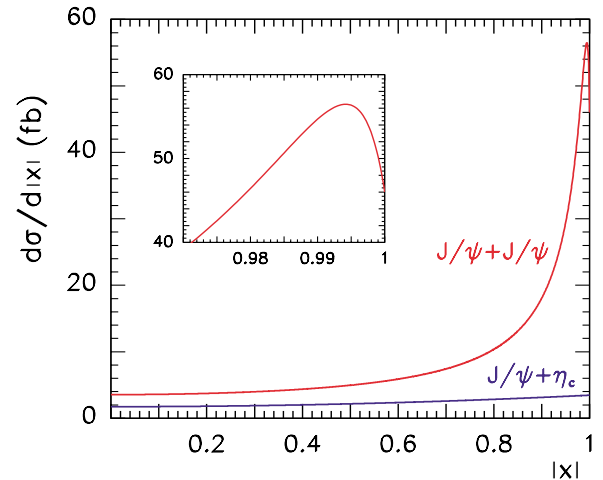


FIG. 2 (color online). Differential cross sections  $d\sigma/d|x|$  for  $e^+e^-$  annihilation into  $J/\psi + J/\psi$  and  $J/\psi + \eta_c$  at  $E_{\text{beam}} = 5.3$  GeV. The areas under the curves are the integrated cross sections 8.70 and 2.31 fb. There are large errors associated with perturbative-QCD and relativistic corrections, as described in the text.

cross section [3]. However, the large discrepancy between the NRQCD predictions and the Belle measurement is still disturbing. This discrepancy can be decreased by taking into account the process  $e^+e^- \rightarrow J/\psi + J/\psi$ . In the Belle fit to the  $J/\psi$  momentum distribution, the full width at half maximum of the  $\eta_c$  peak is about 0.11 GeV. Since the mass difference between the  $J/\psi$  and the  $\eta_c$  is about 0.12 GeV, there are probably  $J/\psi + J/\psi$  events that contribute to the  $J/\psi + \eta_c$  signal observed by Belle.

The Belle Collaboration also saw evidence for  $J/\psi + \chi_{c0}(1P)$  and  $J/\psi + \eta_c(2S)$  events. The  $\eta_c(2S)$  was recently discovered by the Belle Collaboration at a mass  $M_{\eta_c(2S)} = 3654 \pm 6 \pm 8$  MeV [9]. Since the mass difference between the  $\psi(2S)$  and the  $\eta_c(2S)$  is only about 0.03 GeV, it is likely that any signal for  $J/\psi + \eta_c(2S)$  in the  $J/\psi$  momentum spectrum is contaminated by  $J/\psi + \psi(2S)$  events. A three-peak fit to the data for the momentum spectrum of the  $J/\psi$  gives approximately 67, 39, and 42 events with an accompanying  $\eta_c$ ,  $\chi_{c0}(1P)$ , or  $\eta_c(2S)$ , with an uncertainty of 12–15 events for each final state. The predictions in Ref. [2] for the relative cross sections for production of a  $J/\psi$  with an accompanying  $\eta_c$ ,  $\chi_{c0}(1P)$ , or  $\eta_c(2S)$  are 1.00, 0.99, and 0.42, respectively. The observed proportion of events is only marginally compatible with the NRQCD predictions. If significant fractions of the  $J/\psi + \eta_c$  and  $J/\psi + \eta_c(2S)$  signals are actually  $J/\psi + J/\psi$  and  $J/\psi + \psi(2S)$  events, then the data would be more compatible with the NRQCD predictions.

There are also some unresolved puzzles in the inclusive  $J/\psi$  cross sections that have been measured by the Belle and BABAR Collaborations [10,11]. There are significant discrepancies between their measurements of both the production-angle distributions of the  $J/\psi$  and the  $J/\psi$  polarization. There are also significant discrepancies between the measurements and predictions based on NRQCD [12]. These discrepancies may be decreased by taking into account photon-fragmentation contributions to the process  $e^+e^- \rightarrow J/\psi + q\bar{q}$ , which allow the order- $\alpha^4$  QED contribution to compete with the order- $\alpha^2\alpha_s^2$  QCD contribution. The two experiments impose different cuts to decrease the background from the two-photon processes  $e^+e^- \rightarrow e^+e^-\gamma^*\gamma^*$  and from initial-state-radiation processes such as  $e^+e^- \rightarrow \psi(2S) + \gamma$  and  $e^+e^- \rightarrow J/\psi + \ell^+\ell^-$ . These cuts remove substantial parts of the photon-fragmentation contribution to  $J/\psi + q\bar{q}$ . The differences between the experiments could be due to differences between the cuts. The discrepancies with the NRQCD predictions could be due partly to the neglect of the order- $\alpha^4$  QED terms in the theoretical predictions.

Another puzzling result comes from the measurement by the Belle Collaboration of the fraction of the inclusive  $J/\psi$  cross section that is attributable to the final state  $J/\psi + c\bar{c}$  [1]. That fraction is much larger than predic-

tions based on NRQCD. The discrepancies between the predictions and the measurements would be decreased by taking into account the contributions to the processes  $e^+e^- \rightarrow J/\psi + q\bar{q}$  and  $e^+e^- \rightarrow J/\psi + c\bar{c}$  from photon fragmentation into  $J/\psi$ . The experimental cuts allow the photon-fragmentation contribution to  $J/\psi + c\bar{c}$  to survive, but they remove substantial parts of the photon-fragmentation contribution to  $J/\psi + q\bar{q}$ . Thus, the net effect of the photon-fragmentation processes is to increase the ratio of  $J/\psi + c\bar{c}$  events to  $J/\psi + X$  events.

In summary, we have calculated the cross section for  $e^+e^-$  annihilation into  $J/\psi + J/\psi$ . The calculated cross section is larger than that for  $e^+e^- \rightarrow J/\psi + \eta_c$  by about a factor of 3.7 because it receives contributions from the process in which two virtual photons fragment into two  $c\bar{c}$  pairs. The inclusion of this process in the analysis may decrease the large discrepancy between the Belle measurement of the production cross section for  $J/\psi + \eta_c$  and the predictions based on NRQCD.

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