

## Saturation of Nuclear Matter and Short-Range Correlations

Y. Dewulf,<sup>1</sup> W. H. Dickhoff,<sup>2,1</sup> D. Van Neck,<sup>1</sup> E. R. Stoddard,<sup>3</sup> and M. Waroquier<sup>1</sup>

<sup>1</sup>Laboratory of Theoretical Physics, Ghent University, Proeftuinstraat 86, B-9000 Gent, Belgium

<sup>2</sup>Department of Physics, Washington University, St. Louis, Missouri 63130

<sup>3</sup>Department of Physics, University of Missouri at Kansas City, Kansas City, Missouri 64110

(Received 17 October 2002; published 17 April 2003)

A fully self-consistent treatment of short-range correlations in nuclear matter is presented. Different implementations of the determination of the nucleon spectral functions for different interactions are shown to be consistent with each other. The resulting saturation densities are closer to the empirical result when compared with (continuous choice) Brueckner-Hartree-Fock values. Arguments for the dominance of short-range correlations in determining the nuclear matter saturation density are presented. A further survey of the role of long-range correlations suggests that the inclusion of pionic contributions to ring diagrams in nuclear matter leads to higher saturation densities than empirically observed. A possible resolution of the nuclear matter saturation problem is suggested.

DOI: 10.1103/PhysRevLett.90.152501

PACS numbers: 21.65.+f, 21.30.Fe, 21.60.-n, 24.10.-i

A correct description of the saturation properties of nuclear matter has remained an unresolved issue for a very long time. The Brueckner-Bethe-Goldstone expansion [1] supplies a converged result for the energy per particle in the relevant density range, for a given realistic interaction, at the level of three hole-line contributions [2,3]. Such calculations fail to reproduce the empirical saturation properties which require a minimum in the equation of state at a density corresponding to a Fermi momentum,  $k_F$ , of about  $1.33 \text{ fm}^{-1}$  with a binding energy of about 16 MeV. The authors of Ref. [3] obtain for the Argonne  $v_{14}$  interaction [4] a saturation density corresponding to  $1.565 \text{ fm}^{-1}$  with about the correct amount of binding. This corresponds to an overestimation of the empirical density by about 60% but appears completely consistent with corresponding variational calculations [5] for the same interaction.

Several different remedies for this serious problem have been proposed over the years. The intrinsic structure of the nucleon and its related strong coupling to the  $\Delta$  isobar inevitably require the consideration of three-body (or more-body) forces. When three-body forces are considered in variational calculations it is possible to achieve better saturation properties only when an *ad hoc* repulsive short-range component of this three-body force is added [6,7]. It has also been suggested that a relativistic treatment of the nucleon in the medium using a Dirac-Brueckner approach provides the necessary ingredients for a better description of saturation [8–11].

All many-body methods developed for nuclear matter have focused on a proper treatment of short-range correlations (SRC) without the benefit of experimental information on the influence of these correlations on the properties of the nucleon in the medium. This influence can now be clearly identified by considering recent results from  $(e, e'p)$  reactions [12–14] and theoretical calculations of the nucleon spectral function in nuclear matter [15–17]. A recent analysis of the  $(e, e'p)$  reaction on  $^{208}\text{Pb}$

in a wide range of missing energies and for missing momenta below  $270 \text{ MeV}/c$  yields information on the occupation numbers of all the deeply bound proton orbitals. These data indicate that all these orbitals are depleted by the same amount of about 15% [18]. These occupation numbers are associated with the orbits which yield an accurate fit to the  $(e, e'p)$  cross section. The properties of these occupation numbers suggest that the main effect of the global depletion of these mean-field orbitals is due to SRC. Indeed, the effect of the coupling of hole states to low-lying collective excitations affects only occupation numbers of states in the immediate vicinity of the Fermi energy [19]. In addition, nuclear matter momentum distributions display such an overall global depletion due to short-range and tensor correlations [17,20,21]. The latter results formed the basis of the now corroborated prediction [22,23] for the occupation numbers in  $^{208}\text{Pb}$  [18].

Most of this depleted single-particle (sp) strength is located at energies more than 100 MeV above the Fermi energy [17,20,22]. This appearance of strength at high energy is another important aspect of the influence of short-range and tensor correlations. Yet another characteristic feature of these SRC is that this depletion of the sp strength must be compensated by the admixture of a corresponding number of particles with high-momentum components. These high-momentum components have not yet been unambiguously identified but are currently studied experimentally [24]. Solid theoretical arguments [25] and calculations clearly pinpoint this strength at high excitation energy in the hole spectrum both for nuclear matter [17] and finite nuclei [26]. Indeed, experiment confirms that no substantial admixture of these high-momentum components is observed in the vicinity of the Fermi energy [27].

We now present an argument showing that SRC are the dominant factor in determining the empirical saturation density of nuclear matter. We recall that elastic electron

scattering from  $^{208}\text{Pb}$  [28] accurately determines the value of the central charge density in this nucleus. By multiplying this number by  $A/Z$  one obtains the relevant central density of heavy nuclei, corresponding to 0.16 nucleons/ $\text{fm}^3$  or  $k_F = 1.33 \text{ fm}^{-1}$ . Since the presence of nucleons at the center of a heavy nucleus is confined to  $s$ -wave nucleons, and, as discussed above, their depletion is dominated by SRC, one may therefore conclude that the same is true for the actual value of the empirical saturation density of nuclear matter. While this argument is particularly appropriate for the deeply bound  $1s_{1/2}$  and  $2s_{1/2}$  protons, it continues to hold to a large extent for the  $3s_{1/2}$  protons which are depleted predominantly by short-range effects (up to 15%) and by at most 10% due to long-range correlations [13,29]. These considerations demonstrate clearly that one may expect SRC to have a decisive influence on the actual value of the nuclear matter saturation density.

High-momentum components due to SRC also have a considerable impact on the binding energy of nuclear matter. This result can be inferred from the energy sum rule [30]

$$\frac{E}{A} = \frac{2}{\rho} \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\varepsilon_F} d\omega \left( \frac{k^2}{2m} + \omega \right) S_h(k, \omega), \quad (1)$$

where a notation with relative momenta  $p, q, q'$  and the conserved total momentum  $K$  has been used. The propagator  $\bar{g}_f^{\text{II}}$  in Eq. (2) has been obtained by an angle-averaging procedure of the noninteracting two-particle propagator

$$g_f^{\text{II}}(k, k'; \Omega) = \int_{\varepsilon_F}^{\infty} d\omega \int_{\varepsilon_F}^{\infty} d\omega' \frac{S_p(k, \omega) S_p(k', \omega')}{\Omega - \omega - \omega' + i\eta} - \int_{-\infty}^{\varepsilon_F} d\omega \int_{-\infty}^{\varepsilon_F} d\omega' \frac{S_h(k, \omega) S_h(k', \omega')}{\Omega - \omega - \omega' - i\eta}, \quad (3)$$

in order to allow a partial wave decomposition of the ladder equation. Note that  $sp$  momenta  $k, k'$  are used in Eq. (3). In turn, the spectral functions are needed to determine Eq. (3) and thereby the effective interaction  $\Gamma$  through Eq. (2). They can be obtained from the imaginary part of the  $sp$  propagator which solves the Dyson equation

$$g(k, \omega) = g^{(0)}(k, \omega) + g^{(0)}(k, \omega) \Sigma(k, \omega) g(k, \omega), \quad (4)$$

where the self-energy  $\Sigma$  includes the contribution of SRC through  $\Gamma$ , to complete the self-consistency loop.

The implementation of this self-consistency scheme is numerically quite involved and has been attempted by several groups [31–36]. In the present paper two different approaches have been used to generate results for different interactions. In the continuous scheme, a representation of the imaginary part of the self-energy in terms of four

where  $\rho = (2k_F^3)/(3\pi^2)$  is the density. Equation (1) illustrates the link between the energy of the system and the hole spectral function,  $S_h(k, \omega)$ . Results for the momentum distribution and true potential energy based on the spectral function show that enhancements as large as 200% for the kinetic and potential energy over the mean-field values can be obtained for both nuclear matter [20] and finite nuclei [26]. These large attractive contributions to the potential energy of nuclear matter are mainly from weighing the high-momentum components in the spectral function with large negative energies in Eq. (1). The location of these high-momentum components as a function of energy is therefore an important ingredient in the determination of the energy per particle as a function of density. So far, the determination of this location has relied only on quasiparticle properties in the construction of the self-energy. A self-consistent determination of the spectral function including the location of these high-momentum components therefore includes the dominant physics of SRC in the description of nuclear matter and is consistent with the experimental observations of the nucleon spectral function in nuclei.

Such a determination requires the solution of the ladder equation for the effective interaction in the medium

$$\langle q | \Gamma_{\ell\ell'}^{JST}(K, \Omega) | q' \rangle = \langle q | V_{\ell\ell'}^{JST} | q' \rangle + \sum_{\ell''} \int_0^{\infty} dp p^2 \langle q | V_{\ell\ell''}^{JST} | p \rangle \bar{g}_f^{\text{II}}(p; K, \Omega) \langle p | \Gamma_{\ell''\ell'}^{JST}(K, \Omega) | q' \rangle, \quad (2)$$

Gaussians is used to completely describe the  $sp$  propagator. The parameters of these Gaussians are then determined self-consistently [32] for the Reid potential [37] by solving Eq. (2) with the convolution of spectral functions in Eq. (3) as input, constructing the self-energy, and then solving the Dyson equation (4). In the discrete scheme we used a representation of the propagator in terms of three discrete poles [33], which avoids a full continuum solution of Eq. (2). The latter approach is equivalent to a continuous version as far as the energy per particle is concerned, since it requires a reproduction of the relevant energy-weighted moments of the hole and particle spectral function [33]. This is substantiated by comparing the results of this discrete scheme with the results of the continuous self-consistency scheme used in Ref. [34] for the Mongan-type separable interaction [38] and in Ref. [36] for the separable Paris interaction [39]. We find that the binding energies correspond to within 5% over the relevant  $k_F$  range around the minimum, and moreover that the location of the minimum agrees to within 3%.

In Fig. 1 we report the saturation points obtained within the discrete scheme of Ref. [33] for the updated Reid potential (Reid93), the NijmI and NijmII interactions [40], and the separable Paris interaction [39]. The results demonstrate an important and systematic change of the saturation properties with respect to continuous choice Brueckner-Hartree-Fock (ccBHF) calculations, leading to about 4–6 MeV less binding, and

reduced values of the saturation density, closer to the empirical one. Such a trend is entirely consistent with the observations in Ref. [36] made for a separable  $NN$  interaction and is now extended to more realistic (non-separable) interactions.

The discrete scheme [33] could not be used with the original Reid (Reid68) potential because of its slow decay in momentum space, but some results are available in the continuous scheme of Ref. [32]. In Fig. 1 the binding energy is shown at two densities ( $k_F = 1.33$  and  $1.45 \text{ fm}^{-1}$ ); the error bars are an estimate of the remaining uncertainty due to incomplete convergence and the non-self-consistent treatment of some higher order partial waves [32]. The results again seem to indicate a substantial shift in the saturation density for the Reid68 potential, from the ccBHF value of about  $1.6 \text{ fm}^{-1}$ , to a value below  $1.45 \text{ fm}^{-1}$ , without seriously underbinding nuclear matter.

The present self-consistent treatment of SRC (scSRC) differs in two main aspects from the ccBHF approach. First, hole and particle lines are treated on an equal footing, thereby ensuring thermodynamic consistency [34]. Intermediate hole-hole propagation in the ladder diagrams is included to all orders. This feature provides, compared to ccBHF, a substantial repulsive effect in the  $k < k_F$  contribution to Eq. (1), and comes primarily from an upward shift of the quasiparticle energy spectrum as a result of including  $\omega < \epsilon_F$  contributions to the imaginary part of the self-energy. The effect increases with density and is the dominant factor in the observed shift of the saturation point. Secondly, the realistic spectral functions, generated through Eqs. (2)–(4) and used in the evaluation of the in-medium interaction  $\Gamma$  and self-

energy  $\Sigma$ , are in agreement with experimental information obtained from  $(e, e'p)$  reactions. For the Reid93 interaction at  $k_F = 1.37 \text{ fm}^{-1}$  we find  $z = 0.74$  for the quasiparticle strength at the Fermi momentum, whereas the hole strength for  $p = 0$ , integrated up to 100 MeV missing energy, equals 83%; similar values are found for the other interactions. The depletion of the quasiparticle peaks is primarily important to suppress unrealistically large pairing instabilities around normal density. The improved treatment of the high-momentum components does affect the binding energy, through the  $k > k_F$  contribution to Eq. (1). This feature, studied in [33], provides a sizable attraction, but is smaller than the aforementioned repulsive effect.

The inclusion of  $hh$  propagation in scSRC also leads to a somewhat stiffer equation of state than in ccBHF. A recent analysis of the giant monopole resonance in heavy nuclei [41] yields an experimental estimate  $K_{nm} = 210 \pm 30 \text{ MeV}$  for the nuclear matter compression modulus,

$$K_{nm} = k_F^2 \left. \frac{d^2 E/A}{dk_F^2} \right|_{k_F=k_{F,0}}. \quad (5)$$

At the saturation points in Fig. 1 we find ccBHF values  $K_{nm} = 154 \text{ MeV}$  for Reid93 and  $K_{nm} = 148 \text{ MeV}$  for the separable Paris interaction, which are enhanced to  $K_{nm} = 177 \text{ MeV}$  and  $K_{nm} = 216 \text{ MeV}$ , respectively, in our scSRC calculation. These values agree reasonably well with the experimental estimate. Note that reasonable values for  $K$  imply that the Reid68 energies in Fig. 1 may still deviate by 1–1.5 MeV from numerically exact scSRC values, as indicated by the error bars.

The present results indicate that a sophisticated treatment of SRC lowers the ccBHF saturation densities, bringing them closer to the empirical one. It remains to be understood why apparently converged hole-line calculations [3] yield higher saturation densities. The three hole-line terms obtained in Ref. [3] indicate reasonable convergence properties compared to the two hole-line contribution. One may therefore assume that these results provide an accurate representation of the energy per particle of nuclear matter as a function of density for the case of nonrelativistic nucleons and two-body forces. At this point it is useful to identify an underlying assumption when the nuclear matter problem is posed [32]. This assumption asserts that the influence of long-range correlations in finite nuclei and nuclear matter are commensurate. We would like to point out that this underlying assumption is questionable. Three hole-line contributions include a third-order ring diagram characteristic of long-range correlations. The effect of long-range correlations on nuclear saturation properties is sizable, as shown by the results for three- and four-body ring diagrams calculated in Ref. [42] (see also Refs. [2,3]). The results of Ref. [42] demonstrate that such ring-diagram terms are dominated by attractive contributions involving pion quantum numbers propagating around the rings and

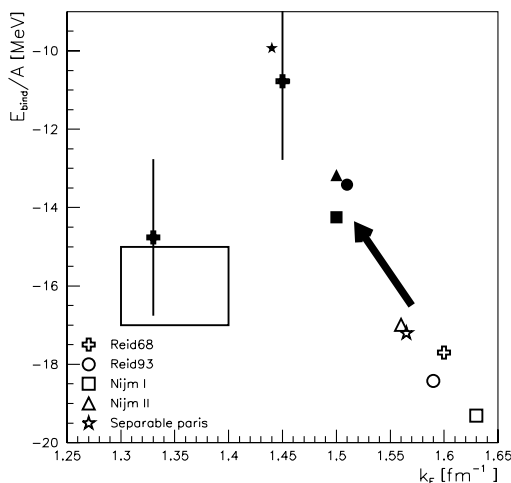


FIG. 1. Nuclear matter saturation points calculated with various realistic  $NN$  interactions. The open symbols refer to continuous choice Brueckner-Hartree-Fock results. The filled symbols refer to self-consistent results and represent saturation points calculated in the discrete scheme, except for the Reid68 interaction where the binding energy at two densities is shown in the continuous scheme.

increase in importance with increasing density. Such long-range pion-exchange contributions to the binding energy appear due to the possibility to coherently sample the attractive interaction in a given ring diagram at momenta  $q$  above  $0.7 \text{ fm}^{-1}$ . This feature is related to momentum conservation in nuclear matter and is not available in finite nuclei, in which no such collective pion degrees of freedom are actually observed [43]. It seems therefore reasonable to call into question the relevance of these coherent long-range pion-exchange contributions to the binding energy per particle since their behavior is so markedly different in finite and infinite systems. One may consider the salient difference of the ratio of spin-longitudinal and spin-transverse response functions in nuclear matter and finite nuclei as another indication of the relevance of our suggestion [44]. We also like to point out that experimental information of these response functions [45–47] suggests no characteristic enhancement of the (pionic) spin-longitudinal response as expected on the basis of nuclear matter calculations.

Clearly, the assertion that long-range pion-exchange contributions to the energy per particle need not be considered in explaining nuclear saturation properties needs to be further investigated. At this point it appears that a fully self-consistent treatment of SRC has substantially different saturation properties than a conventional (continuous choice) Brueckner-Hartree-Fock treatment and is capable of yielding saturation densities close to the empirical one.

This work is supported by the U.S. National Science Foundation under Grants No. PHY-9900713 and No. PHY-0140316 and by the Fund for Scientific Research-Flanders (FWO-Vlaanderen), and the Research Council of Ghent University.

- 
- [1] B. D. Day, *Rev. Mod. Phys.* **39**, 719 (1967).
  - [2] B. D. Day, *Phys. Rev. C* **24**, 1203 (1981).
  - [3] H. Q. Song, M. Baldo, G. Giansiracusa, and U. Lombardo, *Phys. Rev. Lett.* **81**, 1584 (1998).
  - [4] R. B. Wiringa, R. A. Smith, and T. L. Ainsworth, *Phys. Rev. C* **29**, 1207 (1984).
  - [5] B. D. Day and R. B. Wiringa, *Phys. Rev. C* **32**, 1057 (1985).
  - [6] J. Carlson, V. R. Pandharipande, and R. B. Wiringa, *Nucl. Phys.* **401**, 59 (1983).
  - [7] R. B. Wiringa, V. Fiks, and A. Fabrocini, *Phys. Rev. C* **38**, 1010 (1988).
  - [8] M. R. Anastasio, L. S. Celenza, W. S. Pong, and C. M. Shakin, *Phys. Rep.* **100**, 327 (1983).
  - [9] B. ter Haar and R. Malfliet, *Phys. Rev. Lett.* **56**, 1237 (1986).
  - [10] R. Brockmann and R. Machleidt, *Phys. Rev. C* **42**, 1965 (1990).
  - [11] A. Amorim and J. A. Tjon, *Phys. Rev. Lett.* **68**, 772 (1992).

- [12] A. E. L. Dieperink and P. K. A. deWitt Huberts, *Annu. Rev. Nucl. Part. Sci.* **40**, 239 (1990).
- [13] I. Sick and P. K. A. deWitt Huberts, *Comments Nucl. Part. Phys.* **20**, 177 (1991).
- [14] L. Lapikás, *Nucl. Phys.* **A553**, 297c (1993).
- [15] A. Ramos, A. Polls, and W. H. Dickhoff, *Nucl. Phys.* **A503**, 1 (1989).
- [16] O. Benhar, A. Fabrocini, and S. Fantoni, *Nucl. Phys.* **A505**, 267 (1989).
- [17] B. E. Vonderfecht, W. H. Dickhoff, A. Polls, and A. Ramos, *Phys. Rev. C* **44**, R1265 (1991).
- [18] L. Lapikás *et al.*, *Phys. Rev. Lett.* (to be published); M. F. van Batenburg, Ph.D. thesis, University of Utrecht, 2001.
- [19] G. A. Rijsdijk, K. Allaart, and W. H. Dickhoff, *Nucl. Phys.* **A550**, 159 (1992).
- [20] B. E. Vonderfecht, W. H. Dickhoff, A. Polls, and A. Ramos, *Nucl. Phys.* **A555**, 1 (1993).
- [21] S. Fantoni and V. R. Pandharipande, *Nucl. Phys.* **A427**, 473 (1984).
- [22] W. H. Dickhoff and H. Mütter, *Rep. Prog. Phys.* **55**, 1947 (1992).
- [23] V. R. Pandharipande, C. N. Papanicolas, and J. Wambach, *Phys. Rev. Lett.* **53**, 1133 (1984).
- [24] D. Rohe, in *Proceedings of the 5th Workshop on "e-m Induced Two-Hadron Emission," 2001, Lund, Sweden* (University of Lund, 2001), p. 67 (published on CDROM, ISBN: 91-631-1612-X).
- [25] C. Ciofi degli Atti, S. Liuti, and S. Simula, *Phys. Rev. C* **41**, R2474 (1990).
- [26] H. Mütter, A. Polls, and W. H. Dickhoff, *Phys. Rev. C* **51**, 3040 (1995).
- [27] I. Bobeldijk *et al.*, *Phys. Rev. Lett.* **73**, 2684 (1994).
- [28] B. Frois *et al.*, *Phys. Rev. Lett.* **38**, 152 (1977).
- [29] P. Grabmayr, *Prog. Part. Nucl. Phys.* **29**, 251 (1992).
- [30] V. M. Galitski and A. B. Migdal, *Sov. Phys. JETP* **34**, 96 (1958).
- [31] F. de Jong and H. Lenske, *Phys. Rev. C* **56**, 154 (1997).
- [32] W. H. Dickhoff and E. P. Roth, *Acta Phys. Pol. B* **33**, 65 (2002); E. P. Roth, Ph.D. thesis, Washington University, St. Louis, 2000.
- [33] Y. Dewulf, D. Van Neck, and M. Waroquier, *Phys. Rev. C* **65**, 054316 (2002).
- [34] P. Božek and P. Czerski, *Eur. Phys. J. A* **11**, 271 (2001).
- [35] P. Božek, *Phys. Rev. C* **65**, 054306 (2002).
- [36] P. Božek, *Eur. Phys. J. A* **15**, 325 (2002).
- [37] R. V. Reid, *Ann. Phys. (N.Y.)* **50**, 411 (1968).
- [38] T. R. Mongan, *Phys. Rev.* **178**, 1597 (1969).
- [39] J. Haidenbauer and W. Plessas, *Phys. Rev. C* **30**, 1822 (1984); **32**, 1424 (1985).
- [40] V. G. J. Stoks, R. A. M. Klomp, C. P. F. Verheggen, and J. J. de Swart, *Phys. Rev. C* **49**, 2950 (1994).
- [41] J. P. Blaizot, J. F. Berger, J. Dechargé, and M. Girod, *Nucl. Phys.* **A591**, 435 (1995).
- [42] W. H. Dickhoff, A. Faessler, and H. Mütter, *Nucl. Phys.* **A389**, 492 (1982).
- [43] P. Czerski, W. H. Dickhoff, A. Faessler, and H. Mütter, *Phys. Rev. C* **33**, 1753 (1986).
- [44] W. Alberico, *et al.*, *Phys. Rev. C* **34**, 977 (1986).
- [45] T. A. Carey *et al.*, *Phys. Rev. Lett.* **53**, 144 (1984).
- [46] T. N. Taddeucci *et al.*, *Phys. Rev. Lett.* **73**, 3516 (1994).
- [47] T. Wakasa *et al.*, *Phys. Rev. C* **59**, 3177 (1999).