Scheme for Teleportation of Quantum States onto a Mechanical Resonator

Stefano Mancini, David Vitali, and Paolo Tombesi

INFM and Dipartimento di Fisica, Università di Camerino, I-62032 Camerino, Italy

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We propose an experimentally feasible scheme to teleport an unkown quantum state onto the vibrational degree of freedom of a macroscopic mirror. The quantum channel between the two parties is established by exploiting radiation pressure effects.

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Teleportation of an unknown quantum state is its immaterial transport through a classical channel [1], employing one of the most puzzling resources of quantum mechanics: entanglement [2]. A variety of possible experimental schemes have been proposed and few of them partially realized in the discrete variable case involving the polarization state of single photons [3–5]. A successful achievement has been then obtained in the continuous variable case of an optical field [6]. However, the tantalizing problem of extending quantum teleportation at the macroscopic scale still remains open.

Recently, in the perspective of demonstrating and manipulating the quantum properties of bigger and bigger objects [7], it has been shown [8] how it is possible to entangle two massive macroscopic oscillators, such as movable mirrors, by using radiation pressure effects. The creation of such an entanglement at the macroscopic level suggests an avenue for achieving teleportation of a continuous variable state of a radiation field onto the vibrational state of a mirror.

We consider the usual situation where an unknown quantum state of a radiation field is prepared by a verifier (Victor) and sent to an analyzing station (Alice). Here we shall provide a protocol which enables Alice to teleport the continuous variable quantum state of the radiation onto a collective vibrational degree of freedom of a macroscopic, perfectly reflecting, mirror placed at a remote station (Bob) (see Fig. 1). The mirror could also represent the cantilever of a micro-electro-mechanical system (MEMS) [9].

For simplicity we consider only the motion and the elastic deformations of the mirror taking place along the spatial direction x, orthogonal to its reflecting surface. Then we consider an intense laser beam impinging on the surface of the mirror, whose radiation pressure realizes an optomechanical coupling [10]. In fact, the electromagnetic field exerts a force on the mirror proportional to its intensity and, at the same time, it is phase shifted by the mirror displacement from the equilibrium position [11]. In the limit of small mirror displacements, and in the interaction picture with respect to the free Hamiltonian of the electromagnetic field and the mirror displacement field $\hat{x}(\mathbf{r}, t)$ ($\mathbf{r}$ is the coordinate on the mirror surface), one has the following Hamiltonian [12]

$$\hat{H} = -\int d^2\mathbf{r}\, \hat{P}(\mathbf{r}, t)\hat{x}(\mathbf{r}, t), \qquad (1)$$

where $\hat{P}(\mathbf{r}, t)$ is the radiation pressure force [10]. All the continuum of electromagnetic modes with positive longitudinal wave vector q, transverse wave vector $\mathbf{k}$, and frequency $\omega = \sqrt{c^2(k^2 + q^2)}$ (c being the light speed in the vacuum) contributes to the radiation pressure force. The mirror displacement $\hat{x}(\mathbf{r}, t)$ is generally given by a superposition of many acoustic modes [12]; however, a single vibrational mode description can be adopted whenever detection is limited to a frequency bandwidth including a single mechanical resonance. In particular,

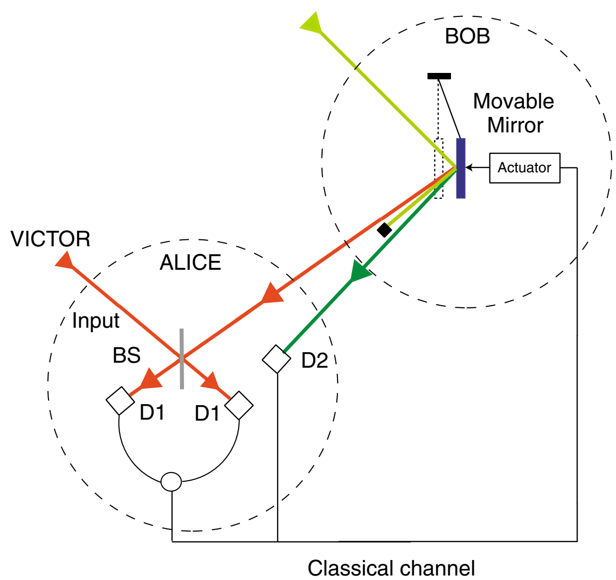

FIG. 1 (color online). Schematic description of the system. A laser field at frequency ω_0 impinges on the mirror oscillating at frequency Ω. In the reflected field two sideband modes are excited at frequencies $\omega_1 = \omega_0 - \Omega$ and $\omega_2 = \omega_0 + \Omega$. These two modes then reach Alice's station. The mode at frequency ω_2 is subjected to a heterodyne measurement $D2$, while the mode at frequency ω_1 is mixed in the 50-50 beam splitter BS with the unknown input given by Victor. A Bell-like measurement $D1$ is then performed on this combination and the result, combined with the heterodyne one, is fed-forward to Bob. Finally, he actuates the displacement in the phase space of the moving mirror.

focused light beams are able to excite Gaussian acoustic modes, in which only a small portion of the mirror, localized at its center, vibrates. These modes have a small waist w, a very large mechanical quality factor Q, and a small effective mass M [12]. The simplest choice is to choose the fundamental Gaussian mode with frequency Ω and ladder operators $\hat{b}$, $\hat{b}^\dagger$, $[b, b^\dagger] = 1$, so that

$$\hat{x}(\mathbf{r}, t) = \sqrt{\frac{\hbar}{2M\Omega}}[\hat{b}e^{-i\Omega t} + \hat{b}^\dagger e^{i\Omega t}]\exp(-r^2/w^2). \quad (2)$$

The interaction Hamiltonian (1) assumes a simple form if we average it over the time interval corresponding to the inverse of the spectral resolution of the optical detection apparatus at Alice station $\Delta\nu_{\rm det}$, and if we choose $\Delta\nu_{\rm det} \ll \Omega$. This implies neglecting all the terms oscillating faster than Ω (rotating wave approximation, RWA). We also consider an intense driving laser polarized parallel to the mirror surface, with power $\wp$, bandwidth $\Delta\nu_{\rm mode}$ centered around the carrier frequency ω_0, impinging on the mirror with an angle of incidence ϕ_0. If it is sufficiently intense, it can be treated as classical, and it is reflected undisturbed by the mirror. At the same time, it strongly couples the acoustic mode with the two back-scattered sideband modes at frequencies $\omega_1 = \omega_0 - \Omega$ (with annihilation operator $\hat{a}_1$) and $\omega_2 = \omega_0 + \Omega$ (with annihilation operator $\hat{a}_2$). Thus, we end up with the three-mode, effective interaction Hamiltonian [13]

$$\hat{H}_{\rm eff} = -i\hbar\chi(\hat{a}_1\hat{b} - \hat{a}_1^\dagger\hat{b}^\dagger) - i\hbar\theta(\hat{a}_2\hat{b}^\dagger - \hat{a}_2^\dagger\hat{b}), \quad (3)$$

where $\chi = \cos\phi_0\sqrt{\wp\Delta\nu_{\rm det}^2\omega_1/2M\Delta\nu_{\rm mode}c^2\Omega}$ and $\theta = \chi\sqrt{\omega_2/\omega_1}$. The physical process described by this interaction Hamiltonian is very similar to a stimulated Brillouin scattering [14], even though, in the present case, the Stokes ($\hat{a}_1$) and anti-Stokes ($\hat{a}_2$) components are backscattered by the acoustic waves at reflection, and the optomechanical coupling is provided by the radiation pressure and not by the dielectric properties of the mirror.

Equation (3) contains two interaction terms: the first one, between modes $\hat{a}_1$ and $\hat{b}$, is a parametric-type interaction leading to squeezing in phase space [15], and it is able to generate the EPR-like entangled state which has been used in the continuous variable teleportation experiment of Ref. [6]. The second interaction term, between modes $\hat{a}_2$ and $\hat{b}$, is a beam-splitter-type interaction [15], which may disturb the entanglement between modes $\hat{a}_1$ and $\hat{b}$ generated by the first term.

The system dynamics can be easily studied through the (normally ordered) characteristic function $\Phi(\mu, \nu, \zeta)$ [14], where μ, ν, ζ are the complex variables corresponding to the operators $\hat{a}_1, \hat{b}, \hat{a}_2$, respectively. Realistic initial conditions are given by the vacuum state for the sideband modes, and a thermal state at temperature T, with mean vibrational number $\overline{n} = [\exp(\hbar\Omega/k_BT) - 1]^{-1}$ for the mechanical mode (k_B being the Boltzmann constant). Then, the state of the three-mode system is Gaussian, with characteristic function

$$\Phi = \exp[-\mathcal{A}|\mu|^2 - \mathcal{B}|\nu|^2 - \mathcal{E}|\zeta|^2 + \mathcal{C}\mu\nu + \mathcal{C}\mu^*\nu^* + \mathcal{F}\mu\zeta + \mathcal{F}\mu^*\zeta^* + \mathcal{D}\nu\zeta^* + \mathcal{D}\nu^*\zeta], \quad (4)$$

where the coefficients $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$, $\mathcal{F}$ depend on the interaction time t [13], which is determined by the time duration of the driving laser pulse. The idea is now to find an experimentally feasible, *modified* version of the standard protocol for the teleportation of continuous quantum variables [16,17], able to minimize the disturbing effects of the beam-splitter-type term in Eq. (3). After reflection on the mirror, the sideband modes $\hat{a}_1$ and $\hat{a}_2$ reach Alice's station. Then Alice performs a heterodyne measurement [18] on the mode $\hat{a}_2$, projecting it onto a coherent state of complex amplitude α. Alice and Bob are left with an entangled state for the optical Stokes mode a_1 and the vibrational mode b, conditioned to this measurement result. In this case, the conditioned entangled state is still Gaussian, and characterized by the following correlation matrix (independent of α, affecting only first order moments)

$$\Gamma = \begin{pmatrix} \mathcal{A} + \frac{1}{2} - \frac{\mathcal{F}^2}{\mathcal{E}+1} & 0 & \mathcal{C} + \frac{\mathcal{F}\mathcal{D}}{\mathcal{E}+1} & 0 \\ 0 & \mathcal{A} + \frac{1}{2} - \frac{\mathcal{F}^2}{\mathcal{E}+1} & 0 & -\mathcal{C} - \frac{\mathcal{F}\mathcal{D}}{\mathcal{E}+1} \\ \mathcal{C} + \frac{\mathcal{F}\mathcal{D}}{\mathcal{E}+1} & 0 & \mathcal{B} + \frac{1}{2} - \frac{\mathcal{D}^2}{\mathcal{E}+1} & 0 \\ 0 & -\mathcal{C} - \frac{\mathcal{F}\mathcal{D}}{\mathcal{E}+1} & 0 & \mathcal{B} + \frac{1}{2} - \frac{\mathcal{D}^2}{\mathcal{E}+1} \end{pmatrix}, \quad (5)$$

where $\Gamma_{i,j} = \langle\hat{\mathbf{v}}_i\hat{\mathbf{v}}_j + \hat{\mathbf{v}}_j\hat{\mathbf{v}}_i\rangle/2 - \langle\hat{\mathbf{v}}_i\rangle\langle\hat{\mathbf{v}}_j\rangle$ with $\hat{\mathbf{v}} = (\hat{X}_{a_1}, \hat{P}_{a_1}, \hat{X}_b, \hat{P}_b)$ and $\hat{X}_{a_1} = (\hat{a}_1 + \hat{a}_1^\dagger)/\sqrt{2}$, $\hat{P}_{a_1} = -i(\hat{a}_1 - \hat{a}_1^\dagger)/\sqrt{2}$, $\hat{X}_b = (\hat{b} + \hat{b}^\dagger)/\sqrt{2}$, $\hat{P}_b = -i(\hat{b} - \hat{b}^\dagger)/\sqrt{2}$.

Once Alice has performed the heterodyne measurement on $\hat{a}_2$, one then follows the standard protocol for continuous variables quantum teleportation [16,17]. The quantum channel between Alice and Bob is established via the two-mode entangled state described by the correlation matrix (5). Alice mixes the radiation mode provided by Victor, whose unknown state she wants to teleport, with her part of entangled state (the Stokes a_1 mode), on a balanced beam splitter (see Fig. 1). She then carries out a homodyne detection at each output port, thereby measuring two commuting quadratures $\hat{X}_+ = (\hat{X}_{\rm in} + \hat{X}_{a_1})/\sqrt{2}$ and $\hat{P}_- = (\hat{P}_{\rm in} - \hat{P}_{a_1})/\sqrt{2}$, with measurement results X_+ and P_-. The final step at the sending station is to transmit the classical information, corresponding to the result of the heterodyne and homodyne measurements she performed, to the receiving terminal. Upon receiving this information, Bob displaces his part of entangled state (the mirror acoustic mode) as follows: $\hat{X}_b \to \hat{X}_b + \sqrt{2}X_+ + \sqrt{2}{\rm Re}\{\alpha\}(\mathcal{F} - \mathcal{D})/(\mathcal{E} + 1)$, $\hat{P}_b \to \hat{P}_b - \sqrt{2}P_- + \sqrt{2}{\rm Im}\{\alpha\}(\mathcal{F} + \mathcal{D})/(\mathcal{E} + 1)$. Notice that, in our protocol, Bob's local operation depends on all Alice's measurements (X_+, P_-, α). To actuate the phase-space displacement, Bob can use again the radiation pressure force. In fact, if the mirror is shined by a bichromatic intense laser field with frequencies ϖ_0 and $\varpi_0 + \Omega$, employing again Eq. (1) and the RWA, one is left with an effective interaction Hamiltonian $H_{act} \propto \hat{b}e^{-i\varphi} + \hat{b}^\dagger e^{i\varphi}$, where φ is the relative phase between the two frequency components. Any phase-space displacement of the mirror vibrational mode can be realized by adjusting this relative phase and the intensity of the laser beam.

In the case when Victor provides an input Gaussian state characterized by a 2×2 symmetric covariance matrix Γ^{in}, the output state at Bob site is again Gaussian, with a covariance matrix Γ^{out}. The input-output relation for these matrices is given by [19]

$$\Gamma^{out}_{11} = \Gamma^{in}_{11} + (\Gamma_{11} + 2\Gamma_{13} + \Gamma_{33}), \quad (6)$$

$$\Gamma^{out}_{12} = \Gamma^{in}_{12} + (\Gamma_{14} - \Gamma_{12} + \Gamma_{34} - \Gamma_{23}), \quad (7)$$

$$\Gamma^{out}_{22} = \Gamma^{in}_{22} + (\Gamma_{22} - 2\Gamma_{24} + \Gamma_{44}). \quad (8)$$

The fidelity F of the described teleportation protocol, defined as the overlap between the input and the output state, can be written, with the help of Eqs. (5)–(8), as

$$F = \{1 + [1 + \mathcal{A} + \mathcal{B} + 2\mathcal{C} - (\mathcal{F} - \mathcal{D})^2/(\mathcal{E} + 1)]\}^{-1}, \quad (9)$$

where we have specialized to the case of an input coherent state. In such a case, the upper bound for the fidelity achievable with only classical means and no quantum resources is $F = 1/2$ [20]. Figure 2 shows the fidelity as a function of the (rescaled) interaction time t for different values of the initial mean thermal phonon number of the mirror acoustic mode $\overline{n}$. The fidelity F is periodic in the interaction time t, and we show only one of all possible time windows where F reaches its maximum. The remarkable result shown in Fig. 2 is that this maximum value, $F_{max} \simeq 0.85$, is well above the classical bound $F = 0.5$ and that it is surprisingly independent of the initial temperature of the acoustic mode. This is apparently in contrast with previous results [21] showing that entanglement is no longer useful above one thermal photon (or phonon). This effect could be ascribed to quantum interference phenomena, and opens the way for the demonstration of quantum teleportation of states of macroscopic systems. However, thermal noise has still important effects so that, in practice, any experimental implementation needs an acoustic mode cooled at low temperatures (see, however, Refs. [22,23] for effective cooling mechanism of acoustic modes). In fact, we see from Fig. 2 that by increasing $\overline{n}$, the useful time interval becomes narrower. That means the necessity of designing precise driving laser pulses in order to have a well-defined interaction time. Furthermore, the time interval within which the classical communication from Alice to Bob, and the phase-space displacement by Bob have to be made, becomes shorter and shorter with increasing temperature, because the vibrational state projected by Alice's Bell measurement heats up in a time of the order of $(\gamma_m \overline{n})^{-1}$, where γ_m is the mechanical damping constant. The effects of mechanical damping can be instead neglected during the backscattering process stimulated by the intense laser beam. In fact, mechanical damping rates of about $\gamma_m \simeq 1$ Hz are available, and therefore negligible with respect to the typical values of the coupling constants $\chi \simeq \theta \simeq 5 \times 10^5$ Hz, and $\Theta = \sqrt{\theta^2 - \chi^2} \simeq 10^3$ Hz, determining the Hamiltonian dynamics. Such values are obtained with the following choice of parameters: $\wp = 10$ W, $\omega_0 \sim 2 \times 10^{15}$ Hz, $\Omega \sim 5 \times 10^8$ Hz, $\Delta\nu_{det} \sim 10^7$ Hz, $\Delta\nu_{mode} \sim t^{-1} \sim 10^3$ Hz, and $M \sim 10^{-10}$ Kg, which are those used in Fig. 2. These parameters are slightly different from those of already performed optomechanical experiments [22,24]. However, using a thinner silica crystal and considering higher frequency modes, the parameters we choose could be obtained. These choices show the difficulties one meets in trying to extend genuine quantum effects as teleportation into the macroscopic domain. So far we have mainly focused on the effects of the thermal noise acting on the mechanical resonator. Other fundamental noise sources are shot noise and radiation pressure noise [15]. Shot noise affects the detection of the sideband modes but it has been already taken into account, as it is responsible for the 1/2 terms in the diagonal entries of the correlation matrix of Eq. (5). The radiation pressure noise of the incident laser beam yields Poissonian distributed fluctuations of the optomechanical coupling. With the chosen parameter values we get $\Delta\wp/\wp = \Delta(\Theta t)/2\Theta t \simeq 10^{-8}$, which is negligibly small with respect to the width of the window where the fidelity attains its maximum (see Fig. 2). On the other hand, technical noise sources can be made negligibly small due to recent progress on low-noise

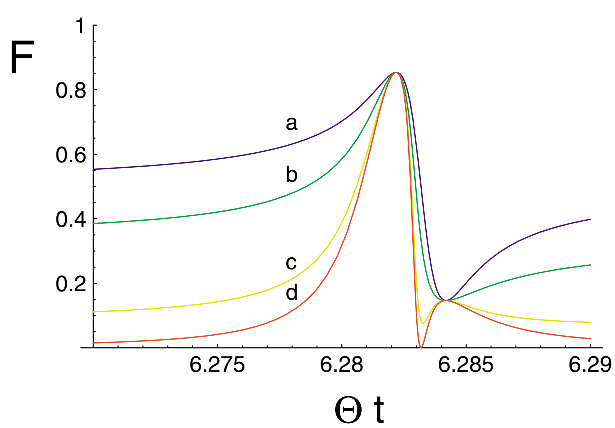

FIG. 2 (color online). Fidelity F vs the scaled time Θt, being $\Theta = \sqrt{\theta^2 - \chi^2}$. Curves a, b, c, d are for $\overline{n} = 0$, 1, 10, 10^3, respectively. The values of parameters are $\wp = 10$ W; $\Omega = 5 \times 10^8$ Hz; $\Delta\nu_{det} = 10^7$ Hz; $M = 10^{-10}$ Kg; $\omega_0 = 2 \times 10^{15}$ Hz, $\Delta\nu_{mode} = 10^3$ Hz.

laser sources, low-loss mirrors, and high detection efficiency [22,24].

The continuous variable teleportation protocol presented here modifies the standard one of Refs. [16,17] by adding a heterodyne measurement on the "spectator" mode $\hat{a}_2$. This additional measurement performed by Alice is important because it significantly improves the teleportation protocol. In fact, it is easy to see that if no measurement is performed on the anti-Stokes mode, the resulting fidelity for the teleportation of coherent states is always smaller with respect to that with the heterodyne measurement. In particular, there is still a maximum value of the fidelity, $F_{\max} = 0.80$ in this case, independent of temperature, but the useful interaction time interval becomes much narrower for increasing temperature.

It is worth remarking that the present teleportation scheme provides also a very powerful *cooling* mechanism for the acoustic mode (see also [22,23]). As matter of fact, the effective number of thermal excitations of the mirror state conditioned to the homodyne measurements at Alice station becomes $\overline{n}_{\rm eff} = 1 + \mathcal{A} + \mathcal{B} + 2\mathcal{C} - (\mathcal{F} - \mathcal{D})^2/(\mathcal{E} + 1)$. It reduces to $\overline{n} + 1$ in the absence of entanglement, where 1 represents the noise introduced by the protocol. Instead, the optomechanical interaction for a proper time permits to achieve $\overline{n}_{\rm eff} = 0.17$, i.e., an 80% reduction of thermal noise.

Finally, for what concerns the experimental verification of teleportation, that is, the measurement of the final state of the acoustic mode, one can consider a second, intense "reading" laser pulse, and exploit again the optomechanical interaction given by Eq. (3), where now a_1 and a_2 are meter modes. It is in fact possible to perfom a heterodyne measurement of the combined sidebands mode $\hat{a}_1(t) - \hat{a}_2^\dagger(t)$, where t is the time duration of the second laser pulse. Solving the Heisenberg equations one obtains

$$\hat{a}_1(t) - \hat{a}_2^\dagger(t) = \frac{1}{\Theta}[\chi + \theta]\sin(\Theta t)\hat{b}^\dagger(0) + \frac{1}{\Theta^2}[\theta^2 - \chi^2\cos(\Theta t) - \chi\theta + \chi\theta\cos(\Theta t)]\hat{a}_1(0) - \frac{1}{\Theta^2}[\chi\theta + \chi\theta\cos(\Theta t) - \chi^2 - \theta^2\cos(\Theta t)]\hat{a}_2^\dagger(0). \quad (10)$$

Then, it is easy to see that for $\cos(\Theta t) = 0$ and $\Theta(\theta + \chi) \gg \theta(\theta - \chi)$ (as it is for the parameters values employed in Fig. 2), the measured quantity practically coincides with the mode operator of the acoustic mode, thus revealing information on its state (see also [25]).

In conclusion, we have proposed a simple scheme to teleport an unknown quantum state of a radiation field onto a macroscopic, collective vibrational degree of freedom of a massive mirror. The basic resource of entanglement is attained by means of the optomechanical coupling provided by the radiation pressure. Here we have shown the teleportation of the quantum information contained in an unknown quantum state of a radiation field to a collective degree of freedom of a massive object. This scheme could be easily extended in principle to realize a transfer of quantum information between two massive objects. In fact Victor could use tomographic reconstruction schemes, again based on the ponderomotive interaction (see [26]), to "read" the quantum state of a vibrational mode of another mirror and use this information to prepare the state of the radiation field to be sent to Alice. The present result could be challenging tested with present technology, and opens new perspectives towards the use of quantum mechanics in the macroscopic world. For example, we recognize possible technological applications such as the preparation of nonclassical states of MEMS [9], where the oscillation frequency could be higher and, consequently, the working temperature can be raised.

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