Using the Chern-Simons Action for a Self-Consistent Determination of the Magnetic Screening Mass in Thermal QCD

G. M. von Hippel and R. R. Horgan

Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Centre for Mathematical Sciences, Cambridge CB3 0WA, United Kingdom
(Received 19 July 2002; published 2 April 2003)

We present a self-consistent determination of the screening mass for chromomagnetic fields in QCD within the framework of dimensional reduction. The three-dimensional Chern-Simons density is used as a mass term for a self-consistent perturbative calculation that yields a value of $m \approx 1.604(g^2 N/2\pi)T$ for the magnetic screening mass.

The perturbative expansion of thermal gauge theories is well known to be plagued by infrared divergences that render perturbative computations of most quantities impossible, as first noted by Linde [1]. In order to overcome these difficulties, several effective field theories have been proposed [2,3]. These are based on the concept of dimensional reduction, where the static Matsubara modes of the thermal theory are described by an effective theory in three-dimensional Euclidean space, with the other modes (and any fermions) integrated out. For the chromomagnetostatic modes of QCD, the lowest-order effective theory is the conventional Yang-Mills theory in three dimensions, with dimensionful coupling $g_3 = g\sqrt{T}$.

In three dimensions, the dynamical generation of a gluon mass is expected. Unfortunately, a perturbative evaluation of this mass is not possible, for similar reasons as in the finite-T case: The superrenormalizability of the theory in three dimensions leads to infrared divergences beyond the one-loop level, and no mass for the chromomagnetic modes is generated at one-loop. In order to determine the magnetic screening mass within the framework of perturbation theory, one will therefore have to resort to self-consistent approximations.

In this Letter, we show how the Chern-Simons Lagrangian can be used in the context of a self-consistent resummation system, and discuss some of the potential problems with our approach, and why some of these problems might actually be considered as advantages.

Methods.—The Chern-Simons Lagrangian in three (Euclidean) dimensions is given by [4]

$$\mathcal{L}_{CS} = -im\epsilon^{\mu\nu\rho}\mathrm{tr}\left(A_\mu \partial_\nu A_\rho + \frac{2g_3}{3}A_\mu A_\nu A_\rho\right), \quad (1)$$

and is gauge-invariant (up to a total divergence) under small gauge transformations. For invariance of the action under large gauge transformations one needs the condition

$$\frac{4\pi m}{g_3^2} \in \mathbb{Z}. \quad (2)$$

For the purposes of doing perturbation theory, however, small gauge invariance is all that is needed, as perturbation theory does not experience contributions from nontrivial vacua. We will therefore regard m as an arbitrary parameter with no *a priori* relation to g_3.

If a Chern-Simons term is added to the standard Yang-Mills Lagrangian, the gauge fields acquire a mass from the quadratic part of $\mathcal{L}_{CS}$, and the bare gluon propagator in a covariant gauge becomes

$$D_{\mu\nu}^{ab}(p) = \left[\frac{p^2\delta_{\mu\nu} - p_\mu p_\nu - m\epsilon_{\mu\nu\rho}p^\rho}{p^2(p^2 + m^2)} + \xi\frac{p_\mu p_\nu}{(p^2)^2}\right]\delta^{ab}, \quad (3)$$

while the three-gluon vertex receives a momentum-independent extra contribution $igmf^{abc}\epsilon_{\mu\nu\rho}$ from the cubic part of the Chern-Simons Lagrangian. The propagator is manifestly infrared safe only in the Landau gauge $\xi = 0$, while in other covariant gauges spurious infrared divergences appear. While a cautious handling should be able to remove these infrared divergences from all physical quantities, we will not concern ourselves with those subtleties, and work exclusively in Landau gauge for reasons of convenience.

It is easily apparent that the Chern-Simons Lagrangian is odd under parity, hence adding it to a Yang-Mills action violates the parity invariance of the latter. This implies that the mass can have either a positive or a negative sign; as we are only interested in the position of the pole, which depends only on the square of the mass, we are free to chose the mass to be positive in all our calculations.

A self-consistent resummation scheme is normally based on adding a quadratic (parity-even) mass term to the Lagrangian and resubtracting it as a counterterm fixing the pole of the propagator at $p^2 = -m^2$:

$$\mathcal{L} = \mathcal{L}_0 + m^2\mathcal{L}_m - \delta m^2\mathcal{L}_m, \quad (4)$$

while demanding $\delta m^2 = m^2$ in order not to change the original theory, leading to a gap equation

$$m^2 = \Pi(-m^2), \quad (5)$$

where $\Pi(p^2)$ is the perturbative shift of the inverse propagator from its free form.

For the Chern-Simons mass term, which is only linear in the mass, a slightly modified scheme is needed. The Lagrangian with the mass term added and subtracted reads

$$\mathcal{L} = \mathcal{L}_0 + m\mathcal{L}_{CS} - \delta m\mathcal{L}_{CS}, \qquad (6)$$

where $\mathcal{L}_0$ is the standard Yang-Mills Lagrangian, and we demand

$$\delta m = m \qquad (7)$$

while considering δm as a counterterm that fixes the pole of the propagator at $p^2 = -m^2$.

By adding the Chern-Simons term to the Lagrangian, we have changed the admissible tensorial structure of the self-energy, which may now have a parity-odd part:

$$\Pi_{\mu\nu}(p) = (p^2\delta_{\mu\nu} - p_\mu p_\nu)A(p^2) + m\epsilon_{\mu\nu\rho}p^\rho B(p^2), \quad (8)$$

where the parity-even part $A(p^2)$ can be isolated by contracting with a symmetric tensor such as $\delta_{\mu\nu} - \lambda(p_\mu p_\nu/p^2)$, while the parity-odd part $B(p^2)$ may be isolated by contraction with $\epsilon_{\mu\nu\lambda}$.

Taking into account that $\delta m = m$, the inverse propagator turns out to be

$$\mathcal{D}_{\mu\nu}^{-1}(p) = (p^2\delta_{\mu\nu} - p_\mu p_\nu)[1 + A(p^2)] + m\epsilon_{\mu\nu\rho}p^\rho B(p^2), \qquad (9)$$

such that the resummed propagator becomes

$$\mathcal{D}_{\mu\nu}(p) = \frac{[1 + A(p^2)](p^2\delta_{\mu\nu} - p_\mu p_\nu) - B(p^2)m\epsilon_{\mu\nu\rho}p^\rho}{[1 + A(p^2)]^2 p^2\{p^2 + [\frac{B(p^2)}{1+A(p^2)}m]^2\}}. \qquad (10)$$

To keep the pole of the propagator fixed at $p^2 = -m^2$, we have to demand

$$B(-m^2) = 1 + A(-m^2) \qquad (11)$$

as the appropriate gap equation for the self-consistent determination of the magnetic mass with a Chern-Simons mass term.

Results.—Adding the Chern-Simons Lagrangian does not change the number or topology of the Feynman diagrams for the theory. The one-loop self-energy is therefore given by the sum of the same three diagrams as in the nonmassive pure gauge theory.

Let us first compute $B(-m^2)$. The only contributions to the parity-odd part of the gluon self-energy come from the diagram with two three-gluon vertices, because the external index structure of the other diagrams has not been changed by the addition of the Chern-Simons term. The contraction of this diagram with the ϵ tensor yields

$$B(p^2) = \frac{g_3^2 N}{p^2} \int \frac{d^3k}{(2\pi)^3}$$
$$\times \frac{(k^2 p^2 - (k \cdot p)^2)(5k^2 + 5k \cdot p + 4p^2 + 2m^2)}{k^2(k^2 + m^2)(k + p)^2((k + p)^2 + m^2)}. \qquad (12)$$

This integral is convergent both in the infrared and the ultraviolet. For our purposes, we need to continue it to $p^2 = -m^2$. Since there is a threshold and corresponding cut in the complex plane at $p^2 = 0$, coming from the p^2 term in the denominator of the propagator (3), we have to assure the analyticity of the integral by making sure that no poles of the integrand cross the axis of integration. Since the problematic poles are those from the $(k + p)^2$ term in the denominator of the integrand, this can be done by rotating the axis of integration by the same phase as p: For $p = e^{i\chi}m$, take $\arg(k) = \chi - \epsilon$, with ϵ infinitesimally small. Using this prescription, we can evaluate the integral numerically as

$$B(-m^2) = [0.1677(3) + 0.0160(5)i]\frac{g_3^2 N}{m}.$$

For an analytic calculation this prescription is slightly awkward. We can, however, still calculate the integral without recourse to any regulator by noticing that, since k and θ enter the integral only via the combinations k^2, $\cos^2\theta$ and $k\cos\theta$, we can replace the $\int_0^\infty dk \int_{-1}^1 d\cos\theta$ integration by a $\int_{-\infty}^\infty dk \int_0^1 d\cos\theta$ one, which can be performed analytically using the residue theorem. Alternatively, we could use partial fractions for a straightforward evaluation of the integral (where, however, a regulator such as dimensional regularization needs to be employed). Continuing $p \to im$, we finally find the value

$$B(-m^2) = \left(\frac{1}{8} + \frac{27}{32}\log 3\right)\frac{g_3^2 N}{2\pi m} + i\frac{g_3^2 N}{64m}$$

for the odd part of the self-energy at the pole, which agrees perfectly with the numerical result obtained using the rotated contour.

The computation of $A(-m^2)$ is a bit more subtle. Superficially, $A(-m^2)$ contains linear divergences, and while these would disappear using, e.g., dimensional regularization, it seems that the value of $A(-m^2)$ would be dependent on the regularization scheme used. Since $A(-m^2)$ enters the gap equation (11), this would render the self-consistent mass regulation dependent, thus invalidating the entire procedure. Fortunately, the situation is yet a bit more subtle: On dimensional grounds, it is known that the factor multiplying $p_\mu p_\nu$ in the self-energy must be ultraviolet convergent in three dimensions. Gauge invariance, however, implies that the parity-even part of the self-energy is of the form $(\delta_{\mu\nu}p^2 - p_\mu p_\nu)A(p^2)$, and hence the divergences in $A(p^2)$ must disappear for any gauge-invariant regulator, so as to render the value of $A(p^2)$ regulator independent. We choose to evaluate $A(-m^2)$ in a manifestly regulation-independent fashion

by identifying the self-energy integral multiplying $p_\mu p_\nu$, which can be isolated by contracting with the projector $\delta_{\mu\nu} - 3(p_\mu p_\nu / p^2)$.

The integral which then needs to be evaluated is

$$A(p^2) = \frac{g_3^2 N}{4p^4} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{k^2(k^2+m^2)(k+p)^2((k+p)^2+m^2)}$$
$$\times [-8k^6 p^2 + 24k^4(k\cdot p)^2 + 8k^4(k\cdot p)p^2 + 36k^2(k\cdot p)^3 - 8k^4 p^2 m^2 - 14k^4 p^4 + 24k^2(k\cdot p)^2 m^2$$
$$+ 66k^2(k\cdot p)^2 p^2 - 12(k\cdot p)^4 + 14k^2(k\cdot p)p^2 m^2 + 12k^2(k\cdot p)p^4 + 18(k\cdot p)^3 - 4k^2 p^2 m^4 - 4k^2 p^6$$
$$+ 12(k\cdot p)^2 m^4 + 24(k\cdot p)^2 p^2 m^2 + 4(k\cdot p)^2 p^4 + 8(k\cdot p)p^2 m^4 + 8(k\cdot p)p^4 m^2]$$

As above, we have to either rotate the integration contour for a numerical evaluation yielding

$$A(-m^2) = [-0.0881(2) + 0.0165(5)i] \frac{g_3^2 N}{m}$$

or perform the integral analytically (making use of a computer algebra program for obvious reasons) and take the limit $p \to im$ afterwards to obtain

$$A(-m^2) = \left(\frac{3}{8} - \frac{27}{32}\log 3\right) \frac{g_3^2 N}{2\pi m} + i\frac{g_3^2 N}{64m},$$

which again agrees with the numerical result.

The self-energies of Chern-Simons massive three-dimensional Yang-Mills theory have previously been evaluated within the frameworks of dimensional regularization [4–6] and differential regularization [7], respectively. It was found that, despite some controversy as to whether ambiguities might arise from the continuation of the totally antisymmetric ϵ-tensor to d dimensions, the results in both regularization schemes agreed. Here, we have shown that the self-energies of Chern-Simons massive three-dimensional Yang-Mills theory are indeed finite and regularization independent for any gauge-invariant regulator.

With both parts of the self-energy at hand, we can now proceed to solve the gap equation (11). Multiplying both sides by m yields a value of

$$m = \left(\frac{27}{16}\log 3 - \frac{1}{4}\right) \frac{g_3^2 N}{2\pi} \approx 1.604 \frac{g_3^2 N}{2\pi} \qquad (13)$$

for the magnetic mass. We note that the Ward identities of the theory [5] ensure $\Im(A(-m^2)) = \Im(B(-m^2))$ so that the self-consistent mass will always be real.

Discussion.—There have been several self-consistent studies of the magnetic mass using parity-even quadratic mass terms:

Alexanian and Nair [8] have proposed a mass term that is related to the generating functional for hard thermal loops, and have obtained the mass

$$m_{\mathrm{AN}} = \left(\frac{21}{16}\log 3 - \frac{1}{4}\right) \frac{g_3^2 N}{2\pi} \approx 1.192 \frac{g_3^2 N}{2\pi}. \qquad (14)$$

Buchmüller and Philipsen [9] have coupled a Higgs field in the symmetric phase to the gluons and have found

a gluon mass of

$$m_{\mathrm{BP}} = \left(\frac{63}{64}\log 3 - \frac{3}{16}\right) \frac{g_3^2 N}{2\pi} \approx 0.894 \frac{g_3^2 N}{2\pi}. \qquad (15)$$

The results of Buchmüller and Philipsen have been extended to two loops by Eberlein [10], who found

$$m_{\mathrm{Eb}} \approx 1.052 \frac{g_3^2 N}{2\pi}, \qquad (16)$$

which indicates a small, though nonnegligible, contribution from higher loop orders to the one-loop results.

Cornwall [11] has used a "pinch" technique for the determination of the gluon mass and has obtained

$$m_{\mathrm{Co}} = \left(\frac{15}{16}\log 3 - \frac{1}{4}\right) \frac{g_3^2 N}{2\pi} \approx 0.780 \frac{g_3^2 N}{2\pi}. \qquad (17)$$

A purely nonperturbative determination of the mass gap in $(2+1)$-dimensional Yang-Mills theory has been carried out by Karabali, Kim, and Nair [12]. Using a functional Hamiltonian approach, they find that, to lowest-order in an enhanced perturbative expansion, three-dimensional Yang-Mills theory is a theory of massive interacting colored particles with mass

$$m_{\mathrm{KKN}} = \frac{g_3^2 N}{2\pi}. \qquad (18)$$

This agrees well with a recent lattice study by Philipsen [13], who uses the mass splittings between 0^{++} and 1^{--} bound states of heavy scalars in the static limit to determine a parton mass of

$$m_P = 0.360(19)g_3^2 \sim 1.131(59)\frac{g_3^2 N}{2\pi} \qquad (19)$$

for the three-dimensional SU(2) gluon.

The similarity between these nonperturbative results and the perturbative calculations above lends credibility to the assumption that a magnetic screening mass of order $(g_3^2 N/2\pi)$ is generated for the chromomagnetostatic modes in the QCD plasma. We also note the tantalizing fact that the nonperturbative result m_{KKN} obeys the condition (2). The result obtained with the Chern-Simons term is almost a factor of 2 larger than those obtained using parity-even mass terms.

Compared to quadratic mass terms, the Chern-Simons term has the apparent disadvantage of being odd under parity, thereby breaking the parity invariance of the original theory. However, we do not necessarily consider this a disadvantage, for the following reason: The thermal ground state of the plasma is not the vacuum. Parity-odd terms made up from chromomagnetic and chromoelectric fields may have a nonvanishing expectation value if something like a chromodynamic dynamo effect takes place in the plasma, breaking the parity invariance of the action. The fact that in this case the symmetry breaking necessary to have a nonvanishing Chern-Simons term in the action has to be caused by effects in the plasma, opens the interesting possibility of having a domain structure within the plasma, where different regions correspond to different signs of the Chern-Simons mass. If this were the case, one would have to sum over both signs in calculating long-distance contributions to thermodynamic quantities with a Chern-Simons massive gluon. The existence of a domain structure would also imply the existence of domain walls separating the two phases, that would have their own (purely nonperturbative) mass scale associated with them, which would also have to be taken into account in determining the long-range dynamics of the plasma. The question of a possible domain structure in the plasma, and possible observable consequences of parity breaking in the plasma certainly deserve further research.

Another apparent problem lies in the appearance of the p^2 term in the numerator of the propagator (3), which as seen above leads to a branch point at $p^2 = 0$, and seems to preclude exponential screening of chromomagnetic fields. However, as noted by Deser, Jackiw, and Templeton [4], and by Pisarski and Rao [5], no massless intermediate states may contribute to the expectation values of gauge-invariant operators, so that the physical long-range inter-actions would still be screened. In principle, this could be demonstrated by computing the static quark potential, which should not contain a massless contribution.

We conclude that the Chern-Simons term cannot be excluded *a priori* on the grounds that it is forbidden by symmetry, and therefore must be included in the effective Lagrangian. From there, our self-consistent calculation shows that it is possible to have a nonzero value for the Chern-Simons mass. There are, however, many problems with the Chern-Simons term as a mass term for the gluon, and further research is certainly needed in order to arrive at any definite conclusions about its possible role in chromomagnetic screening.

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