# Rotons in Gaseous Bose-Einstein Condensates Irradiated by a Laser

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A gaseous Bose-Einstein condensate irradiated by a far off-resonance laser has long-range interatomic correlations caused by laser-induced dipole-dipole interactions. These correlations, which are tunable via the laser intensity and frequency, can produce a "roton" minimum in the excitation spectrum—behavior reminiscent of the strongly correlated superfluid liquid He II.

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According to the celebrated Bijl-Feynman formula [1] for the excitation spectrum of He II,

$$E(k) \leq \frac{\hbar^2 k^2}{2mS(k)}, \tag{1}$$

the peculiar roton minimum at $k \approx 2\pi/r_0$, where $r_0$ is the average atomic separation, is due to a corresponding peak in the static structure factor $S(k) \equiv \langle 0|\hat{\rho}_k \hat{\rho}_k^\dagger|0\rangle/N$. Here $N$ is the number of atoms of mass $m$, $|0\rangle$ is the ground state of the system, and $\hat{\rho}_k \equiv \sum_q \hat{c}_q^\dagger \hat{c}_{q+k}$ is the density fluctuation operator. $S(k)$ is the Fourier transform of the pair correlation function and hence provides a measure of the degree of pair (2nd order) correlation between the atoms. The existence of strong pair correlations in He II may at first seem surprising since it remains a liquid even at temperatures approaching absolute zero precisely because of weak interatomic interactions (in combination with a small atomic mass) [2]. However, despite their apparent weakness, these interactions are very effective because the density of the liquid state is such that the average atomic separation, $r_0 = 4.44$ Å, is close to the minimum of the attractive interatomic potential well at 3 Å.

Contrast this now with an ultracold alkali atom gas in which the Bose-Einstein condensed fraction can be very nearly 100% [3]. The interactions in $^{87}$Rb, for example, are repulsive and characterized by an $s$-wave scattering length, $a \approx 5.5$ nm. This is between 1 and 2 orders of magnitude smaller than the average atomic spacing at typical densities. Steinhauer *et al.* [4] recently measured the bulk excitation spectrum of a $^{87}$Rb Bose-Einstein condensate (BEC) and found excellent agreement with Bogoliubov theory [5] (appropriate for a degenerate almost ideal Bose gas). No roton minimum was observed, a consequence of the diluteness with respect to $a$. Indeed, since Eq. (1) becomes an equality within the Bogoliubov theory [1], one sees that the pair correlation is small compared to He II. Significant pair correlation might exist in gaseous BECs at the very small scale of $a$, but this is fairly inaccessible in such a delicate system.

A marvelous feature of atoms, though, is that their interactions can be manipulated using external fields, allowing us to microscopically engineer the macroscopic properties of a many-body system. Thus, the experiment of Inouye *et al.* [6] took advantage of a Feshbach resonance to change the $s$-wave scattering length using magnetic fields. We recently proposed the use of off-resonant lasers to induce long-range dipole-dipole interactions whose characteristic length is the laser wavelength. These interactions can cause laser-induced "self-gravity" in a BEC, leading to three-dimensional self-trapping and electrostriction accompanied by unusual excitation spectra [7], as well as "supersolid" structures [8]. The aim here is to explore how the excitation spectrum and, by virtue of (1), the correlations of a gaseous BEC are changed when the interatomic potential is modified via laser-induced dipole-dipole interactions. This task requires a knowledge of the Fourier transform (FT) of the total interatomic potential, so we turn to this first.

Consider a BEC confined by a potential $H_{\rm trap} = \frac{m}{2}\omega_r^2(x^2+y^2) + \frac{m}{2}\omega_z^2 z^2$ into a very elongated cigar shape ($\omega_r \gg \omega_z$), irradiated by a far off-resonance plane-wave laser (Fig. 1, inset). The laser polarization is along the long $z$ axis of the condensate to suppress collective ("superradiant") Rayleigh scattering [9] or coherent atomic recoil lasing [10] that are forbidden along the direction of polarization. The far off-resonance condition, together with the small extent of the BEC along the laser propagation direction, enables us to treat the electromagnetic field inside the BEC in the Born approximation (field at each point is the sum of the incident plus once-scattered fields). Then the dipole-dipole potential between two atoms of separation $\mathbf{r}$, induced by far-off resonance light of intensity $I$, wave vector $\mathbf{k}_L = k_L\hat{\mathbf{y}}$ (along the $y$ axis), and polarization $\hat{\mathbf{e}} = \hat{\mathbf{z}}$ (along the $z$ axis) is [11]

$$U_{\rm dd}(\mathbf{r}) = \frac{I\alpha^2(\omega)k_L^3}{4\pi c\varepsilon_0^2} V_{zz}(k_L, \mathbf{r})\cos(k_L y). \tag{2}$$

Here $\alpha(\omega)$ is the isotropic, dynamic, polarizability of the

  

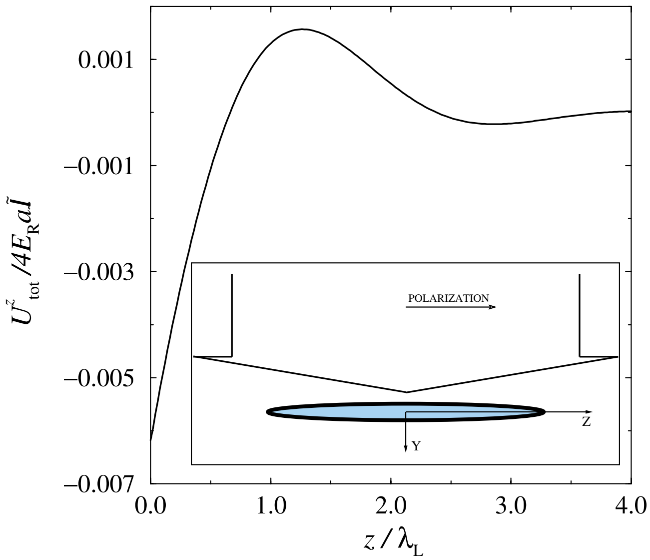


FIG. 1 (color online). The 1D interatomic potential $U^z_{tot}(z)$ [FT of (6)], for $w_r = 3.5\lambda_L$. A repulsive contact term $4E_R a(k_L w_r)^{-2}(1 + 4\tilde{I}/3)\delta(k_L z)$ is not shown. Inset: The laser beam and condensate geometry.

atoms at frequency $\omega = ck_L = 2\pi c/\lambda_L$. The prefactor can be expressed in terms of the Rayleigh scattering rate, $\gamma_R$, as $I\alpha^2 k_L^3/(4\pi c\varepsilon_0^2) = (3/2)\hbar\gamma_R$. $V_{zz}$ is the component of the retarded dipole-dipole interaction tensor generated by the linearly $\hat{z}$-polarized laser light

$$V_{zz} = \frac{1}{k_L^3 r^3}[(1 - 3\cos^2\theta)(\cos k_L r + k_L r \sin k_L r) - \sin^2\theta k_L^2 r^2 \cos k_L r], \quad (3)$$

$\theta$ being the angle between the interatomic axis and the $z$ axis. For large separations between atoms along the $z$ axis, $U_{dd}$ is asymptotically proportional to $-\sin(k_L r)/(k_L r)^2$. Many atoms (400 at densities of $8 \times 10^{14}$ atoms/cm$^3$) may lie within the characteristic interaction volume ($\lambda_L^3$) of this attractive long-range potential. As for the electron gas and charged Bose gas, mean-field (here Bogoliubov) theory applies in this *high density* regime [12].

The laser (dynamically) induced dipole-dipole potential is distinguished from the static field ($r^{-3}$) case [13,14] by a longer range and a huge enhancement of atomic polarizability around a resonance. For example, in Ref. [4], $^{87}$Rb atoms are magnetically trapped in the maximally stretched $|5s\,^2S_{1/2}, F = 2, M = 2\rangle$ state. A laser polarized along $\hat{z}$ is then $\pi$ polarized and only $\Delta M = 0$ dipole transitions are allowed. If the light is detuned by, say, $\delta = 2\pi \times (6.5$ GHz) (i.e., 1134 natural linewidths) below the $D1$ line (795.0 nm), then only virtual transitions to the $|5p\,^2P_{1/2}, F = 2, M = 2\rangle$ state need be considered. We calculate $\alpha \approx 5.0 \times 10^{-35}$ C m$^2$/V (cf. the static value $5.3 \times 10^{-39}$ C m$^2$/V).

In terms of the condensate density $n(\mathbf{r})$ at zero temperature, we account for atom-atom interactions using a mean-field energy functional of the form $H_{dd} + H_s$, where $H_{dd} = (1/2)\int n(\mathbf{r})U_{dd}(\mathbf{r} - \mathbf{r}')n(\mathbf{r}')d^3r d^3r'$, and $H_s = (1/2)(4\pi a\hbar^2/m)\int n(\mathbf{r})^2 d^3r$ is due to short-range interactions, which are described, as is usual, by a delta function pseudopotential (we take here the *repulsive* case for which $a > 0$). By working with the bare dipole-dipole interaction, we assume the Born approximation also for *atom-atom* scattering by this long-range part of the total potential. We note that the short-range (static) part of the laser-induced dipole-dipole interaction can cause a shift in $a$. For the laser intensities and detunings considered here, this shift is small according to existing estimates [14].

In a radially tight trap it is reasonable to assume a cylindrically symmetric ansatz for the density profile of radial width $w_r$: $n(\mathbf{r}) \equiv N(\pi w_r^2)^{-1} n^z(z)\exp[-(x^2 + y^2)/w_r^2]$, where $N$ is the total number of atoms and $n^z(z)$ is normalized to 1 and left general. Denoting the FT of the atomic density by $\tilde{n}(\mathbf{k}) = \int \exp[-i\mathbf{k}\cdot\mathbf{r}]n(\mathbf{r})d^3r$, then we have $H_{dd} = (1/2)(2\pi)^{-3}\int \tilde{U}_{dd}(\mathbf{k})\tilde{n}(\mathbf{k})\tilde{n}(-\mathbf{k})d^3k$, where the FT of the dipole-dipole potential (2), $\tilde{U}_{dd}(\mathbf{k}) = \int \exp[-i\mathbf{k}\cdot\mathbf{r}]U_{dd}(\mathbf{r})d^3r$, is the real part of

$$\tilde{U}_{dd}(\mathbf{k}) = \frac{I\alpha^2}{2\epsilon_0^2 c}\left(\frac{k_z^2 - k_L^2}{k_x^2 + (k_y - k_L)^2 + k_z^2 - k_L^2 - i\eta} + \frac{k_z^2 - k_L^2}{k_x^2 + (k_y + k_L)^2 + k_z^2 - k_L^2 - i\eta} - \frac{2}{3}\right). \quad (4)$$

The principal value of the radial integration in $H_{dd}$ can be evaluated analytically so that the dipole-dipole energy reduces to a one-dimensional functional along the axial direction $H_{dd} = (N^2/2)\int n^z(z)n^z(z')U^z_{dd}(z - z')dzdz' = (N^2/4\pi)\int \tilde{n}^z(k_z)\tilde{n}^z(-k_z)\tilde{U}^z_{dd}(k_z)dk_z$, where $\tilde{n}^z(k_z)$ is the FT of the axial density $n^z(z)$. The one-dimensional (1D) axial potential that appears in this expression has the form

$$\tilde{U}^z_{dd}(k_z) = \frac{I\alpha^2 k_L^2}{4\pi\epsilon_0^2 c}Q(w_r, k_z), \qquad Q(w_r, k_z) = -\frac{2}{3}\frac{1}{k_L^2 w_r^2} + \frac{k_z^2 - k_L^2}{k_L^2}e^{(k_z^2 - 2k_L^2)w_r^2/2} \times \sum_{j=0}^{\infty}\frac{(k_L w_r)^{2j}}{2^j j!}\mathrm{Re}\left\{E_{j+1}\left[\frac{(k_z^2 - k_L^2)w_r^2}{2}\right]\right\}, \quad (5)$$

where $\mathrm{Re}\{E_j[z]\}$ is the real part of the generalized exponential integral [15]. The FT of the total ($s$-wave plus dipole-dipole) 1D reduced interatomic potential is

$$\tilde{U}^z_{\mathrm{tot}}(k_z) = 4E_R a[(k_\mathrm{L} w_r)^{-2} + \mathcal{I} Q(w_r, k_z)], \tag{6}$$

where $E_R = \hbar^2 k_L^2/2m$ is the photon recoil energy of an atom and $\mathcal{I}$ is the dimensionless "intensity" parameter

$$\mathcal{I} = \frac{I\alpha^2(\omega)m}{8\pi\epsilon_0^2 c\hbar^2 a}, \tag{7}$$

in which $a$ is the $s$-wave scattering length in the absence of the laser-induced interaction. It is emphasized that the radial degree of freedom is contained in (6) via the radius $w_r$. The coordinate space potential, $U^z_{\mathrm{tot}}(z)$, is shown in Fig. 1.

We now have the essential ingredients to compute the excitation spectrum of the BEC, as it is the FT of the effective interatomic interaction potential that appears in the Bogoliubov dispersion formula [5]. Since the influence of radial excitations upon the low-energy spectrum can be largely frozen out under tight radial confinement, we consider only axial phononic excitations, neglecting the mixing with radial modes, and assume that the system is infinite along the $\hat{z}$ direction. In terms of the phonon momentum $p_z = \hbar k_z$, the axial Bogoliubov spectrum is [cf. Eq. (1)]

$$E_\mathrm{B} = \sqrt{c_z^2 p_z^2 + (p_z^2/2m)^2} = p_z^2/[2mS(k_z)], \tag{8}$$

where $c_z^2 = \pi n(0) w_r^2 \tilde{U}^z_{\mathrm{tot}}(k_z)/m$. $n(0)$ is the central density in the cigar, so $\pi n(0) w_r^2$ is the linear density along the cigar. For the linear parts of the spectrum, $c_z$ can be interpreted as the speed of sound in the gas. Shining a 795.0 nm laser upon a $^{87}$Rb BEC of density $n(0) = 8 \times 10^{20}$ atoms/m$^3$ and radius $w_r = 3.5\lambda_L = 2.78$ $\mu$m, a roton minimum appears when $\mathcal{I} \geq 0.051$ (i.e., $I \geq 0.506$ W/cm$^2$), although the dispersion relation is considerably altered far before this. The change in the dispersion relation could be observed using Bragg spectroscopy as performed in [4]. Figure 2 plots the Bogoliubov dispersion for $\mathcal{I} = 0.057$ ($I = 0.565$ W/cm$^2$).

Local to the roton minimum at $k = k_{\mathrm{roton}}$ one can write $E = \Delta + \hbar^2(k - k_{\mathrm{roton}})^2/2m^*$, and for the parameters above with $\mathcal{I} = 0.057$, one finds $m^* = 0.06m$. He II has $m^* = 0.16m$ [5]. The static structure factor is plotted in Fig. 3. The peak in $S(k_z)$ corresponds to the minimum in the energy spectrum. The model described here predicts that when $\mathcal{I} \geq 0.066$ ($I \geq 0.654$ W/cm$^2$) the minimum touches the zero energy axis. At this point the system is unstable to a periodic, supersolidlike, density modulation [8,16].

The laser-induced dipole-dipole potential can lead to electrostriction (compression) of a condensate [7]. In the present regime of low laser intensity/large detuning, the electrostriction is negligible (on a scale set by the collapse threshold $\mathcal{I} = 3/2$ [7,13]). This regime also ensures the absence of two-body bound states in the 1D reduced potential shown in Fig. 1, a necessary condition for the validity of the Born approximation for atom-atom scattering by this potential. Only when $\mathcal{I} > 1.3$ do bound states appear.

The interaction (3) arises from the forward scattering of laser photons by atom pairs. At large detunings there are two main competing processes that can heat a dense gas. (A) Light-induced transfer of pairs of colliding atoms to a quasimolecular excited state followed by dissociation, releasing $\approx \hbar\delta$ into the kinetic energy of the pair [17]. This is a density

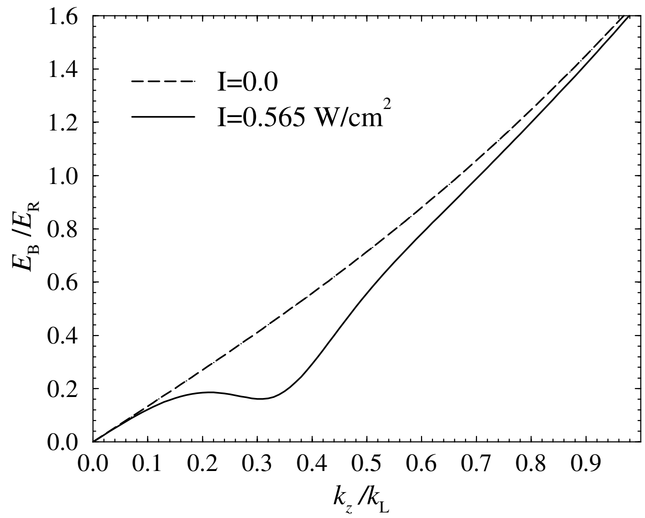


FIG. 2. The modification of the Bogoliubov dispersion relation of a $^{87}$Rb BEC by a laser detuned by 6.5 GHz below the $D1$ line. The radial width of the BEC is $w_r = 3.5\lambda_L$ and the central density is $n(0) = 8 \times 10^{20}$ atoms/m$^3$. For pure $s$-wave scattering ($\mathcal{I} = 0$) the inverse healing length $1/\xi_0 = \sqrt{8\pi a n(0)} = 1.32k_L$.

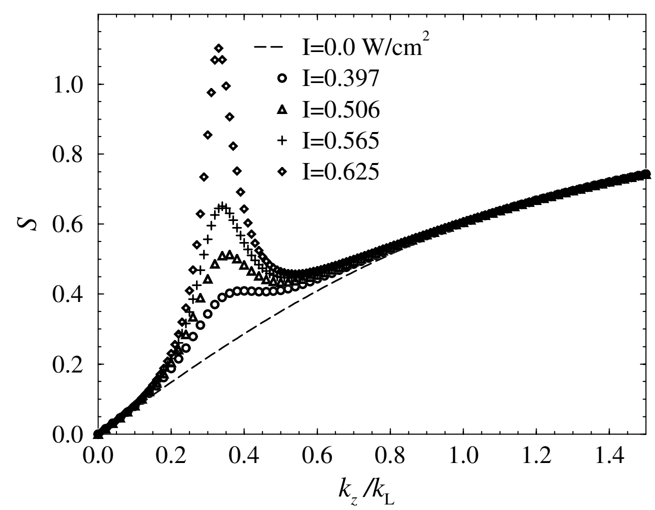


FIG. 3. The static structure factor $S(k_z)$ for various laser intensities. Same parameters as Fig. 2.

dependent effect whose rate can therefore be high. Even when the laser is red-detuned from an atomic resonance, when two atoms collide the energy separation between the ground state and a molecular excited state ($-C_3/r^3$) comes into resonance at small distances. However, by choosing $\delta$ so that the resonance point occurs between two molecular vibrational states, this process is suppressed [17]. Below the $D1$ line there are only discrete molecular vibrational states (i.e., no continuum states) so a detuning can be selected which is between these molecular resonances [18], which are narrow at ultracold temperatures. (B) Incoherent light scattering by single atoms occurs at approximately the Rayleigh scattering rate which can be written $\gamma_R = (8/3)E_R k_L a I/\hbar$. Applying the $f$-sum rule for the dynamic structure factor, one can show [19] that Rayleigh scattering transfers energy to the gas at a rate $\frac{d}{dt}E_{\text{tot}} = 2E_R N \gamma_R$ which, surprisingly, is independent of the interactions between the atoms. Comparing this heating rate with the energy of the ground state of the gas, $E_{\text{tot}} \approx H_s + H_{\text{trap}} + H_{\text{kin}}$, where $H_{\text{kin}}$ is the kinetic energy of the atoms, one can estimate a heating time via $\tau_{\text{heat}} = E_{\text{tot}}/(dE_{\text{tot}}/dt)$. To measure a roton the BEC must survive for longer than the roton period $\tau_{\text{roton}} \propto 2\pi\hbar/E_R$ (cf. Fig. 2). For the density and radius stated above, then for $I = 0.051$ one finds $\tau_{\text{heat}} \approx 8\tau_{\text{roton}}$, making an experiment challenging but feasible. The situation improves for larger, denser condensates since the polarization increases as $N^2$, whereas $\gamma_R$ is a single atom effect. Finally, we note that reducing the $s$-wave scattering length via a Feshbach resonance allows the laser intensity (and hence the Rayleigh scattering) to be reduced by an equal factor—see Eq. (7)—and still obtain the same effects.

In conclusion, atom-atom correlations due to laser-induced dipole-dipole interactions in a gaseous condensate can give a roton minimum in the Bogoliubov dispersion relation. The correlations are tunable via parameters such as radial width, laser intensity, and wavelength.

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*Note added.*—A maxon-roton spectrum has just been predicted for a pancake-shaped BEC with *static* dipolar interactions [20].

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[14] S. Yi and L. You, Phys. Rev. A **63**, 053607 (2001). Figure 1 shows shape resonances in $^{41}$K for *static* electric fields of $E > 15 \times 10^5$ V/cm. The relevant quantity to compare with the present case is the atom's response $P = \alpha E$. See also P. O. Fedichev, Yu. Kagan, G. V. Shlyapnikov, and J. T. M. Walraven, Phys. Rev. Lett. **77**, 2913 (1996).
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