Nonballistic Spin-Field-Effect Transistor

John Schliemann, J. Carlos Egues,* and Daniel Loss
Department of Physics and Astronomy, University of Basel, CH-4056 Basel, Switzerland
(Received 26 November 2002; published 8 April 2003)

We propose a spin-field-effect transistor based on spin-orbit coupling of both the Rashba and the Dresselhaus types. Different from earlier proposals, spin transport through our device is tolerant against spin-independent scattering processes. Hence the requirement of strictly ballistic transport can be relaxed. This follows from a unique interplay between the Dresselhaus and the Rashba coupling; these can be tuned to have equal strengths, leading to *k-independent* eigenspinors even in two dimensions. We discuss two-dimensional devices as well as quantum wires. In the latter, our setup presents strictly *parabolic* dispersions which avoids complications from anticrossings of different bands.

DOI: 10.1103/PhysRevLett.90.146801 PACS numbers: 73.63.–b, 71.70.Ej

In the recent years research in semiconductor physics has been focused on the emerging field of spintronics. This key word refers to the variety of efforts to use the electron spin rather than, or in combination with, its charge for information processing or, even more ambitiously, quantum information processing [1]. Among the most prominent device proposals is the spin-field-effect transistor (FET) due to Datta and Das [2]. This proposal uses the Rashba spin-orbit (s.o.) coupling to perform controlled rotations of spins of electrons passing through an FET-type device. This particular spin-orbit interaction is due to the inversion-asymmetry of the confining potential and is of the form [3]

$$\mathcal{H}_R = \frac{\alpha}{\hbar}(p_x\sigma^y - p_y\sigma^x), \tag{1}$$

where $\vec{p}$ is the momentum of the electron confined in a two-dimensional geometry, and $\vec{\sigma}$ the vector of Pauli matrices. The coefficient α is tunable in strength by the external gate of the FET. Because of the dependence on the momentum, the Rashba spin-orbit coupling can be viewed as a wave vector-dependent Zeeman field which can change drastically if the electron is scattered into a different momentum state. Therefore, such scattering events readily randomize the electron spin, thus limiting the range of operation of the Datta-Das spin-FET to the regime of *ballistic* transport where such processes do not occur.

In the present work we propose a modified version of the spin-FET in which the electrons are not only subject to spin-orbit interaction of the Rashba but also of the Dresselhaus type [4]. The latter is present in semiconductors lacking bulk inversion symmetry. When restricted to a two-dimensional semiconductor nanostructure with appropriate growth geometry, this coupling is of the form [5,6]

$$\mathcal{H}_D = \frac{\beta}{\hbar}(p_x\sigma^x - p_y\sigma^y), \tag{2}$$

where the coefficient β is determined by the semiconductor material and the geometry of the sample. Below we show that our proposed device is robust against spin-independent scattering and hence can also operate in a nonballistic (or diffusive) regime. This unique feature follows from the possibility of tuning the Rashba (via proper gating) and the Dresselhaus terms so that they have equal strengths $\alpha = \beta$. In this case, we show quite generally below that the electron spinor is k independent in two dimensions—even in the presence of (spin-independent) scatterers.

Tuned Rashba and Dresselhaus terms.—Consider the Hamiltonian $\mathcal{H} = \vec{p}^2/2m + V(\vec{r}) + \mathcal{H}_R + \mathcal{H}_D$, where m is the effective mass of the semiconductor and $V(\vec{r})$ an *arbitrary* scalar potential. For $\alpha = \pm\beta$ the operator $\Sigma = (\sigma^x \pm \sigma^y)/\sqrt{2}$ provides an additional conserved quantity, and a general eigenstate of $\mathcal{H}$ and Σ reads (for $\alpha = +\beta$)

$$\psi_\pm(\vec{r}) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm e^{i\pi/4} \end{pmatrix}\varphi(\vec{r})e^{\mp i\sqrt{2}\alpha m(x-y)/\hbar^2}, \tag{3}$$

where the function $\varphi(\vec{r})$ fulfills the usual spin-independent Schrödinger equation

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r})\right)\varphi(\vec{r}) = \left(\varepsilon + \frac{2\alpha^2 m}{\hbar^2}\right)\varphi(\vec{r}), \tag{4}$$

and ε is the energy eigenvalue of the wave function $\psi_\pm(\vec{r})$ with $\Sigma = \pm 1$. Since Eq. (4) is independent of the quantum number Σ all eigenstates are generally twofold degenerate. Such two degenerate states differing in Σ are related by time reversal. Note that the eigenvalue problem (4) is invariant under a formal time reversal, and the function $\varphi(\vec{r})$ can be taken to be real. The potential $V(\vec{r})$ can provide further confinement of the quantum well into a quantum wire (see below) or a quantum dot; it can also possibly describe *nonmagnetic* scatterers due to imperfections or impurities.

A robust two-dimensional spin-FET.—We consider an FET setup given by a two-dimensional quantum well which is laterally contacted by two spin-polarized contacts. In the vertical direction across the well, an electric

field tuning the Rashba coefficient α is provided by a gate. The spin-polarized leads can be realized by ferromagnetic metals or by ferromagnetic semiconductors. The latter version appears to be preferable with respect to the spin injection properties of the interfaces [1].

Within the two-dimensional channel the Hamiltonian is $\mathcal{H} = \vec{p}^2/2m + \mathcal{H}_R + \mathcal{H}_D$ whose eigenstates are

$$\psi_{\vec{k}}^{\pm}(\vec{r}) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \pm e^{i\phi(\vec{k})} \end{pmatrix} \frac{e^{i\vec{k}\vec{r}}}{2\pi}, \tag{5}$$

with $\phi(\vec{k}) = \arg[-\alpha k_y + \beta k_x + i(\alpha k_x - \beta k_y)]$ and eigenenergies

$$\varepsilon(\vec{k}) = \frac{\hbar^2 \vec{k}^2}{2m} \pm \sqrt{(\alpha k_y - \beta k_x)^2 + (\alpha k_x - \beta k_y)^2}. \tag{6}$$

For general α and β the spinor in the eigenstates (5) depends via $\phi(\vec{k})$ on the wave vector, and the dispersion (6) is nonparabolic. However, as described above, the case $\alpha = \beta$ is particular. Here the spin state of the wave functions is independent of the wave vector, and the dispersion is perfectly parabolic. The first observation is crucial for our device proposal.

Device operation.—The "off state" of our transistor corresponds to a gate bias such that the Rashba and the Dresselhaus coupling strengths are unequal, i.e., $\alpha \neq \beta$. In this case, the spinor of an injected electron is k dependent, Eq. (5), and hence becomes randomized due to momentum scattering. For strong enough spin relaxation, the drain current is that of an unpolarized beam. The predominant spin-dephasing mechanism in spin transistors is that of the Dyakonov-Perel type [7,8], due to the largest of the two spin-orbit terms [9]. The "on state" of our device operates with a gate bias for which $\alpha = \beta$. Here an injected electron with arbitrary momentum and in one of the k-independent spin states $(1, \pm \exp(i\pi/4))$ traverses the transistor channel with its spin state unchanged. The current at the drain would be the same as the injected one, assuming that the ferromagnetic source and drain have parallel polarizations. Note that spin-independent scattering events (provided by lattice imperfections, phonons, and nonmagnetic impurities) cannot change the spin state of the traversing electron. Moreover, as it will become clear from the discussion below, further device setups can be thought of as switching between the two points $\alpha = \pm\beta$ and/or using different combinations of magnetic polarizations in the contacts.

Absence of spin relaxation.—The Elliot-Yafet spin-flip mechanism is completely suppressed for $\alpha = \beta$ (on state of our device) since the spinor is k independent in this case. In addition, the Rashba-Dresselhaus rotation axis is fixed for equal couplings, and hence no Dyakonov-Perel spin relaxation is operative either. This can be seen from the general stationary solution (3): particles injected into the device with spin components in one of the eigenspinor states $(1, \pm \exp(i\pi/4))$ do not get altered at all (up to an unimportant global phase). Moreover, inspection of Eq. (3) shows that particles injected in a general spin state do not undergo a randomization of their spin but a controlled rotation around the $(1, 1, 0)$ axis by an angle η given by $\eta = 2\sqrt{2}\alpha m(a_x - a_y)/\hbar^2$, where $\vec{a}$ connects the locations where the particles are injected and detected, respectively. Therefore, if the locations of injection and detection are defined with sufficient precision, an electron injected in a general spin state will not suffer a randomization of its spin. However, uncertainties in those locations will translate to an uncertainty in the rotation angle. A way to avoid this problem is to inject electrons in the eigenspinor states $(1, \pm \exp(i\pi/4))$ (as discussed above) where the rotation has only a trivial effect, or to inject and detect electrons in a general spin state through quantum point contacts leading to well-defined distance vector $\vec{a}$. To enable a higher efficiency of the device, arrays of such quantum point contact pairs, separated by barriers, can be used in parallel as shown schematically in Fig. 1.

A similar finding was obtained numerically by Kiselev and Kim [10] who studied an effective spin model of the form $\tilde{\mathcal{H}} = \mathcal{H}_{\mathcal{R}} + \mathcal{H}_{\mathcal{D}}$, where the momentum $\vec{p}(t) = m\dot{\vec{r}}$ is a classical variable (not an operator) whose dependence on time t is generated by a Markovian process. The general time evolution operator reads $\mathcal{U}(t) = \mathcal{T} \exp\{-i \int_0^t dt' \tilde{\mathcal{H}}[\vec{p}(t')]/\hbar\}$, where $\mathcal{T}$ denotes the time-ordering symbol. For $\alpha = \beta$ the time ordering becomes trivial, and $\mathcal{U}(t)$ reads, up to a global phase,

$$\mathcal{U}(t) = \mathbf{1} \cos\left(\frac{\eta(\vec{a})}{2}\right) - i \frac{\sigma^x + \sigma^y}{\sqrt{2}} \sin\left(\frac{\eta(\vec{a})}{2}\right), \tag{7}$$

with, as above, $\eta(\vec{a}) = 2\sqrt{2}\alpha m(a_x - a_y)/\hbar^2$ and $\vec{a} = \vec{r}(t) - \vec{r}(0)$. Note that this finding is independent of whether or not the energy $\tilde{\mathcal{H}}[\vec{p}(t)]$ is conserved along the path $\vec{p}(t)$, as it was assumed in Ref. [10]. Thus, also this simplified effective spin model (with the orbital degrees of freedom treated classically) leads to the same controlled spin rotation as the full quantum mechanical solution (3). The above findings are in contrast with

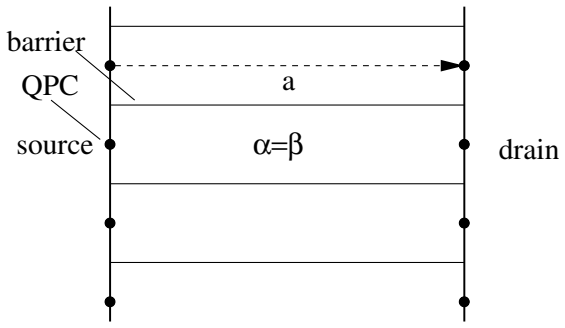

FIG. 1. Schematic of the spin-FET setup using quantum point contacts (QPC) to source and drain. The pairs of QPCs are separated by barriers to avoid cross talk.

earlier assertions where a randomization of the spin was predicted to occur even for $\alpha = \beta$ [11], or at least for $\alpha = \beta$ and a general spin state of the injected electron differing from $(1, \pm \exp(i\pi/4))$ [12]. These conclusions are due to the weak-coupling treatment performed in [11,12] and become invalid in the presence of the additional conserved quantity Σ arising at $\alpha = \beta$.

A quantitative description of the transport in the on state of our device should include possible spin-independent scatterers in the potential $V(\vec{r})$ of Eq. (4). This equation describes the *orbital* part of the single-particle eigenstates whose spin part is independent of the momentum for $\alpha = \beta$. Solutions to this equation can be matched with wave functions in the leads according to the appropriate boundary conditions in the presence of spin-orbit coupling [13,14]. Transmission coefficients can then be determined from the stationary solutions.

Ballistic regime.—In the strictly ballistic case [$V(\vec{r}) = 0$] and for source and drain with parallel polarizations chosen along either of the spinor directions $(1, \pm \exp(i\pi/4))$, we find the transmission amplitude

$$T^{\pm} = \frac{e^{-ika \pm i\sqrt{2}\alpha m_2 a/\hbar^2}(4q\frac{m_2}{m_1}k)}{(\frac{m_2}{m_1}k + q)^2 e^{-iqa} - (\frac{m_2}{m_1}k - q)^2 e^{iqa}}, \qquad (8)$$

for an electron injected at energy ε and wave vector $\vec{k} = k\vec{e}_y$. In (8) a is the length of s.o. active region, m_1, m_2 are the band masses in the contacts and the two-dimensional channel, respectively, and $q = \sqrt{2m_2(\varepsilon - V_0)/\hbar^2}$ with V_0 being a possible band offset between the contacts and the FET channel (also including a contribution from s.o. coupling). From the above expression one can find the conductance using Landauer's formula. Concerning the phase of transmitted electrons, one finds $T^+/T^- = \exp(i2\sqrt{2}\alpha m_2 a/\hbar^2)$. As discussed above, this phase factor is also obvious from the general form of eigenstates (3) and corresponds to a controlled rotation of the spin of the injected particle around the $(1, 1, 0)$ direction. The rotation angle is, up to a factor of $\sqrt{2}$, due to the presence of both Rashba and Dresselhaus coupling, the same as the one in the original proposal by Datta and Das [2]. Note, however, that here the spin part of the wave functions is independent of the wave vector. Moreover, according to the general form of eigenstates given by Eq. (3), the same phase factor occurs if spin-independent scatterers encompassed in the potential $V(\vec{r})$ are included. Therefore, as discussed above, the range of operation of our device is not limited to the strictly ballistic regime.

Magnitudes of α and β.—The largest values for α observed in III-V semiconductors are of the order of a few 0.1 eV Å [15–20]. An estimate for the Dresselhaus coefficient in a confined geometry is obtained from $\beta = \gamma\langle k_z^2\rangle$, where $\langle k_z^2\rangle$ is the expectation value of the square wave-vector component in the growth direction. A typical value for the coefficient γ is $\gamma \approx 25$ eV Å^3 [21–23]. For an infinite well with width w we find $\langle k_z^2\rangle = (\pi/w)^2$, which yields $\beta \approx 0.09$ eV Å for $w = 50$ Å. Hence there should be no principle difficulty to achieve the situation $\alpha = \beta$ even in comparatively narrow wells. Note also that small deviations from the case $\alpha = \beta$, i.e., $\alpha = \beta + \delta$ with $|\delta/\alpha| \ll 1$, lead (using Fermi's golden rule) to spin scattering rates which are quadratic in δ. Thus, spin dephasing due to spin-orbit coupling is completely suppressed in first order in δ. This is in accordance with the results of Ref. [10] studying an effective time-dependent Hamiltonian where the inverse dephasing time has a minimum equal to zero at $\alpha = \beta$ and is differentiable around this point.

Quantum wire with spin-orbit coupled bands.—We now consider a quantum wire formed by an additional confining potential $V(x)$ along the x direction. For $\alpha = \beta$ single-particle wave functions are of the form (3) with $\varphi_n(\vec{r}) = \chi_n(x)\exp(i(k \mp \sqrt{2}\alpha m/\hbar^2)y)$, where $\chi_n(x)$ obeys the usual Schrödinger equation for the transverse variable x with quantized eigenvalues $\tilde{\varepsilon}_n$, and n labels the energy levels. The single-particle eigenenergies are then $\varepsilon_n^{\pm}(k) = \tilde{\varepsilon}_n + (\hbar^2/2m)(k \mp \sqrt{2}\alpha m/\hbar^2)^2 - 2\alpha^2 m/\hbar^2$. Note that, similar to the two-dimensional case discussed earlier, the wire energy dispersions here are also parabolic—for *any* strength of the $\alpha = \beta$ coupling. In addition, as we discuss below, there are no avoided crossings in the energy dispersions. These results are significantly different from the usual case of a quantum wire with only the Rashba s.o. interaction [24–26]; there the bands are highly nonparabolic and anticross for strong Rashba couplings.

Figure 2 illustrates the wire dispersions for a two-band model. Note in Fig. 2(a) the features mentioned above for the case with tuned couplings: parabolic dispersions with no anticrossings. For differing coupling constants $\alpha \neq \beta$, the bands are nonparabolic and display avoided crossings. The contrasting features of the $\alpha = \beta$ and $\alpha \neq \beta$ cases are crucial for spin injection across a no-s.o./s.o. active interface.

Spin injection in quasi-1D channels.—Strong Rashba s.o. interaction can greatly affect the spin conductance of wires [25] and even suppress spin injection [26]. The wires we consider here, with tuned s.o. couplings $\alpha = \beta$, should not present any obstacle to spin injection since the bands are parabolic with no avoided crossings. The

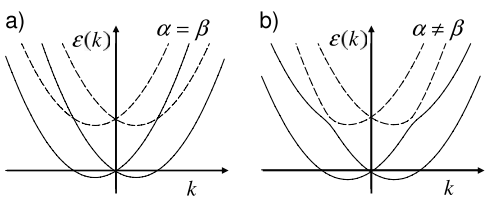

FIG. 2. Quantum wire dispersions $\varepsilon(k)$ in the presence of both Rashba and Dresselhaus s.o. interactions. For equal s.o. strengths $\alpha = \beta$ (a) the dispersions are parabolic with no anticrossings. For differing coupling strengths $\alpha \neq \beta$ (b) the bands are nonparabolic and avoided crossings occur.

problem with tuned couplings is similar to that of a quantum wire with *uncoupled* s.o. bands, where spin injection is always possible in the ballistic regime [2].

We can also consider a spin-FET setup with a wire as the connecting channel between the source and the drain. Since the spin part of the wire eigenstates are wave-vector dependent for $\alpha \neq \beta$, this quasi-1D spin-FET operates similarly to the nonballistic two-dimensional one discussed earlier. That is, elastic and/or inelastic scattering processes changing the wave vector also randomize the spin state of transmitted electrons (off state) for wires with many bands. These effects are absent for $\alpha = \beta$ and the spin state is preserved (on state).

Influences beyond the effective Hamiltonian.—Our analysis assumes that the effects of spin-orbit interaction are entirely described by the contributions (1) and (2) to the Hamiltonian. In a realistic semiconductor system there are additional corrections to these dominant terms. It is instructive to consider the influence of possible nonparabolicity in the band structure described by a contribution to the Hamiltonian of higher order in the momentum. For instance, as it was argued theoretically [21] and confirmed experimentally [18], particularly large values of α are typically accompanied by a sizable quartic nonparabolicity of the form $(\vec{p}^{\,2})^2$. This is due to the similar dependence of both terms on the band gap. However, also in this case Σ is still a conserved quantity at $\alpha = \beta$, and the Hamiltonian is invariant under time reversal if only even powers of the momentum occur. In particular, if $(1, \exp(i\pi/4))\Phi(\vec{r})$ solves the stationary Schrödinger equation, so does the orthogonal state $(1, -\exp(i\pi/4))\Phi^*(\vec{r})$ with the same energy, thus leading to the same general degeneracy pattern as in the parabolic case. Moreover, transmission amplitudes for such eigenspinor states are as before related by complex conjugation. Thus, for injection into eigenspinor states $(1, \pm\exp(i\pi/4))$ the device operation is completely unaltered. For injection into linear combinations of them the same controlled rotation around the $(1, 1, 0)$ axis occurs. The rotation angle, however, is more difficult to determine since an elegant transformation as described in Eqs. (3) and (4) does not seem to exist. Note that the result for the spin evolution operator (7) obtained within the classical approximation remains the same if additional spin-independent terms are included, and the rotation angle is again given by the distance $\vec{a}$. In summary, in situations where a sizable quartic term is present and the classical approximation to the orbital motion appears problematic, injection in directions close to the eigenspinor directions is favorable in order to ensure good device operation. Thus, even with such corrections like nonparabolicity being included, our spin-transistor proposal—which benefits from a unique "cancellation" of the Rashba and the Dresselhaus terms for tuned couplings—should provide a substantial increase in performance and stability of a spin-FET device as compared with the original proposal [2]. We stress that this cancellation occurs for both signs in the relation $\alpha = \pm\beta$. Therefore further devices can be envisioned switching between these two points and/or using different combinations of magnetic polarizations in the contacts.

This work was supported by NCCR Nanoscience, the Swiss NSF, DARPA, and ARO.

*Permanent address: Department of Physics and Informatics, University of São Paulo at São Carlos, 13560-970 São Carlos/SP, Brazil.

[1] *Semiconductor Spintronics and Quantum Computation*, edited by D. D. Awschalom, D. Loss, and N. Samarth (Springer, Berlin, 2002).
[2] S. Datta and B. Das, Appl. Phys. Lett. **56**, 665 (1990).
[3] E. I. Rashba, Fiz. Tverd. Tela (Leningrad) **2**, 1224 (1960) [Sov. Phys. Solid State **2**, 1109 (1960)]; Y. A. Bychkov and E. I. Rashba, J. Phys. C **17**, 6039 (1984).
[4] G. Dresselhaus, Phys. Rev. **100**, 580 (1955).
[5] M. I. Dyakonov and V. Y. Kachorovskii, Sov. Phys. Semicond. **20**, 110 (1986).
[6] G. Bastard and R. Ferreira, Surf. Sci. **267**, 335 (1992).
[7] M. I. Dyakonov and V. I. Perel, Sov. Phys. JETP **33**, 1053 (1971).
[8] G. E. Pikus and A. N. Titkov, in *Optical Orientation*, edited by F. Meier and B. P. Zakharchenya (Elsevier, Amsterdam, 1984).
[9] A. Bournel *et al.*, Eur. Phys. J. Appl. Phys. **4**, 1 (1998); see also M. A. Brand *et al.*, Phys. Rev. Lett. **89**, 236601 (2002).
[10] A. A. Kiselev and K. W. Kim, Phys. Status Solidi (b) **221**, 491 (2000).
[11] F. G. Pikus and G. E. Pikus, Phys. Rev. B **51**, 16 928 (1995).
[12] N. S. Averkiev and L. E. Golub, Phys. Rev. B **60**, 15 582 (1999).
[13] L. W. Molenkamp, G. Schmidt, and G. E. W. Bauer, Phys. Rev. B **64**, 121201 (2001).
[14] T. Matsuyama *et al.*, Phys. Rev. B **65**, 155322 (2002).
[15] J. Nitta *et al.*, Phys. Rev. Lett. **78**, 1335 (1997).
[16] G. Engels *et al.*, Phys. Rev. B **55**, 1958 (1997).
[17] J. P. Heida *et al.*, Phys. Rev. B **57**, 11 911 (1998).
[18] C.-M. Hu *et al.*, Phys. Rev. B **60**, 7736 (1999).
[19] D. Grundler, Phys. Rev. Lett. **84**, 6074 (2000).
[20] Y. Sato *et al.*, J. Appl. Phys. **89**, 8017 (2001).
[21] G. Lommer, F. Malcher, and U. Rössler, Phys. Rev. Lett. **60**, 728 (1988).
[22] B. Jusserand *et al.*, Phys. Rev. Lett. **69**, 848 (1992).
[23] B. Jusserand *et al.*, Phys. Rev. B **51**, 4707 (1995).
[24] A. V. Moroz and C. H. W. Barnes, Phys. Rev. B **60**, 14 272 (1999); A. V. Moroz, K. V. Samokhin, and C. H. W. Barnes, Phys. Rev. Lett. **84**, 4164 (2000).
[25] F. Mireles and G. Kirczenow, Phys. Rev. B **64**, 024426 (2001).
[26] M. Governale and U. Zülicke, Phys. Rev. B **66**, 073311 (2002).