Measuring the Transmission Phase of a Quantum Dot in a Closed Interferometer

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The electron transmission through a *closed* Aharonov-Bohm mesoscopic solid-state interferometer, with a quantum dot (QD) on one of the paths, is calculated exactly for a simple model. Although the conductance is an even function of the magnetic flux (due to Onsager's relations), in many cases one can use the measured conductance to extract both the amplitude and the phase of the "intrinsic" transmission amplitude $t_D = -i|t_D|e^{i\alpha_D}$ through the "bare" QD. We also propose to compare this indirect measurement with the (hitherto untested) direct relation $\sin^2(\alpha_D) \equiv |t_D|^2/\max(|t_D|^2)$.

Recent advances in nanoscience raised interest in quantum dots (QDs), which represent artificial atoms with experimentally controllable properties [1,2]. Connecting the QD via metallic leads to electron reservoirs yields resonant transmission through the QD, with peaks whenever the Fermi energy in the leads crosses a resonance on the QD. The energies of the latter are varied by controlling the plunger gate voltage on the QD, V. The quantum information on the tunneling of an electron is contained in the complex transmission amplitude, $t_D = -i\sqrt{\mathcal{T}_D}e^{i\alpha_D}$. The phase α_D is particularly interesting, given its relation to the additional electron occupation in the system via the Friedel sum rule [3,4]. This phase is also predicted to exhibit interesting behavior, e.g., near a Kondo-like resonance [5]. This motivated experimental attempts to measure α_D [6,7], using the Aharonov-Bohm interferometer (ABI) [8].

In the ABI, the QD is placed on one branch, in parallel to a "reference" branch (both connecting the two external leads). A magnetic flux Φ in the area between the two branches creates a phase difference $\phi = e\Phi/\hbar c$ between the wave functions in the two branches [9]. In the *two-slit limit*, the total ABI transmission is

$$\mathcal{T} = |t|^2 = |t_D e^{i\phi} + t_B|^2 = A + B\cos(\phi + \beta), \quad (1)$$

with $\beta = \alpha_D - \kappa$, where κ contains V-independent contributions from the reference transmission, $t_B = -i|t_B|e^{i\delta_B}$, and from the electron "optical" paths on the two branches. However, for the "closed" two-terminal geometry, unitarity (conservation of current) and time reversal symmetry imply the Onsager relations [10], which state that the two-terminal conductance, $\mathbf{G} = (e^2/h)\mathcal{T}$, is an *even function* of ϕ. Therefore, a naive fit of the experimental transmission to Eq. (1) *must* yield $\beta = 0$ or π—with no relation to α_D. Indeed, the experimental data [6] for $\mathcal{T}$ depend only on $\cos\phi$ [11].

Aiming to measure a *nontrivial* AB phase shift β then led to experiments with "open" interferometers [7,12],

which contain additional "leaky" channels, breaking the Onsager symmetry. A fit to Eq. (1) then yields a phase β which increases (with V) *gradually* from 0 to π through each resonance. However, the detailed V dependence of β *depends on the strength* of the coupling to the additional terminals [13]. Although it is possible to optimize this strength, and reproduce the two-slit conditions [14], this involves large uncertainties.

In this Letter, we present exact results for the total transmission of the closed ABI, $\mathcal{T}$. Although $\mathcal{T}$ is even in ϕ, contradicting the simple two-slit Eq. (1), it *does* depend on *both* $\mathcal{T}_D$ and α_D. Under appropriate conditions (see below), one can thus extract α_D from the measured $\mathcal{T}$, *eliminating the need to open the interferometer*. This possible extraction was not noticed in earlier discussions of the closed ABI. Theoretical analyses used the Keldysh technique, combined with the wideband and related approximations [15,16], or ignored electron-electron interactions [17]. These approximations, which sometimes miss important features of the results (see below), are avoided in our calculation, which is done in the linear response limit, and at temperature $T = 0$.

We demonstrate our results for a simple lattice model, shown in Fig. 1: For $\Phi = 0$, each (unit length) segment in the figure represents a real tight-binding hopping matrix element $-J$, $-I_L$, $-I_R$, $-J_L$, and $-J_R$, as indicated. All
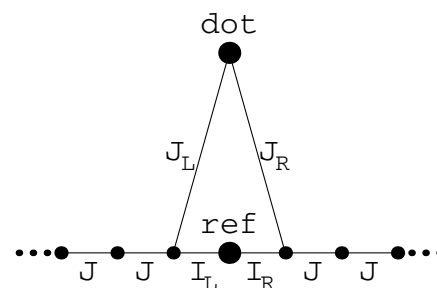

FIG. 1.   Model for the closed ABI.

the on-site energies are zero, except ϵ_D on the site "dot" and ϵ_0 on the site "ref" (which sits on the reference path and represents a simple point contact, a tunnel junction, etc.). The latter two energies can be varied experimentally by the plunger (or point contact) gate voltages $\epsilon_D \equiv V$ and $\epsilon_0 \equiv V_0$ [11]. As usual for such models, electron-electron interactions are included only via an on-site Hubbard interaction U on the QD. The AB phase in the triangle, $\phi = \phi_L + \phi_R$, is included by attaching a factor $e^{i\phi_L}$ ($e^{i\phi_R}$) to the hopping matrix element J_L (J_R). At $T = 0$, the electron energy $\epsilon_k = -2J \cos k$ is equal to the Fermi energy on the leads, ϵ_F, and we calculate the transmission for electrons with spin σ.

We start by reviewing the intrinsic transmission through the QD, without the reference path (e.g., for large $|V_0| = |\epsilon_0|$, or with $I_L = I_R = 0$). Adapting the results of Ref. [18], one has

$$t_D = -i\gamma_D \sin\alpha_D e^{i\alpha_D} \equiv 2i \sin|k| J_L J_R g_D(\epsilon_k)/J, \quad (2)$$

with the QD asymmetry factor $\gamma_D = 2J_L J_R/(J_L^2 + J_R^2)$ and the intrinsic Green function on the QD, $g_D(\epsilon_k) = 1/[\epsilon_k - \epsilon_D - \Sigma_D(\epsilon_k)]$. Here, $\Sigma_D(\epsilon_k)$ is the self-energy on the QD, which contains contributions from the leads, $\Sigma_{D,\text{ext}} = -e^{i|k|}(J_L^2 + J_R^2)/J$ (which exists also for the non-interacting case [13]), and from the electron-electron interactions on the QD itself, $\Sigma_{D,\text{int}}(\omega)$ (which vanishes when $U = 0$). As $\epsilon_D \equiv V$ increases, α_D grows gradually from zero (far below the resonance), through $\pi/2$ (at the resonance), towards π (far above the resonance).

Interestingly, for this one-dimensional model, normalizing the measured $\mathcal{T}_D = |t_D|^2 = \gamma_D^2 \sin^2(\alpha_D)$ by its (V-independent) maximum γ_D^2 yields the value of α_D. Assuming coherence, this (hitherto ignored) method for measuring α_D directly from $\mathcal{T}_D$ eliminates the need for any complicated interferometry [19]. In the remainder of this Letter, we discuss ways of extracting α_D indirectly, from the closed ABI measurements. Comparing results from $\sin^2(\alpha_D) = \mathcal{T}_D/\gamma_D^2$, from the closed ABI (below), and from the open ABI [14] (all with the same QD) should serve as consistency checks for this conclusion.

The same analysis yields the transmission amplitude through the reference path (when, e.g., $J_L = J_R = 0$),

$$t_B = -i\gamma_B \sin\delta_B e^{i\delta_B} \equiv 2i \sin|k| I_L I_R g_B(\epsilon_k)/J, \quad (3)$$

with the bare reference site Green function $g_B = 1/[\epsilon_k - \epsilon_0 + e^{i|k|}(I_L^2 + I_R^2)/J]$, and $\gamma_B = 2I_L I_R/(I_L^2 + I_R^2)$. In the two-slit situation, Eqs. (2) and (3) suffice to determine the overall transmission, as in Eq. (1). However, the situation is more complicated for the closed ABI. The main result of this Letter concerns the exact transmission amplitude through the closed ABI,

$$t = A_D t_D e^{i\phi} + A_B t_B, \quad (4)$$

where we find $A_D = g_B(\epsilon_k - \epsilon_0)G_D(\epsilon_k)/g_D(\epsilon_k)$ and $A_B = 1 + G_D(\epsilon_k)\Sigma_{\text{ext}}(\epsilon_k)$. Here, $G_D(\omega) = 1/[\omega - \epsilon_D - \Sigma(\omega)]$

is the fully "dressed" Green function on the QD, with the dressed self-energy $\Sigma = \Sigma_{\text{int}} + \Sigma_{\text{ext}}$. Both terms here differ from their counterparts in the intrinsic Σ_D, by contributions due to the reference path. Equation (4) looks like the two-slit formula, $t = t_D e^{i\phi} + t_B$. However, each of the terms is now renormalized: A_D contains all the additional processes in which the electron "visits" the reference site ($A_D = 1$ when $I_L = I_R = 0$), and A_B contains the corrections to t_B due to visits on the dot. In fact, a physical derivation of Eq. (4) amounts to starting from Eq. (2), and adding an infinite power series in I_L and I_R. We now discuss the ϕ dependence of $\mathcal{T} \equiv |t|^2$, in connection with the Onsager relations and with the possible indirect extraction of α_D.

We first note that both parts in $\Sigma(\epsilon_k)$ are even in ϕ, due to additive contributions (with equal amplitudes) from clockwise and counterclockwise motions of the electron around the ring (see, e.g., Refs. [8,13,17,20]). In order that $\mathcal{T}$ also depends only on $\cos\phi$, as required by the Onsager relations, the ratio $K \equiv A_B t_B/(A_D t_D) \equiv \tilde{x}[G_D(\epsilon_k)^{-1} + \Sigma_{\text{ext}}(\epsilon_k)]$, with the real coefficient $\tilde{x} = I_L I_R/[J_L J_R(\epsilon_k - \epsilon_0)]$, must be real, i.e.,

$$\text{Im}[G_D(\epsilon_k)^{-1} + \Sigma_{\text{ext}}(\epsilon_k)] \equiv \text{Im}\,\Sigma_{\text{int}} \equiv 0. \quad (5)$$

The same relation follows from the unitarity of the 2×2 scattering matrix of the ring. This relation already appeared for the special case of single impurity scattering, in connection with the Friedel sum rule [4], and was implicitly contained in Eq. (2), where $\text{Im}\,\Sigma_{D,\text{int}} = 0$ [18]. Equation (5) implies that (at $T = 0$ and $\omega = \epsilon_k$) the interaction self-energy $\Sigma_{\text{int}}(\epsilon_k)$ is real, and therefore the width of the resonance, $\text{Im}\,G_D(\epsilon_k)^{-1}$, is fully determined by the noninteracting self-energy $\text{Im}\,\Sigma_{\text{ext}}(\epsilon_k)$.

Since $\Sigma_{\text{ext}}(\omega)$ depends only on the (noninteracting) tight-binding terms, it is easy to calculate it explicitly. We find $\Sigma_{\text{ext}}(\epsilon_k) = \Sigma_{D,\text{ext}}(\epsilon_k) + \Delta_{\text{ext}}$, where

$$\Delta_{\text{ext}} = e^{2i|k|} g_B(J_L^2 I_L^2 + J_R^2 I_R^2 + 2J_L J_R I_L I_R \cos\phi)/J^2. \quad (6)$$

The term proportional to $\cos\phi$ comes from the electron clockwise and counterclockwise motion around the ABI "ring." Similarly, one can write $\Sigma_{\text{int}}(\epsilon_k) = \Sigma_{D,\text{int}}(\epsilon_k) + \Delta_{\text{int}}$, and thus $G_D(\epsilon_k)^{-1} = g_D(\epsilon_k)^{-1} - \Delta$, with $\Delta = \Delta_{\text{ext}} + \Delta_{\text{int}}$. Hence, $t = A_D t_D(e^{i\phi} + K)$. Writing also $A_D = C/[1 - g_D(\epsilon_k)\Delta]$, with $C = (\epsilon_k - \epsilon_0)g_B$, we find

$$\mathcal{T} = |C|^2 \mathcal{T}_D \frac{1 + K^2 + 2K \cos\phi}{1 - 2\,\text{Re}[g_D\Delta] + |g_D\Delta|^2}. \quad (7)$$

Equation (7) presents an alternative form of our main result. Although the numerator looks like the two-slit Eq. (1), with $\beta = 0$ or π (depending on sign K), the new physics is contained in the denominator — which becomes important in the vicinity of a resonance. The central term in this denominator depends explicitly on the phase of the complex number g_D. Since this number is directly related to t_D, via Eq. (2), one may expect to

extract α_D from a fit to Eq. (7), taking advantage of the dependence of the denominator on $\cos\phi$. Physically, this dependence originates from the infinite sum over electron paths which circulate the ABI ring. The rest of this Letter is devoted to the conditions for such an extraction. Generally, this is not trivial, as one needs the detailed dependence of Δ on $\cos\phi$ and on the various parameters. We have presented this dependence for Δ_{ext}, but not for Δ_{int}.

The extraction of α_D becomes easy when one may neglect Δ_{int}. The simplest case for this is for single-electron scattering, when $\Sigma_{int} = 0$. Interactions (i.e., U) are also negligible for a relatively *open* dot, with small barriers at its contacts with the leads [21]. Another effectively single-electron scattering case arises near a Coulomb blockade resonance, when the effect of interactions can simply be absorbed into a Hartree-like shift, $\epsilon_D + \Sigma_{int} \to \epsilon_D + NU$, if one *assumes* that N depends smoothly on the number of electrons on the QD, and not on ϕ [20]. If one may neglect Δ_{int}, then $\Delta \approx \Delta_{ext}$ is given in Eq. (6). Using also Eqs. (2) and (3), we find

$$\mathcal{T} = |C|^2 \mathcal{T}_D \frac{1 + K^2 + 2K\cos\phi}{1 + 2P(z + \cos\phi) + Q(z + \cos\phi)^2}, \quad (8)$$

where $z = (J_L^2 I_L^2 + J_R^2 I_R^2)/(2J_L J_R I_L I_R)$, $P = \text{Re}[\upsilon t_B t_D]$, $Q = |\upsilon t_B|^2 \mathcal{T}_D$, and $\upsilon = e^{2i|k|}/(2\sin^2|k|)$ depends only on the Fermi wave vector k, independent of any detail of the ABI. A five-parameter fit to the explicit ϕ dependence in Eq. (8) for given values of V and V_0 then yields $|C|^2 \mathcal{T}_D, K, z, P$, and Q, and, thus, $\cos(\alpha_D + \delta_B + 2|k|) = P/\sqrt{Q}$, from which one can extract the V dependence of α_D. The same V dependence of α_D is also contained in $K \propto (\cot\alpha_D + \cot|k|)$. As discussed after Eq. (2), our model also implies that $\mathcal{T}_D = \gamma_D^2 \sin^2(\alpha_D)$. Since the V dependence of $\mathcal{T}_D$ can also be extracted from the fitted values of either $|C|^2 \mathcal{T}_D$ or Q, we end up with several consistency checks for the determination of α_D. Additional checks arise from direct measurements of $\mathcal{T}_D$ and $\mathcal{T}_B = |t_B|^2$, by taking the limits $|V_0| = |\epsilon_0| \to \infty$ or $|V| = |\epsilon_D| \to \infty$.

The left frame in Fig. 2 shows an example of the V and ϕ dependence of $\mathcal{T}$ for this limit (no interactions), with $k = \pi/2$ and $J_L = J_R = I_L = I_R = 1$, $V_0 = 4$ (in units of J), implying $K = \epsilon_D/\epsilon_0 = V/V_0$. Far away from the resonance $\mathcal{T} \ll 1$, $Q \ll |P| \ll 1$ and $|K| \gg 1$, yielding the two-slit-like form $\mathcal{T} \approx A + B\cos\phi$, dominated by its first harmonic, with $B/A \approx 2[K^{-1} - P]$. However, close to the resonance $\mathcal{T}$ shows a rich structure; the denominator in Eq. (8) generates higher harmonics, and the two-slit formula is completely wrong. This rich structure may be missed if one neglects parts of the ϕ dependence of Δ, as done in Ref. [16]. Note also the Fano vanishing [22] of $\mathcal{T}$ for $V \sim 10$ at $\phi = 2n\pi$, with integer n. Without interactions, everything can be extended to a QD with many resonances, e.g., due to Coulomb blockade shifts in the effective ϵ_d with the number of electrons. Using a gen-
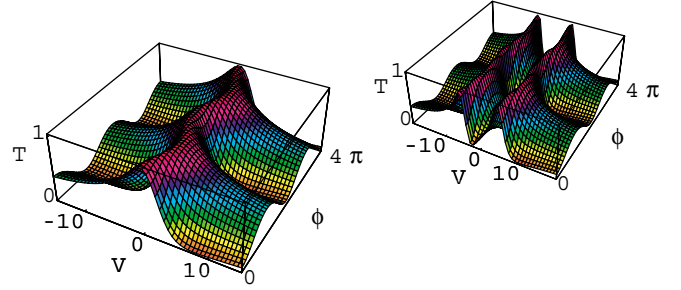

FIG. 2 (color). AB transmission $\mathcal{T}$ versus the AB phase ϕ and the gate voltage V, for one (left) and two (right) non-interacting resonances.

eralization to Eq. (8), given in Ref. [14], the right frame in Fig. 2 shows results for two resonances, with $\epsilon_D = \pm 5$. Interestingly, Fig. 2 is qualitatively similar to the experimentally measured transmission in Ref. [11]. However, thus far there has been no quantitative analysis of the experimental data.

To treat the general case, we need information on Δ_{int}. First of all, we emphasize *that a successful fit to Eq. (8) justifies the neglect of the ϕ dependence* of Δ_{int}. If the various procedures to determine α_D from Eq. (8) yield the same V dependence, this would also confirm that Δ_{int} is negligibly small. A failure of this check, or a more complicated dependence of the measured $\mathcal{T}$ on $\cos\phi$, would imply that Δ_{int} is not negligible.

As seen from Eq. (6), Δ_{ext} is fully determined by a single "visit" of the electron at "ref." For small $\mathcal{T}_B$, or large $|V_0| = |\epsilon_0|$, it is reasonable to conjecture that Δ_{int} is also dominated by such processes. In that case, we expect Δ_{int} to be proportional to the same brackets as in Eq. (6); i.e., $\Delta_{int} \approx w(z + \cos\phi)$, with a real coefficient w. This yields the same dependence of $\mathcal{T}$ on $\cos\phi$ as in Eq. (8), with a shifted coefficient υ. If w depends only weakly on V, then this shift has little effect on the determination of α_D. Again, the validity of this approach relies on getting the same V dependence of α_D from all of its different determinations.

The situation becomes more complicated near a Kondo-like resonance. Maintaining the (nontrivial) assumption that $G_D = 1/[\omega - \epsilon_D - \Sigma_D(\omega)]$, the Kondo peak at the Fermi energy must be generated by Σ_D. For the intrinsic QD, this yields $\alpha_D = \pi/2$ and $t_D = \gamma_D$, resulting in a V independent plateau for $\mathcal{T}_D$. A priori, it is not obvious what happens in the presence of the reference path. Hofstetter et al. identified the Kondo region by requiring that the phase δ_{res} of the fully dressed Green function G_D be equal to $\pi/2$. Our result for G_D shows that this is impossible: The phase δ_{res} *depends* on ϕ, via the ϕ dependence of Δ, and thus *cannot* be set at the constant value $\pi/2$. (Apparently, this ϕ dependence was neglected in the analytic parts, and was weak for the numerical parameters used in Ref. [16].) Alternatively, one might assume that the bare QD sticks to the Kondo resonance,

and thus $\alpha_D = \pi/2$ (independent of V) even in the ABI. Equation (7) then replaces the Kondo plateau by a complicated dependence on ϕ (including the first harmonic), which differs significantly from that of Ref. [16]. Clearly, this limit requires more research.

Finally, we give some more details of our derivation. Our Hamiltonian, which simply adds the reference path to that of Ng and Lee [18], is

$$
\mathcal{H} = \epsilon_D \sum_\sigma d_\sigma^\dagger d_\sigma + \frac{U}{2} \sum_\sigma n_{d\sigma} n_{d\overline\sigma} + \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma}
$$
$$
+ \sum_{k\sigma} (\mathcal{V}_k d_\sigma^\dagger c_{k\sigma} + \mathcal{V}_k^* c_{k\sigma}^\dagger d_\sigma) + \epsilon_0 \sum_\sigma c_{0\sigma}^\dagger c_{0\sigma}
$$
$$
+ \sum_{k\sigma} (\mathcal{U}_k c_{0\sigma}^\dagger c_{k\sigma} + \mathcal{U}_k^* c_{k\sigma}^\dagger c_{0\sigma}), \tag{9}
$$

where $c_{k\sigma}^\dagger$ creates single-particle eigenstates (with spin σ) on the unperturbed "background" chain (with $I_L = I_R = J$, $J_L = J_R = 0$), with eigenenergy $\epsilon_k = -2J\cos k$, while $c_{0\sigma} = \sum_k c_{k\sigma}/\sqrt{N}$, $\mathcal{U}_k = -[(I_L - J)e^{-ik} + (I_R - J)e^{ik}]/\sqrt{N}$, and $\mathcal{V}_k = -(J_L e^{i\phi_\ell - ik} + J_R e^{-i\phi_r + ik})/\sqrt{N}$. The operators on the dot, d_σ and $d_\sigma^\dagger$, anticommute with $c_{k\sigma}$, $c_{k\sigma}^\dagger$. Also, $n_{d\sigma} = d_\sigma^\dagger d_\sigma$, and $\overline\sigma \equiv -\sigma$.

As stated above, one can derive Eq. (4) by adding to Eq. (2) a power series in I_L and I_R. A more general approach uses the standard relation between the 2×2 scattering matrix $T_{kk'}$ and the matrix of retarded single-particle Green functions, $G_{kk'}(\omega) = \delta_{kk'} g_k^0 + g_k^0 T_{kk'}^\sigma g_{k'}^0$, with $g_k^0(\omega) = 1/(\omega - \epsilon_k)$, evaluated on the energy shell, $\omega = \epsilon_F = \epsilon_k = \epsilon_{k'}$ [5]. The equation-of-motion (EOM) method is then used to express $(\omega - \epsilon_k)G_{kk'}(\omega)$ and $(\omega - \epsilon_k)G_{kd}(\omega)$ as linear combinations of each other and of $G_D(\omega)$, allowing us to express each of them (and thus also $t \propto T_{|k|,|k|}$) in terms of $G_D(\omega)$, yielding Eq. (4). Since we do not use an explicit solution for $G_D(\omega)$ itself, we do not need to deal with the higher order correlation functions (due to U), which appear in its EOM.

We hope that our paper will stimulate attempts to fit experimental data to our Eq. (8), and to compare the resulting α_D with its direct estimate via $\mathcal{T}$. This procedure should work in many cases. We also hope that our paper will stimulate more detailed theoretical calculations of Δ_{int}. As explained, the existing approximate calculations miss the crucial ϕ dependence of these interaction-dependent terms.

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