Spacelike Brane Actions

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We derive effective actions for "spacelike branes" (S-branes) and find a solution describing the formation of fundamental strings in the rolling tachyon background. The S-brane action is a Dirac-Born-Infeld action for Euclidean world volumes defined in the context of time-dependent tachyon condensation of non-BPS (Bogomol'nyi-Prasad-Sommerfield) branes. It includes gauge fields and, in particular, a scalar field associated with translation along the time direction. We show that the BIon spike solutions constructed in this system correspond to the production of a confined electric flux tube (a fundamental string) at late time of the rolling tachyon.

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I. Introduction.—The exploration of time-dependent backgrounds is of theoretical interest and has cosmological applications. In the context of string theory a natural candidate to study is the tachyon condensation process on unstable branes. In investigating this direction, techniques are being developed and, in particular, a new type of "spacelike brane" (S-brane) [1] has appeared. This brane is defined as a time-dependent classical solution of the tachyon system. While S-branes naturally arise in the tachyon system especially with rolling tachyons [2], their role and physical significance in string theory, especially in time-dependent backgrounds and de Sitter conformal field thepry (dS/CFT), is still being developed (see Ref. [3] for an example). In this Letter we derive an effective action for S-branes, and show that the S-brane approach can be successfully applied to show that fundamental strings appear during tachyon condensation. We utilize the correspondence that any solution of the S-brane action has the corresponding tachyon + gauge time-dependent configuration on a non-BPS brane.

II. Derivation of the S-brane action.—The form of general tachyon actions has been shown [4] to guarantee that spatial tachyon kinks are universally governed by the Dirac-Born-Infeld (DBI) form of D-brane actions. We similarly derive the effective action for time-dependent tachyon kinks representing S-branes and show it has a universal form so essential features of the S-brane action are independent of the explicit tachyon action. Examining the tachyon system of a non-BPS $D(p+1)$-brane we now construct time-dependent classical solutions representing Sp-branes. Using the action

$$S = -\int d^{p+2}x\, V(T)\sqrt{1 + (\partial_\mu T)^2} \qquad (1)$$

originally proposed in Ref. [5], we will see how the rolling tachyon picture in Ref. [2] is reconciled with the S-brane picture. Here $T(x^\mu)$ with $\mu = 0, 1, \ldots, p+1$ is the tachyon field and dot will denote a derivative with respect to time x^0. The tachyon potential achieves its maximum at $T = 0$ and asymptotes to zero (closed string vacuum) at large T. This effective action gives the known exponentially decreasing pressure at late times while being consistent with the string theory calculation where $V(T)$ is taken to be an exponential function of T. For simplicity we take the tachyon classical solution to depend only on time x^0. Using the fact that energy $\mathcal{E} = V(T)/\sqrt{1-\dot{T}^2}$ is conserved, we obtain the homogeneous solution $T_{\rm cl}(x^0)$

$$x^0 = \int_0^{T_{\rm cl}} \frac{dT}{\sqrt{1 - V(T)^2/\mathcal{E}^2}}. \qquad (2)$$

When the tachyon approaches its minimum, $V(T) \to 0$, the time dependence of the tachyon simplifies to $T \sim x^0$. This constant behavior characterizes the final state of the rolling tachyon.

The location of a static domain wall is determined by the equation $T_{\rm cl}(x^\mu) = 0$ where $T_{\rm cl}$ is the classical solution of the domain wall, so time-dependent tachyon solutions are analogously characterized by $T = 0$ when the tachyon passes the top of the potential; the S-brane is found wherever $T = 0$. We now see that the physical statement of (2) is that we have chosen the Sp-brane tachyon solution to be the spacelike $p+1$-dimensional space $x^0 = 0$.

Deformations of the S-brane world volume are given by analyzing fluctuations of the tachyon field around its classical solution (2), $T = T_{\rm cl}(x^0) + t(x^\mu)$. Substituting this into the action (1) and keeping terms quadratic in t, one is led to the fluctuation action

$$S_{\rm f} = \frac{-\mathcal{E}}{2}\int dx^0 d^{p+1}x^{\hat{\mu}}\left[\frac{-\mathcal{E}^2}{V^2}(\dot{t})^2 + (\partial_{\hat{\mu}} t)^2 + M^2(x^0)t^2\right],$$

where $\hat{\mu} = 1, 2, \ldots, p+1$, and the time-dependent mass is

$$M^2(x^0) = \left[\frac{V''}{V} - \frac{(V')^2}{V^2}\right]_{T=T_{\rm cl}}. \qquad (3)$$

The factor in front of $(\dot{t})^2$ in the fluctuation action diverges at late time $x^0 \to \infty$, which shows that the fluctuation t is governed by the Carrollian metric [6] and ceases to propagate. This is consistent with the expectation that at the true vacuum of the tachyon theory open string degrees of freedom disappear and we therefore concentrate on the fluctuations around S-branes. Since the solution (2) breaks translation invariance along the time direction, there is a zero mode on the defect S-brane. It is well known that this mode is given by

$$t(x^\mu) = -X^0(x^{\hat{\mu}})\dot{T}_{\rm cl}(x^0), \tag{4}$$

with the function X^0 depending only on the coordinates along the Sp-brane. Once this is substituted into the fluctuation action, one finds that the mass term (3) cancels with the contribution from the term $(\dot{t})^2$. The effective action for a massless displacement field $X^0(x^{\hat{\mu}})$ is

$$S = -\mathcal{T}(\mathcal{E}) \int d^{p+1}x^{\hat{\mu}}\ \frac{1}{2}(\partial_{\hat{\nu}} X^0)^2, \tag{5}$$

with the positive constant $\mathcal{T}$ depending only on the energy $\mathcal{E}$. We have therefore determined the S-brane effective action for a Euclidean world volume to lowest order.

While in the above argument we have introduced only the tachyon field and its fluctuation X^0, it is natural to expect gauge fields on the S-branes, just like on D-branes. Following the procedures developed in Ref. [4], we shall obtain corrections to the S-brane action from higher order terms in X^0, and determine couplings to the gauge field in the limit of slowly varying fields, $\partial\partial X^0 \sim \partial F \sim 0$. First we note that the constant gauge field strength appears in the tachyon action only through the overall Born-Infeld factor $\sqrt{-\det(\eta + F)_{\mu\nu}}$ and the open string metric $G^{\mu\nu} = [(\eta + F)^{-1}_{\rm sym}]^{\mu\nu}$ used for contracting the indices of the derivatives (we work in the units $2\pi\alpha' = 1$). Requiring that the equations of motion for the gauge fields are also satisfied in the time-dependent homogeneous tachyon background, we find that the open string metric should satisfy $G^{00} = -1$, $G^{0\hat{\mu}} = 0$. This condition, allowing us to introduce dynamical gauge fields while also preserving the tachyon equations of motion, essentially states that we cannot turn on electric fields on a Euclidean world volume. The second notable point is that the dependence on the zero mode X^0 in the tachyon action should be

$$S = -\int dx^0 d^{p+1}x\ L\left[T_{\rm cl}\left(\frac{x^0 - X^0(x^{\hat{\mu}})}{\beta(X^0)}\right)\right]. \tag{6}$$

Here $\beta(X^0)$ can be fixed by the global Lorentz invariance in the world volume spacetime. The condition that the Lorentz boost preserves the open string metric is $\Lambda_\mu{}^\nu G_{\nu\rho}(\Lambda^t)^\rho{}_\sigma = G_{\mu\sigma}$, where according to the property (6) we define the Lorentz boost as

$$\Lambda_\mu^\nu = \begin{pmatrix} 1/\beta & -\partial_{\hat{\mu}} X^0/\beta \\ * & * \end{pmatrix}. \tag{7}$$

Then it is determined that $\beta = \sqrt{1 - G^{\hat{\mu}\hat{\nu}}\partial_{\hat{\mu}} X^0 \partial_{\hat{\nu}} X^0}$. Performing the integration over x^0 in Eq. (6) and including the F dependence, we obtain the S-brane action

$$\begin{aligned} S &= S_0(\mathcal{E}) \int d^{p+1}x\ \beta(X^0)\sqrt{\det(\delta + F)_{\hat{\mu}\hat{\nu}}} \\ &= S_0(\mathcal{E}) \int d^{p+1}x \sqrt{\det(\delta_{\hat{\mu}\hat{\nu}} - \partial_{\hat{\mu}} X^0 \partial_{\hat{\nu}} X^0 + F_{\hat{\mu}\hat{\nu}})}. \end{aligned} \tag{8}$$

The factor $S_0(\mathcal{E})$ is the tachyon action evaluated with the classical solution $T_{\rm cl}$. The S-brane action (8) differs from the usual D-brane action (DBI action) in two important respects: first, the action is defined on a Euclidean world volume, and second the kinetic term of the transverse scalar field X^0 has a "wrong" sign since it represents time translation. Covariantizing this action, one easily sees that the Lagrangian is simply $\sqrt{\det(g + F)}$, where g is the induced metric on the brane. It differs from the usual DBI Lagrangian only by a factor of i, and thus has the same equations of motion. This ensures that their solutions correspond to consistent backgrounds in which conformal symmetry is preserved on the open string worldsheet [7].

III. Fundamental strings from S-branes.—The location of the S-brane, given by $X^0(x^{\hat{\mu}})$, corresponds to the time when the tachyon passes its potential maximum $T = 0$. So while in the previous section we have shown that S-branes play an essential role in time-dependent tachyon condensation, it seems rather surprising if S-branes can be used to describe remnants of tachyon condensation, especially in the background $T \to \infty$. To understand how this is possible, let us recall how D-branes are realized in the noncommutative tachyon setup [8]. Those remnants (although in static condensation) are constructed as tachyon lumps at whose core the tachyon sits still around its potential maximum ($T \sim 0$) which we can describe with S-branes. The second obstacle, which is more intrinsically related to this Letter, is that an S-brane is spacelike while any remnants, such as fundamental strings, are timelike. How do we obtain timelike objects from spacelike objects? To see this in detail, let us return to the S-brane action (8). When there are no gauge fields excited on the S-brane, the Lagrangian is $\sqrt{1 - (\nabla X^0)^2}$. To keep this action real we enforce the condition $|\nabla X^0| \leq 1$, in units where $c = 1$, which is the statement that excitations on the S-brane keep the world volume Euclidean in the target space. This is analogous to the original motivation for introducing nonlinear electromagnetism by Born and Infeld [9], if one regards X^0 as an electrostatic potential ϕ. However, we know that for the DBI action, this critical electric field can be exceeded by turning on appropriate fields. For example, turning on transverse scalars Φ we can obtain BIon solutions [10] which have an electric field exceeding the critical value $1/(2\pi\alpha')$. Precisely the same situation can occur for S-branes. Turning on appropriate magnetic fields on the S-brane allows $|\nabla X^0| > 1$, and, in particular, configurations in which X^0 goes to infinity. The physical meaning of having X^0 going to infinity is that the world volume of the S-brane exists for all the time. Equivalently we can, say, that the S-brane has decayed into branes or strings at late times.

We now discuss the BIon-type spike solutions which will represent how fundamental strings appear as remnants in the tachyon condensation process. Let us turn on a single gauge potential A_{p+1} and suppose that all the world volume fields are independent of x^{p+1}. Then the S-brane Lagrangian is rewritten as

$$\sqrt{\det(\delta_{\hat{\mu}\hat{\nu}} - \partial_{\hat{\mu}} X^0 \partial_{\hat{\nu}} X^0 + \partial_{\hat{\mu}} A_{p+1} \partial_{\hat{\nu}} A_{p+1})},$$

which is exactly the usual static DBI Lagrangian for a Dp-brane, under the replacement $(X^0, A_{p+1}) \leftrightarrow (\phi, \Phi)$. Consequently, there are Sp-brane spike solutions (for $p \geq 3$) similar to the BPS BIons,

$$X^0 = A_{p+1} = \frac{c_p}{r^{p-2}}. \tag{9}$$

Here $r = \sqrt{(x^1)^2 + \cdots + (x^p)^2}$ is the radial distance along the Euclidean world volume (except x^{p+1}), but we also see from (9) that r parametrizes time evolution for this S-brane. As time evolves the radius decreases, therefore the remnant becomes a $1 + 1$ dimensional object, a string, parametrized by X^0 and x^{p+1} (see Fig. 1).

One can calculate the induced metric

$$ds^2 = (dx^{p+1})^2 + [1 - (dX^0/dr)^2]dr^2 + r^2 d\Omega_{p-1}^2 \tag{10}$$

and find that it is Euclidean for $0 < X^0 < X_c^0 \equiv c_p^{1/(p-1)}(p-2)^{(2-p)/(p-1)}$. It is amazing that for $X_c^0 < X^0$ the world volume becomes timelike so we are describing an object which is moving slower than the speed of light. This S-brane describes an infinitely long cylindrical world volume, where x^{p+1} is the infinite direction and the radius of the cylinder, r, shrinks with time. This therefore gives the tantalizing possibility that the solution

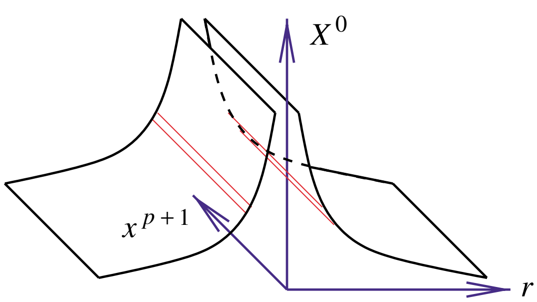

FIG. 1 (color online). Formation of a fundamental string as deformation of an Sp-brane surface, Eq. (9). The thin double lines denote the critical time $X^0 = X_c^0$ when the spacelike world volume becomes timelike.

(9) represents a time-dependent tachyon process which produces fundamental strings.

Let us provide evidence showing that this remnant is a fundamental string. First, the field strength induced on the deformed world volume at late times is

$$E_{p+1} = F_{0p+1} = \frac{\partial r}{\partial X^0} F_{rp+1} = \frac{1}{2\pi\alpha'}. \tag{11}$$

Here we changed the world volume variable r to X^0, and put in the α' dependence. This is the critical electric field induced on the cylindrical world volume and indicates that fundamental string charge is induced on the deformed S-brane. The criticality of the induced electric field is actually expected in tachyon condensation on general grounds. In Ref. [6], it is shown that for general tachyon effective Lagrangians with electric field, the late time behavior of the rolling tachyon is governed by $\dot{T}^2 + E^2 = 1$. When the tachyon stops rolling the electric field E reaches its critical value. Since the S-brane surface is characterized by $T = 0$, and at late times the world volume of the S-brane stops shrinking, we find $\dot{T} \to 0$. This shows that at late times the domain $T = 0$ is supported only locally at $r = 0$ where the electric field becomes critical. This is consistent with the picture given in Ref. [6] for describing fundamental strings after the tachyon condensation.

Next, we demonstrate that this flux tube has fundamental string tension. In coordinates more appropriate to the spacetime point of view, the S-brane action is

$$S = S_0 \int dx^0 d^p x \sqrt{-1 + E_{p+1}^2 + \dot{r}^2}, \tag{12}$$

which is i times usual DBI action. The canonically conjugate momenta are

$$D = S_0 \frac{E}{\sqrt{-1 + E^2 + \dot{r}^2}}, \qquad P_r = S_0 \frac{\dot{r}}{\sqrt{-1 + E^2 + \dot{r}^2}}, \tag{13}$$

and the Hamiltonian density is

$$H = \frac{S_0}{\sqrt{-1 + E^2 + \dot{r}^2}} = \frac{D}{E}. \tag{14}$$

Imposing flux quantization $\int d^{p-1}x\, D = n$ and noting that the electric field takes the critical value $E = 1$ at late times, we find that the energy becomes

$$\int d^p x\, H = \frac{n}{2\pi\alpha'} \int dx^{p+1}, \tag{15}$$

where we put in the α' dependence. This reproduces the Hamiltonian of n static fundamental strings with fundamental string tension. The situation resembles that of Ref. [11] and supertubes [12] which have critical electric fields.

Finally, we show that this remnant has no D-brane charge at future infinity. It is natural to take the Ramond-Ramond (RR) coupling for an S-brane to be the same as that for a D-brane. The coupling of RR fields to the particular S-brane above is

$$\mu \int A = \mu \int A^{p+1\,r\,\Omega}\, r^{p-1} dr\, dx^{p+1}\, d\Omega_{p-1}. \quad (16)$$

Transforming r into the embedding time X^0 we obtain

$$\mu \int A^{p+1\,0\,\Omega} \left(\frac{X^0}{c_p}\right)^{-(p-1)/(p-2)} dX^0\, dx^{p+1}\, d\Omega_{p-1}. \quad (17)$$

Because of the factor $(X^0)^{-(p-1)/(p-2)}$ which goes to zero at late times, we see that the D-brane charge of this solution shrinks to zero at future infinity.

If as suggested from the static cases [13], we assume that any S-brane solution has a corresponding tachyon solution, it is interesting to see why timelike defects are generated from spacelike defects from the viewpoint of tachyon solutions. Recall that S-branes are constructed with the open string tachyon and that the open string light cone always lies inside the closed string light cone [14]. Therefore the region near the spike core $r \sim 0$ can be timelike (10) in the target space, and also spacelike in the sense of open string metric due to the diverging field strength.

T-duality provides another viewpoint of the problem. In the T-dual picture by compactifying x^{p+1}, A_{p+1} becomes a spatial coordinate, and the S-brane solution (9) has no timelike region. The emergence of the timelike domain in the original description is simply due to the general fact that projection of a spacelike trajectory onto a lower dimensional subspace can appear to be timelike.

We have shown that the S-brane can be used to study tachyon condensation, however it is not clear if the validity of the S-brane action breaks down at some point due to effects such as radiation of gravitons, open strings or other modes [3,15]. This point is being investigated.

IV. Conclusions.—The essential point studied in this Letter is that one can make S-branes timelike. Although by definition S-branes are spacelike objects, they are, however, constructed using the open string tachyon and hence governed by the open string metric. Since the light cone of this metric always lies inside the closed string cone, one can boost S-branes to make them timelike relative to the closed string metric. The construction presented here opens up new possibilities in describing time-dependent tachyon condensation with S-branes. Believing that there is correspondence between the solution of the original tachyon system and the solution of the defect effective action [13], the spike solution found in this Letter shows how the fundamental string is formed during time-dependent tachyon condensation. The relation of this process to the confinement picture of the tachyon system [16] is to be investigated.

In addition, there are numerous avenues to explore using S-brane actions. D-brane actions have been truly useful for investigating string/M theory, and it is intriguing to see what will be the result of just replacing D-branes by S-branes. Starting with various brane configurations such as S-brane junctions and spherical S-branes, one may examine their supersymmetric properties, their M-theory lift, non-Abelian S-brane actions, Matrix theory with S-instantons, noncommutative S-branes, S and T-duality on S-brane actions, spacelike fundamental strings, S-branes in cubic string field theory, and numerous subjects. Some of them will be studied in a forthcoming paper.

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