Isospin Violation in $e^+e^- \to B\bar{B}$

Roland Kaiser,[1] Aneesh V. Manohar,[1] and Thomas Mehen[2,3]

[1]*University of California, San Diego, 9500 Gilman Drive, La Jolla, California 92093-0319*
[2]*Department of Physics, Duke University, Durham, North Carolina 27708*
[3]*Jefferson Laboratory, 12000 Jefferson Avenue, Newport News, Virginia 23606*

(Received 23 August 2002; published 9 April 2003)

The ratio of the B^+B^- and $B^0\bar{B}^0$ production rates in e^+e^- annihilation is computed as a function of the B meson velocity and $BB^*\pi$ coupling constant, using a nonrelativistic effective field theory.

DOI: 10.1103/PhysRevLett.90.142001 PACS numbers: 12.39.Hg, 12.39.Fe, 13.20.Gd, 13.66.Bc

The B-physics program reached an important milestone recently with the first observation of CP violation in the B meson system at the e^+e^- collider experiments BaBar and Belle [1,2]. The dominant production mechanism for B mesons at CLEO, BaBar, and Belle is via the P-wave decay of the $\Upsilon(4S)$ state, $e^+e^- \to \Upsilon(4S) \to B\bar{B}$. The final state can contain either a B^+B^- or $B^0\bar{B}^0$, and the ratio of charged to neutral B mesons, defined by

$$R^{+/0} = 1 + \delta R^{+/0} = \frac{\Gamma[\Upsilon(4S) \to B^+B^-]}{\Gamma[\Upsilon(4S) \to B^0\bar{B}^0]}, \qquad (1)$$

is an important input for many measurements. For example, the uncertainty in $R^{+/0}$ is the dominant source of error in the measurement of the $B \to D^* l\nu$ branching ratio [3], which is needed for the extraction of the Cabibbo-Kobayashi-Maskawa matrix element V_{cb}.

The experimental value of $R^{+/0}$ measured by the BaBar Collaboration is $R^{+/0} = 1.10 \pm 0.06 \pm 0.05$ [4] and by the CLEO Collaboration is $1.04 \pm 0.07 \pm 0.04$ [5] and $1.058 \pm 0.084 \pm 0.136$ [6]. In the absence of isospin violation, $R^{+/0} = 1$. Isospin violation is due to electromagnetic interactions and the mass difference of the up and down quarks. While the leading electromagnetic corrections to $R^{+/0}$ can be easily calculated, isospin violation due to the strong interactions has been thought to be under poor theoretical control. In this Letter we compute $R^{+/0}$. The result depends on two parameters: the $B^*B\pi$ coupling and δc, which represents the isospin violating part of the $\Upsilon(4S)$ coupling to $B\bar{B}$ states. The $B^*B\pi$ coupling can be extracted from D meson decays by applying heavy quark symmetry [7]. The parameter δc can be extracted from the energy dependence of $R^{+/0}$, which we calculate in this Letter.

Because of the small up and down quark masses and the weak electromagnetic coupling, isospin violation is usually at the level of a few percent. However, it is possible that there can be significant isospin violation in Υ decay [8–10]. The $\Upsilon(4S)$ is barely above the $B\bar{B}$ threshold; the B mesons are produced with a momentum $p_B \sim 338$ MeV and velocity $v/c = 0.064$ [using $M_{\Upsilon(4S)} = 10.58$ GeV, $M_B = 5.2792$ GeV], so that the final state is nonrelativistic. The electromagnetic contribution to $R^{+/0}$ is a function of v and the fine-structure constant α. In the nonrelativistic limit, there are $1/v$ enhancements, and the leading contribution is a function of α/v,

$$R^{+/0} = \frac{\pi\alpha/v}{1 - e^{-\pi\alpha/v}}\left(1 + \frac{\alpha^2}{4v^2}\right) = 1 + \frac{\pi\alpha}{2v} + \mathcal{O}\left(\frac{\alpha^2}{v^2}\right), \qquad (2)$$

and can be obtained by solving the Schrödinger equation in a Coulomb potential for a P-wave final state [11]. Corrections to this result are suppressed by powers of α without any $1/v$ enhancements. For $\Upsilon(4S)$ decay, this gives $R^{+/0} = 1.19$, a significant enhancement of the charged/neutral ratio [8–10].

Lepage [9] computed corrections to Eq. (2) by assuming a form factor at the meson-photon vertex and found that $\delta R^{+/0}$ could be significantly reduced from 0.19, or even change sign. Recent advances in the study of heavy quark systems and nonrelativistic bound states allow us to improve on this estimate of $\delta R^{+/0}$. Since the final state B mesons are nonrelativistic, and have low momentum, the final state interactions of the B meson can be treated using nonrelativistic field theory combined with chiral perturbation theory [12]. At momentum transfers smaller than the scale of chiral symmetry breaking $\Lambda_\chi \sim 1$ GeV [13], the photon vertex can be treated as pointlike. Since the $B^* - B$ mass splitting (46 MeV) is not much larger than the kinetic energy of the two B mesons ($p_B^2/M_B \approx 22$ MeV), the B^* should be included as an explicit degree of freedom in this problem. In the $m_Q \to \infty$ limit, the B and B^* are degenerate and they form a single multiplet described by the $H^{(b)}$ field of heavy quark effective theory [14]. Similarly, the $\bar{B}$ and $\bar{B}^*$ can be combined into a $H^{(\bar{b})}$ field, whose properties are related to those of $H^{(b)}$ by charge conjugation [15]. At low velocities, the dominant isospin violation is that enhanced by factors of $1/v$, which is obtained by solving the Schrödinger equation with the $H^{(b)} - H^{(\bar{b})}$ interaction potential. The nonrelativistic QCD counting rules [16] show that $B\bar{B}$ annihilation is suppressed and can be neglected. At low momentum transfer, the $H^{(b)} - H^{(\bar{b})}$ potential is dominated by single-pion exchange. Isospin violation in the potential arises from Coulomb photon exchange, and from isospin violation in the pion sector due to the $\pi^+ - \pi^0$ mass difference and $\eta - \pi^0$ mixing.

In perturbation theory, the first contribution to $\delta R^{+/0}$ is from the graphs in Fig. 1. The one-loop photon graph gives the $\pi\alpha/2v$ term in Eq. (2). It is enhanced by π^2/v compared with a typical relativistic radiative correction, which is of order α/π, because of the nonrelativistic nature of the integral. The one-loop pion graph is similarly enhanced by $\pi^2/v \sim 150$ compared with a typical chiral loop correction. As a result, the correction from Fig. 1(b) is not small and cannot be treated in perturbation theory. However, it is possible to sum the multiple pion exchanges by solving the Schrödinger equation using the one-pion plus one-photon exchange potential. This sums the series of graphs shown in Fig. 2. Additional corrections, such as vertex corrections, are not included in the Schrödinger equation. However, these corrections are not enhanced by π^2/v and so are subleading compared with the terms we have retained.

The $\Upsilon(4S)$ is a 1^{--} state and can decay into five possible channels: (i) $B\bar{B}$ with $S=0$, $\ell=1$; (ii) $B^*\bar{B}^*$ with $S=0$, $\ell=1$; (iii) $B^*\bar{B}^*$ with $S=2$, $\ell=1$; (iv) $B^*\bar{B}^*$ with $S=2$, $\ell=3$, and (v) $B\bar{B}^*+B^*\bar{B}$ with $S=1$, $\ell=1$, where ℓ is the orbital angular momentum and S is the total spin. Since the $\Upsilon(4S)$ is below BB^* and B^*B^* threshold, only the first state is allowed as a final state, but all five states need to be included as intermediate states in the calculation. (The actual number of states is double this, since one has both charged and neutral channels.) Let $\eta, \beta = 1$–5 denote one of the five possible ℓS states and $a, b = 1, 2$ denote the charged and neutral sectors, respectively, so that a given channel is labeled by the index pairs ηa or βb. The radial Schrödinger equation has the potential

$$V^{\pi}_{\eta a,\beta b}(r) + V^{\gamma}_{\eta a,\beta b}(r) + V^{\ell}_{\eta a,\beta b}(r) + M_\eta \delta_{\eta\beta}\delta_{ab}, \quad (3)$$

where V^π is the pion potential, V^γ is the Coulomb potential, V^ℓ is the angular momentum potential, and M_η is the contribution due to the B^*-B mass difference Δm,

$$M_1 = 0, \qquad M_2 = M_3 = M_4 = 2M_5 = 2\Delta m. \quad (4)$$

The $B^0 - B^+$ mass difference is 0.33 ± 0.28 MeV [17] and will be neglected in our analysis. Note that a $B^0 - B^+$ mass difference of 0.33 MeV contributes about 0.05 to $\delta R^{+/0}$ from the p^3 dependence of the phase space of the P-wave decay. Because the uncertainty in the mass difference is nearly equal to the central value, this correction to $\delta R^{+/0}$ is highly uncertain and is not included in our analysis; it can trivially be included once the mass difference is determined more accurately.

The angular momentum potential is

$$V^{\ell}_{\eta a,\beta b}(r) = \frac{\ell_\eta(\ell_\eta+1)}{m_B r^2}\delta_{ab}\delta_{\eta\beta}, \quad (5)$$

where $\ell_\eta = (1, 1, 1, 3, 1)$ are the angular momenta of the various channels. The denominator is m_B since $m_B/2$ is the reduced mass of the $B\bar{B}$. The Coulomb potential is

$$V^{\gamma}_{\eta a,\beta b}(r) = -\frac{\alpha}{r}\delta_{\eta\beta}\delta_{a1}\delta_{b1}, \quad (6)$$

where α is the fine-structure constant. It contributes only to the charged sector $a = b = 1$ and does not mix different ℓS states.

The $H^{(b)} - H^{(\bar{b})}$ interaction potential is the same as the $H^{(b)} - H^{(b)}$ potential (by charge conjugation symmetry) and was computed in Ref. [18] which studied $bbqq$ exotic states. The potential depends on the $B^*B\pi$ coupling constant g which is not known. Heavy quark symmetry implies that g is the same as the $D^*D\pi$ coupling. The D^* can decay into $D\pi$ (via the coupling g) or $D\gamma$ (via electromagnetic interactions), and the decay rates can be used to obtain g [19,20]. A fit to the experimental data gives two possible solutions, $g = 0.27^{+0.04}_{-0.02}{}^{+0.05}_{-00.02}$ or $g = 0.76^{+0.03}_{-0.03}{}^{+0.2}_{-0.1}$ [7], with the smaller value being preferred. A recent measurement of the D^{*+} width by the CLEO Collaboration gives $g = 0.59 \pm 0.01 \pm 0.07$ [21]. We will give our results as a function of g. It is important for our calculation to compute the pion potential in the basis of physical states rather than the eigenstates of quark spin, as done in Ref. [18]. The potential is of the form

$$V^{\pi}_{\eta a,\beta b}(r) = \begin{bmatrix} h_+^2 U_{\eta\beta}(m_{\pi^0}, r) & 2U_{\eta\beta}(m_{\pi^+}, r) \\ 2U_{\eta\beta}(m_{\pi^+}, r) & h_0^2 U_{\eta\beta}(m_{\pi^0}, r) \end{bmatrix}_{ab}, \quad (7)$$

where $h_+^2 = 1.01$, $h_0^2 = 0.99$ [22], and $U_{\eta\beta}$ is given below. The values of h_+ and h_0 differ from unity due to $\eta - \pi^0$ mixing.

The computation of the matrix $U_{\eta\beta}(m_\pi, r)$ is nontrivial. The answer is $U(m, r) = T\tilde{U}(m, r)T^t$ where

$$\begin{aligned} \tilde{U}(m, r) &= \frac{g^2 m^2 e^{-mr}}{8\pi f^2 r}\,\mathrm{diag}(1, u_1, u_2, u_2, u_1), \\ u_1(m, r) &= \left(1 + \frac{2}{mr}\right)^2 = -2u_2(m, r) - 1, \end{aligned} \quad (8)$$

$f \sim 132$ MeV is the pion decay constant,

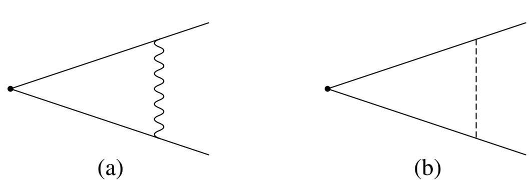

FIG. 1. One-loop correction to $\Upsilon(4S) \to B\bar{B}$ due to (a) photon and (b) pion exchange.

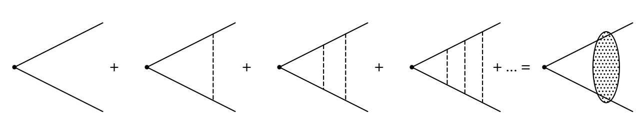

FIG. 2. Series of graphs summed by solving the Schrödinger equation. The dashed line represents pions and photons.

$$T = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2\sqrt{3}} & 0 & -\frac{1}{\sqrt{6}} & 0 \\ 0 & -\frac{2}{\sqrt{15}} & -\sqrt{\frac{3}{10}} & -\sqrt{\frac{2}{15}} & -\sqrt{\frac{3}{10}} \\ 0 & \sqrt{\frac{2}{5}} & -\frac{1}{\sqrt{5}} & \frac{1}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \\ 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}, \qquad (9)$$

and T^t is the transpose of T.

Equation (7) is the leading contribution to the long distance part of the potential. As argued in Ref. [18], Eq. (7) will dominate the potential until $r \sim 1/(2m_\pi)$ at which point two-pion exchange begins to contribute. We introduce a cutoff $r_{\min} = 1/(2m_\pi)$ and use Eq. (7) for $r \geq r_{\min}$ and set $V^\pi = 0$ for $r < r_{\min}$. The short-distance part of the potential can be included into a renormalization of the production vertex. The Coulomb potential will be allowed to act until $r = 0$.

The $\Upsilon(4S)$ is produced by the space component of the electromagnetic current $\bar{b}\gamma^i b$. Heavy quark spin symmetry holds in the Υ system [23,24], so the $\Upsilon(4S)$ decays into $H^{(b)} - H^{(\bar{b})}$ such that the spins of the heavy quarks in the final mesons are combined to form the spin of the $\Upsilon(4S)$, i.e., the polarization of the virtual photon. The orbital angular momentum and spin of the light degrees of freedom are combined to form total angular momentum zero. A little Clebsch-Gordan algebra shows that the amplitude for the $\Upsilon(4S)$ to decay into the five channels is [23]

$$A_{\eta a} = c_a \left(\frac{1}{2\sqrt{3}}, -\frac{1}{6}, \frac{\sqrt{5}}{3}, 0, -\frac{1}{\sqrt{3}} \right)_\eta. \qquad (10)$$

The amplitude for decay to the $\ell = 3$ channel is zero to this order in the velocity expansion. The coefficients c_a, $a = 1, 2$ are unknown, but the absolute values of c_a are irrelevant for the computation of $R^{+/0}$; all that is needed is the ratio c_1/c_2 of the charged to neutral production amplitudes. The dominant production of the B mesons is via the isosinglet $\Upsilon(4S)$ state, in which case $c_1 = c_2$. Isospin violating effects, including direct production of B's not via the $\Upsilon(4S)$ lead to a deviation of c_1/c_2 from unity. As discussed above, cutoff effects in the potential can be absorbed into the production amplitudes c_a. One expects short-distance corrections to introduce isospin violation in the ratio c_1/c_2 of a few percent, the typical size of other isospin violating effects in hadron physics. We will define δc by $c_1/c_2 = 1 + \delta c$. The value of δc is related to the value of $r_{\min}$, since changes in the cutoff induce changes in the Lagrangian coefficients. Since δc is unknown, our computation of $R^{+/0}$ is uncertain at the 5% level; however the uncertainity is much smaller than the expectation that $\delta R^{+/0}$ is 19% from Coulomb interactions alone. Cutting off the Coulomb potential at short distances reduces the value of $\delta R^{+/0}$. Since the Coulomb potential is the dominant source of isospin violation, one expects that δc will be negative.

The method of computation is as follows. One solves the Schrödinger equation with potential Eq. (7). The boundary condition on the wave function as $r \to \infty$ is that one has a plane wave plus an outgoing scattered wave. (One can see this directly from the sum of graphs in Fig. 2.) Only the B^+B^- and $B^0\bar{B}^0$ states exist as propagating modes as $r \to \infty$; the other channels have exponentially decaying wave functions. The plane wave state is chosen to be in the B^+B^- or $B^0\bar{B}^0$ channels to compute the charged or neutral meson production rates, respectively. The overlap of the computed wave function as $r \to 0$ with the production amplitude Eq. (10) gives the final production amplitude, the absolute square of which gives the production rate. (Note that the wave function near $r = 0$ can have all five channels.) The answer for $R^{+/0}$ depends on δc, g, and the velocity v of the outgoing B meson. Provided the dominant production mechanism is via the photon coupling to the heavy quark, the result for $R^{+/0}$ holds even away from the $\Upsilon(4S)$ resonance since it depends only on the quarks being nonrelativistic. The value of c_a will depend strongly on the beam energy, and peak at the resonance, but δc, the isospin violation in the production amplitude should be a smooth function of energy.

In Fig. 3 we have plotted $R^{+/0}$ as a function of g for δc and $v = 0.064$, the value in $\Upsilon(4S)$ decay. $R^{+/0}$ is approximately constant and equal to its value from only Coulomb corrections, Eq. (2), until $g > 0.6$, at which point $R^{+/0}$ starts to decrease. $R^{+/0}$ is approximately constant for small g even though the shifts in the production amplitudes are large. The one-loop pion graph in Fig. 1 is about 3 times the tree-level graph. Summing the pion graphs in Fig. 2 gives about a 20% (for $g \sim 0.6$) shift in the charged and neutral production rates. The rates into the charged and neutral channels vary by about a factor of 2 for the range of Yukawa couplings in Fig. 3, but their

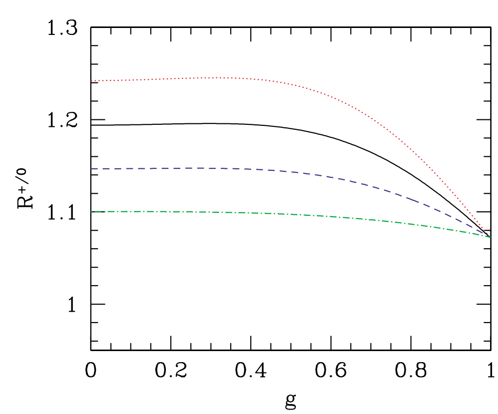

FIG. 3 (color online). $R^{+/0}$ as a function of g for $v = 0.064$ and $\delta c = 0.02$ (dotted line), 0.0 (solid line), -0.02 (dashed line), and -0.04 (dot-dashed line).

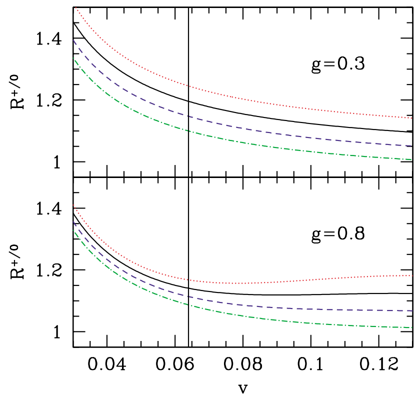

FIG. 4 (color online). $R^{+/0}$ as a function of v for $\delta c = 0.02$ (dotted line), 0.0 (solid line), -0.02 (dashed line), and -0.04 (dot-dashed line), and $g = 0.3, 0.8$.

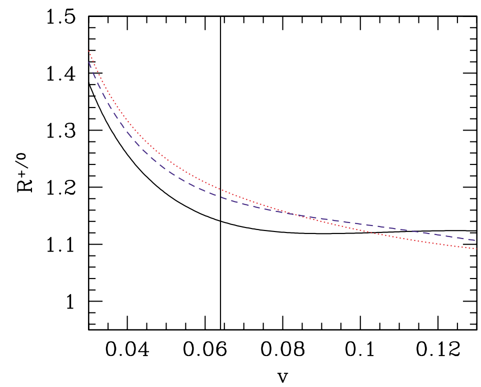

FIG. 5 (color online). $R^{+/0}$ as a function of v for $g = 0.8$, $\delta c = 0$, and cutoffs $r_{\min} = 1/(2m_\pi)$ (solid line), $1/\sqrt{2}m_\pi$ (dashed line), and $1/m_\pi$ (dotted line).

ratio $R^{+/0}$ varies by about 10%. For larger values of g than those shown, $R^{+/0}$ has rapid v dependence due to the formation of meson bound states, because the pion-exchange potential is sufficiently attractive. For our choice of parameters, this occurs for $g \sim 1.3$, well outside the allowed range [7,21].

In Fig. 4 we have plotted $R^{+/0}$ as a function of velocity for different values of δc for two illustrative choices $g = 0.3$ and $g = 0.8$ consistent with the two solutions for g found in Ref. [7]. The vertical line is the velocity at the $\Upsilon(4S)$. At the $\Upsilon(4S)$ peak, for $g = 0.8$, $R^{+/0}$ varies from 1.17 to about 1.09, whereas for $g = 0.3$, $R^{+/0}$ varies between about 1.25 and 1.1.

In Fig. 5, we have plotted $R^{+/0}$ as a function of v for $g = 0.8$, for different values of the cutoff from $r_{\min} = 1/(2m_\pi)$ to $1/m_\pi$. For the smaller value $g = 0.3$, the dependence of $R^{+/0}$ on the cutoff is negligible. For larger values of g, the cutoff variation is consistent with expectations from naive dimensional analysis [8]. A factor of 2 variation in the cutoff introduces a 4% variation in $R^{+/0}$.

The absolute value of $R^{+/0}$ depends on the value of δc and the cutoff $r_{\min}$. If g is small (~ 0.3, the preferred value in Ref. [7]), then for values of δc consistent with expectations from dimensional analysis, one expects $\delta R^{+/0} \gtrsim 0.1$. The Yukawa corrections do not significantly change $R^{+/0}$ from the Coulomb value. We note, however, that this is due to a cancellation in $R^{+/0}$ after summing the graphs in Fig. 2; the one-loop pion correction from Fig. 1 is about 3 and is not small. If g is close to the larger value $g = 0.8$, then $\delta R^{+/0}$ at the $\Upsilon(4S)$ is smaller, but still around 0.1. In this case, there is some cutoff dependence, so $R^{+/0}$ is more uncertain.

The dependence of $R^{+/0}$ on v (or equivalently, $\sqrt{s}$) is calculable. One can see that the curves in Fig. 4 have a different shape for $g = 0.3$ and $g = 0.8$, so measuring $R^{+/0}$ as a function of v can provide information on the $B^*B\pi$ and $D^*D\pi$ coupling g, which is needed for many calculations, such as the ratio of the $B_s - \bar{B}_s$ to $B - \bar{B}$ mixing amplitudes [10].

We thank S. Fleming and V. Sharma for discussions. This work was supported in part by the Department of Energy under Grants No. DOE-FG03-97ER40546, No. DE-FG02-96ER40945, and No. DE-AC05-84ER40150. R. K. was supported by the Swiss National Science Foundation.

[1] B. Aubert *et al.*, Phys. Rev. Lett. **87**, 091801 (2001).
[2] K. Abe *et al.*, Phys. Rev. Lett. **87**, 091802 (2001).
[3] BaBar Collaboration (private communication).
[4] B. Aubert *et al.*, Phys. Rev. D **65**, 032001 (2002).
[5] J. P. Alexander *et al.*, Phys. Rev. Lett. **86**, 2737 (2001).
[6] S. B. Athar *et al.*, hep-ex/0202033.
[7] I.W. Stewart, Nucl. Phys. **B529**, 62 (1998).
[8] D. Atwood and W. J. Marciano, Phys. Rev. D **41**, 1736 (1990).
[9] G. P. Lepage, Phys. Rev. D **42**, 3251 (1990).
[10] N. Byers and E. Eichten, Phys. Rev. D **42**, 3885 (1990).
[11] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill, New York, 1968).
[12] M. Luke and A.V. Manohar, Phys. Rev. D **55**, 4129 (1997).
[13] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).
[14] A.V. Manohar and M. B. Wise, Cambridge Monogr. Part. Phys., Nucl. Phys., Cosmol. **10**, 1 (2000).
[15] B. Grinstein *et al.*, Nucl. Phys. **B380**, 369 (1992).
[16] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995); **55**, 5853(E) (1997).
[17] K. Hagiwara *et al.*, Phys. Rev. D **66**, 010001 (2002).
[18] A.V. Manohar and M. B. Wise, Nucl. Phys. **B399**, 17 (1993).
[19] J. F. Amundson *et al.*, Phys. Lett. B **296**, 415 (1992).
[20] P. L. Cho and H. Georgi, Phys. Lett. B **296**, 408 (1992); **300**, 410(E) (1993).
[21] S. Ahmed *et al.*, Phys. Rev. Lett. **87**, 251801 (2001).
[22] E. Jenkins, Nucl. Phys. **B412**, 181 (1994).
[23] A. F. Falk and B. Grinstein, Phys. Lett. B **249**, 314 (1990).
[24] E. Jenkins *et al.*, Nucl. Phys. **B390**, 463 (1993).