Comment on "Mesoscopic Rectifiers Based on Ballistic Transport"

Recently, Fleischmann and Geisel (FG) [1] proposed a theoretical model to explain the rectifying effect demonstrated by Song *et al.* [2] in an asymmetric mesoscopic ballistic four-terminal structure. FG assume that the experimental setup may be reduced to a combination of narrow and wide conductors. Following a quasiclassical approach [3], they find that the energy dependence of the number of modes of a wide wire is $M(E) \propto \sqrt{E}$, giving rise to a nonlinear current-voltage characteristics, $I \sim \mu_L^{3/2} - \mu_R^{3/2}$, where μ_L and μ_R are the potentials at the left and right contact. This result contradicts the gauge invariance required by overall charge conservation [4]. Here we demonstrate that in the charge neutral limit their effect disappears altogether.

Consider a quasi-1D ballistic conductor connected to two electron reservoirs. In equilibrium, the electrochemical potential μ_0 must be equal throughout the system so that an equilibrium potential $eU_{\rm eq} = \mu_0 - E_F$ builts up inside the wire. Here, E_F is the Fermi energy given by the total number of quantum channels (see below). We neglect the spatial dependence of $U_{\rm eq}$ on the longitudinal direction and assume that the potential drops are accounted for in the interfaces between the wire and the leads. The confinement potential is assumed to consist of hard walls of infinite height. The wire width is w. Thus, the total density of states per unit length is $\nu(E) = \sum_n (m/2\pi^2\hbar^2)^{1/2}\theta(E-\varepsilon_n - eU_{\rm eq})/\sqrt{E-\varepsilon_n - eU_{\rm eq}}$, where $\theta(x)$ is the Heaviside step function and $\varepsilon_n = n^2\pi^2\hbar^2/2mw^2$ are the energy subbands. The total charge per unit length in equilibrium is $q_0 = e\int_0^\infty dE\nu(E)f_0(E)$. In the quasiclassical approximation, at $kT=0$, we find that $q_0 = ew\nu_{2D}(\mu_0 - eU_{\rm eq})$, where $\nu_{2D} = m/\pi\hbar^2$ is the 2D density of states for electrons. Hence, $E_F = q_0/ew\nu_{2D}$.

In a nonequilibrium situation, eU must be determined likewise self-consistently [4]. A calculation similar to that presented aboved yields $q = ew\nu_{2D}(\mu_L - eU)/2 + ew\nu_{2D}(\mu_R - eU)/2$. Notice that $q = q_0$ must be fulfilled to ensure charge neutrality [4]. Thus, the nonequilibrium screening potential is $eU = (\mu_L + \mu_R)/2 - E_F$. On the other hand, the number of modes taking part in the transport up to an energy E results from $M(E) = \sum_n \theta(E - \varepsilon_n - eU)$. Within the quasiclassical approximation, this yields $M(E) = (2mw^2/\pi^2\hbar^2)^{1/2}\sqrt{E - eU}\theta(E - eU)$. This is a crucial point of our discussion. In Ref. [1], $M(E)$ depends solely on the absolute value of E and the role played by the screening potential is neglected.

The electric (nonlinear) current may be now written:

$$I(\mu_L - \mu_R) = \frac{4e}{3h}M(\mu_0)E_F \times\left[\left(1+\frac{\mu_L-\mu_R}{2E_F}\right)^{3/2} - \left(1-\frac{\mu_L-\mu_R}{2E_F}\right)^{3/2}\right]. \quad (1)$$

Our expression shows two significant features. First, Eq. (1) is invariant under a global voltage shift. Second, there arises a natural energy scale, $2E_F$, below which our model makes sense. This energy scale is conspicuously absent in Ref. [1].

The application of Eq. (1) to the electron transport from a source (S) toward a drain (D) with a voltage probe (P) attached between the two wires [sketched in Fig. 1(c) of Ref. [1]] leads to the following current balance: $I(\mu_S - \mu_P) = I(\mu_P - \mu_D)$, which is itself gauge invariant [in contrast to Eq. (15) of Ref. [1]]. For fixed values of μ_S and μ_D, $I(\mu_S - \mu_P)$ is a monotonously decreasing function of μ_P whereas $I(\mu_P - \mu_D)$ is a monotonously increasing function of μ_P. Therefore, the latter equation possesses a unique solution: $\mu_P = (\mu_S + \mu_D)/2$. This result is independent of the wire width and we conclude that in the charge neutral limit the mechanism of FG is absent. The energy dependence of the channel number is irrelevant.

Let us now assume that the wire is in proximity to a gate with capacitance c per unit length. When a gate voltage V_g is applied to the wire, charge conservation leads to $c(U - V_g) + q^+ = c_q(\mu_L - eU)/2e + c_q(\mu_R - eU)/2e$, where $c_q \equiv e^2 w\nu_{2D}$ is the quantum capacitance and $q^+ = q_0$ is the positive background. For $c \to 0$ (the charge neutral limit), this equation gives rise to our previous results. In the opposite limit ($c \gg c_q$), we obtain $E_F = \mu_0 - eV_g$ in the equilibrium state. Only in this noninteracting limit is the model of FG [1] valid since now $I \propto [(\mu_L - eV_g)^{3/2} - (\mu_R - eV_g)^{3/2}]$. This infinite capacitance limit is, however, not a realistic description of the charge state of a ballistic wire [5].

Our discussion demonstrates the importance of a self-consistent treatment of nonlinear transport. For the scattering approach this implies that the scattering matrix is both a function of the incident energy of carriers and a functional of the self-consistent potential [4].

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[1] R. Fleischmann and T. Geisel, Phys. Rev. Lett. **89**, 016804 (2002).
[2] A. M. Song *et al.*, Phys. Rev. Lett. **80**, 3831 (1998); A. M. Song, Phys. Rev. B **59**, 9806 (1999).
[3] C.W. J. Beenakker and H. Van Houten, Solid State Phys. **44**, 1 (1991).
[4] M. Büttiker, J. Phys. Condens. Matter **5**, 9361 (1993); T. Christen and M. Büttiker, Europhys. Lett. **35**, 523 (1996).
[5] V. A. Sablikov *et al.*, Phys. Rev. B **61**, 13763 (2000).

