Self-Induced Hysteresis for Nonlinear Acoustic Waves in Cracked Material

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A new phenomenon of self-induced hysteresis has been observed in the interaction of bulk acoustic waves with a cracked solid. It consists in a hysteretic behavior of material nonlinearity as a function of the incident pump wave amplitude. Hysteresis manifests itself in the self-action of the monochromatic pump wave and in the excitation of its superharmonics and of its subharmonics. The proposed theoretical models attribute the phenomenon to hysteresis in transition of the acoustically forced oscillation of cracks from a nonclapping regime to a regime of clapping contacts.

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Hysteresis in elastic behavior of materials is a well-known (but in many cases much less understood) physical phenomenon. Similar to the studies in ferromagnetic systems [1], efficient phenomenologies have already been proposed [2,3] for some hysteretic phenomena observed at macroscopic level, but an adequate microscopic theory is yet to be developed. The hysteresis of a macroscopic stress/strain relationship in such materials as rocks, microcrystalline metals, and ceramics, for example, might be attributed [3] to hysteresis of an unspecified nature in mechanical behavior of mesoscopic structural two-level elements. In accordance with theory [2,3] the mesoscopic mechanical elements contribute to hysteresis of material nonlinearity. Because of this (in order to identify the mesoscopic elements), it is tempting to investigate the nonlinearity of a material stress/strain relationship by methods of nonlinear acoustics [3,4].

The studies of mechanical systems, which might be considered as prototypes of the individual hysteretic elements, constitute the opposite extreme limit of research activities in nonlinearity hysteresis. Using an atomic force microscope (AFM) and a friction force microscope, hysteresis in tip/surface interaction and sliding friction can be evaluated on a nanoscale (see, for example, Ref. [5] and references therein). However, there is still a significant gap between our understanding of hysteresis of artificial contacts and those of a crack embedded in a solid matrix.

Only recently were the methods of nonlinear acoustics applied for the first time to study the hysteresis phenomenon of individual crack [6]. A new type of hysteresis phenomenon was observed in the case of high-amplitude surface acoustic pulse interaction with surface-breaking cracks. In the first experiment the amplitude of the excited $3\omega/2$ subharmonic exhibited hysteresis as a function of the wave amplitude at fundamental frequency ω (pump wave) incident on the crack. In the second experiment (with different ω) the hysteresis in the amplitude of the second (2ω) and the third (3ω) harmonic amplitudes was observed. Neither transmitted nor reflected pump acoustic wave (at frequency ω) showed signs of the threshold or unstable behavior. The observed effects were attributed [6] to stochastic motion of cracks *parametrically* driven by acoustic wave. In particular, hysteresis in the 2ω signal was attributed to even parametric resonance $4(\omega/2)$.

In this Letter we report experimental observation of self-induced hysteretic behavior for the bulk sinusoidal acoustic wave interacting with a system of cracks inside a glass plate. Besides the effect of the self-induced hysteresis in the pump wave amplitude (at frequency ω), which have not been reported before, our observations differ from those reported earlier [6] in the following important aspect. We observed, for the fixed frequency of the pump wave in a given (unmodified) experimental configuration, different thresholds for the hysteresis of superharmonics (2ω, 3ω, ...) excitation and for the hysteresis of subharmonic ($\omega/2$) excitation [Fig. 1(a)]. The threshold for superharmonics excitation was significantly lower. Consequently, neither hysteresis in harmonics excitation nor self-induced hysteresis can be attributed in our system to parametric nonlinear phenomena. We interpret the phenomena as a hysteresis of an additional mechanism of nonlinearity due to clapping contacts between crack lips. This mechanism "turns-on" and "turns-off" at different amplitudes of the pump wave. The nonlinear oscillations of contacts in our model are *forced* but not parametric.

The experimental study was carried out on a set of glass plates with a different quantity of cracks produced through a thermal shock. Figure 1 illustrates the hysteresis phenomena in the acoustic spectra of two different plates representing two principal cases of in-phase and of out-of-phase observation. The effect of the hysteresis has not been observed in the samples without cracks even at the highest available level of the excitation. The setup consisted of two piezoelectric wide-band transducers firmly attached onto the opposite edges of a rectangular sample plate ($230 \times 190 \times 18$ mm^3). The emitter was driven by a harmonic 111 kHz wave of 150 V$_{p\text{-}p}$ amplitude. The signal induced in the receiver from acoustically excited cracks together with the transmitted pump wave was Fourier transformed by a vector signal analyzer (HP 89410A) having a full 100 dB dynamic range within 20 MHz bandwidth. Because of significant damage of

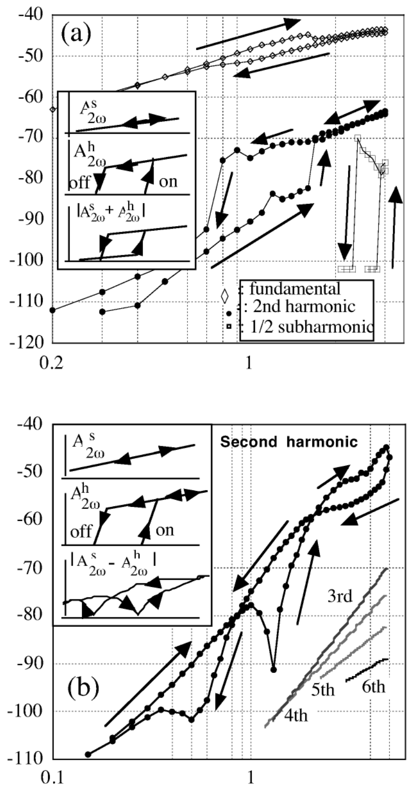

FIG. 1. Harmonics hysteresis loops obtained with intense ultrasonic pump observed in two different samples. Vertical axis: Spectral amplitude (in dB). Horizontal axis: External driving voltage (in arbitrary units, $1\,\text{a.u.} = 30\,\text{V}_{p\text{-}p}$). Insets: $A^s_{2\omega}$ and $A^h_{2\omega}$ denote the amplitude of the smoothly varying second harmonic signal that would occur without contribution from the activated crack, and of the hysteretic part due to nonlinear mode of crack vibration, respectively; the axes are in proportion to those in the basic figure. Figures 1(a) and 1(b) are obtained in the case of in-phase, and out-of-phase superposition of the signals, respectively.

the sample the attenuation length at fundamental frequency (estimated from the additional pulsed-echo experiments) did not exceed the dimensions of the sample, and the resonance phenomenon did not contribute to our observations.

The oscillation mode of each individual crack depends on the local distribution of the acoustical field. Thus the transition from the linear to nonlinear mode of oscillation happens first for a single crack. In the nonlinear mode the crack provides additional localized sources of superharmonics and modifies the acoustic field at fundamental frequency. The localized character of these sources is of great importance because the signal from the "activated" crack arrives to the point of observation with a phase shift depending on the relative position of the observation point and the crack. It is also important that the cracks in different positions are driven by the different superposition of the directly incident acoustic waves and the waves scattered from the boundaries and other cracks. The second harmonic curves in Figs. 1(a) and 1(b) correspond to two different plates where the signal from the crack oscillating in nonlinear mode arrives to the receiver in-phase and out-of-phase with the pump wave, respectively. Pronounced hysteresis is seen both on the second harmonic curve and on the fundamental one. The insets provide a qualitative explanation of the observed shapes for the second harmonic dependence on the pump amplitude in two limiting cases of the phase shift.

Hysteresis phenomena have been observed on all recorded superharmonics, up to 9ω, as well as on subharmonics, $\omega/2$. Analysis of these data provides two trends. First, the hysteresis threshold on the superharmonics is systematically much lower than the pump level for the appearance of subharmonics, which is illustrated by the $\omega/2$ curve in Fig. 1(a). Second, the threshold in hysteresis phenomena on ω and 2ω is accompanied by a very efficient generation of a large number of superharmonics. This fact is illustrated in Fig. 1(b) by four straight lines emerging from the narrow zone of the graph corresponding to the "jump-up" of the 2ω hysteresis loop. Here the experimental points for the third, fourth, fifth, and sixth harmonics are approximated for clarity by a linear function; thus the hysteresis loops are not seen on them.

The qualitative scenario of the observed phenomena proposed in the following is based also on the results of the calibration experiment. By optically measuring the vibration amplitudes at the surface of the plates, it was estimated that strain amplitudes in our experiments never exceeded 10^{-5}. This strain is too small to completely close the crack when the maximum crack length does not exceed a few centimeters (as it is in our samples). Because of this we currently attribute the observed phenomenon to the hysteresis turn-on and turn-off in clapping of some intermittent contacts between crack lips. The distance between the opposite asperities at crack lips can be much smaller than the average crack opening. It is well documented in literature that nonlinearity accompanying contacts' clapping is significantly higher than the elastic nonlinearity of homogeneous materials [7,8] and is even higher than the nonlinearity of nonclapping Hertzian contacts [8,9]. In AFM the distortion of the basic sinusoidal motion of the cantilever due to the tip hitting the sample was observed [10]. From the physics point of view this is due to very abrupt changes in the motion of clapping contacts during the impact.

To gain a further insight into the physical nature of the observed phenomenon we model the local place of possible clapping as an oscillator embedded in the elastic solid matrix (Fig. 2). In this lumped element model the effective masses of the interacting asperities are separated by the local crack opening width $2l_0$. The spring stiffness k models the rigidity of the crack (which is much less than the rigidity of the elastic matrix), and u denotes the local mechanical displacement of the crack surface. The regime of the interaction between the acoustic field and the

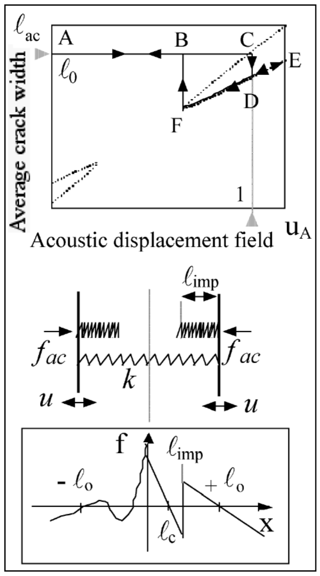

FIG. 2. Lumped element model for the clapping contacts between crack lips. The two dotted curves in the top inset present the result of a numerical simulation of Eq. (1).

crack depends on the ratio of the acoustic wavelength λ to the characteristic crack length L.

In the case $(\lambda/L) \ll 1$, the acoustic field action can be modeled by local sinusoidal forces f_{ac} applied to the interacting asperities. Only the part of these forces, which is symmetric relative to the plane $x = 0$ and which may cause clapping of the contacts, is presented in Fig. 2. In the limiting case $(\lambda/L) \ll 1$ our model is mathematically equivalent to that of a forced impact oscillator (see [11–15] and the references therein). Our problem has also evident similarity to the phenomena occurring with a ball bouncing on a vibrating table [15,16]. Fortunately the theory of impact oscillations is sufficiently developed to be useful for the interpretation of our experimental observations. Both the simplest model using an instantaneous impact rule [13] and a more realistic model [14] of the impact process, the Hertz contact law, demonstrate that sinusoidal nonimpacting oscillation becomes unstable (with increasing force amplitude) when the oscillating masses contact for the first time. The numerical solution [14] demonstrates that a subsequent increase of the force may cause restabilization of the oscillator onto one impact motion with the same period (period one) but nonsinusoidal (nonlinear). If the amplitude of the acoustic pump wave later diminishes, the system exhibits hysteresis in returning to nonimpacting period-one oscillation. The existence of a hysteretic zone where (depending on the excitation procedure) nonlinear impacting or linear nonimpacting motion occurs is confirmed by experiments [17]. The described scenario provides a possible explanation for the clapping onset in our experiments, which takes place at higher acoustic amplitudes than those necessary to stop clapping. As clapping provides the additional strong mechanism of acoustic nonlinearity [7–10], this scenario explains both the threshold increase in superharmonics excitation and the observed hysteresis in superharmonic amplitudes. The hysteresis at fundamental frequency (self-induced hysteresis) may be due to hysteresis in the amplitude of contact vibration at fundamental frequency and/or due to hysteresis in fundamental wave energy losses for superharmonics excitation and intermittent contact heating (pump depletion).

If the amplitude of the acoustic pump wave continues to increase (after the clapping threshold) then at some higher excitation the transition evolves first to a period-two and then to a period-four solution, as numerically predicted [14]. This is the beginning of a period doubling cascade. It is important to note that theoretically, the hysteresis in returning from period-two (subharmonic) to period-one oscillations may take place [12]. Recently the hysteresis in subharmonic excitation was reported for a nanoscale contact [15]. The above-described scenario provides a possible physical explanation for the behavior of the subharmonic $(\omega/2)$ signal in our experiment (Fig. 1).

In the forced impact systems the chaotic oscillations are possible theoretically [12,18,19] and are observed experimentally [19]. Thus a scenario of parametric excitation hypothesized in [6] is not the only possible route to chaos in crack oscillations. The acoustic turbulence in the interaction of high-power sound with liquids was attributed to forced (and not to parametric) excitation of gas bubbles [20]. In our experiments with cracked glass the threshold conditions for chaotic vibrations have not been achieved.

In the opposite limiting case of the acoustically small crack $(\lambda/L) \gg 1$ the scenario of forced motion of the crack explains observed hysteresis phenomenon as well. In this regime the crack lips oscillation precisely follows the displacement u_{ac} in the acoustic field (in Fig. 2 the force f_{ac} can be omitted while $u = u_{ac}$). The nonlinear interaction between the crack lips in this regime causes the variation in the local crack opening depending on the amplitude of the acoustic wave. This regime of the interaction of sound with crack resembles the ultrasonic force mode in the operation of AFM [21]. The clapping acts as a mechanical diode, demodulating the vibrations of crack lips. A process of this type was also observed in sound reflection from the solid-solid interface [8]. In the considered regime the distance between opposite asperities in the presence of the acoustic wave $2l_{ac}$ differs from $2l_0$ and is found from the condition of equilibrium $\langle f(l_{ac} + u_{ac})\rangle = 0$, where f is the total force acting on each of the masses in Fig. 2, $\langle\cdots\rangle$ denotes averaging over wave period T, and only the symmetric part of the acoustic displacement should be substituted. In general, the impact can be described rather precisely by including in the total force one or another model of single-valued (or modified by adhesion hysteresis) atomic interaction forces between the asperities [5]. These additional forces are represented qualitatively in Fig. 2 by the springs with the length $l_{\rm imp}$. For sufficiently soft cracks the force f contains an attractive region (see the part $x < 0$ of the inset of Fig. 2). To get a solution in a compact form we approximate f by a piecewise linear function [$f(x) = -k_0(x - l_0)$ if $x > l_{\rm imp}$, $f(x) = -k_c(x - l_c)$ if $x < l_{\rm imp}$; see the part $x > 0$ of the inset of Fig. 2]. Here $2l_c$ denotes the local separation of the asperities in the second possible equilibrium position ($2l_c \ll 2l_0$), k_c and k_0 denote the stiffness of locally closed and locally open crack, respectively ($k_c > k_0$). We approximate sinusoidal acoustic motion also by a piecewise linear function ($u_{ac} = 4u_A t/T$ if $-T/4 \le t \le T/4$, $u_{ac} = -4u_A(t/T - 1/2)$ if $T/4 \le t \le 3T/4$). The result of our evaluation of average local crack width is

$$l_{ac} = l_0 + \frac{1}{F^2 - E}\left[(F^2 + E)u_A - F(F + E) \pm \sqrt{E^2(F - 1)^2 + 4EF^2(u_A - 1)\left(u_A - \frac{E}{F}\right)}\right]. \quad (1)$$

Here all displacements are normalized to the distance $l_0 - l_{\rm imp}$, the parameter $F \equiv k_c(l_{\rm imp} - l_c)/k_0(l_0 - l_{\rm imp}) = |f(l_{\rm imp} - 0)/f(l_{\rm imp} + 0)|$ characterizes the relative magnitude of attractive and repulsive forces in the impact plane, and the parameter $E \equiv k_c(l_{\rm imp} - l_c)^2/k_0(l_0 - l_{\rm imp})^2$ characterizes the relative position of potential energy minima in closed and open contact states. It is the lower branch of the solution in Eq. (1), which is stable. The analysis demonstrates that if $F > 1$ then the initial nonimpacting regime $l_{ac} = l_0$ becomes unstable when crack lips just start kissing ($u_A = 1$) and local crack width diminishes by a jump. If simultaneously $E < 1$ (that is, if the crack in the open state has lower energy than in the closed state), then the return to open position takes place at smaller amplitudes u_A of acoustic wave. A typical hysteresis loop (*ABCDEDFBA*) for the crack width is presented [based on Eq. (1)] on the inset of Fig. 2. Note that even the sinusoidal motion of the crack lips provides (due to the nonlinearity of the interaction force in the regime of clapping contacts) the source of acoustic superharmonics. So the behavior predicted theoretically qualitatively reproduces hysteresis observed in our experiment.

Thermoelastic strain (induced by acoustic heating of clapping contacts [6,22]) may also bring hysteresis in the clapping process if local width of the crack diminishes due to inhomogeneous heating. In general, this mechanism should operate in parallel with the mechanism associated with nonlinearity of forced crack motion, which was proposed above.

In conclusion, our experimental results and qualitative models indicate prospects for future applications of nonlinear acoustic methods in nondestructive characterization of material damage. Acoustic monitoring of cracks is an important motivation for future research. The development of a quantitative theory for crack probing by strong acoustic waves is challenging.

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