Depinning by Fracture in a Glassy Background

M. B. Hastings, C. J. Olson Reichhardt, and C. Reichhardt

Center for Nonlinear Studies and Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

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We force a single particle through a two-dimensional simulated glass of smaller particles. We find that the particle velocity obeys a robust power law that persists to drive wells above threshold. As the single driven particle moves, it induces cooperative distortions in the surrounding medium. We show theoretically that a fracture model for these distortions produces power-law behavior and discuss implications for experimental probes of soft matter systems.

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Recently, there has been considerable interest in the behavior of systems of interacting particles that resist flow, including vortex glasses in superconductors [1], charge-density-wave metals [2], Wigner crystals [3], dense colloidal suspensions [4,5], and Coulomb-blockade arrays [6,7]. In each of these systems, quenched disorder and repulsive interactions from the surrounding particles prevent any individual particle from moving in response to a small external force. As a result, below a threshold force, the system behaves as a solid, while above the threshold, the system can flow plastically and is soft and disordered. Once the system depins, increasing the force leads to rapidly increasing motion, as the speed and transport response increase faster than linearly with the flow over a range of applied force. Theories [6,8–10] have attributed this anomalous transport to the concerted action of many driven particles across a rough pinscape. They predict power-law growth of the velocity with force in the vicinity of a threshold force $F_c : v \sim (F - F_c)^{\beta}$. In experiments and simulations, the velocity above threshold has been observed to vary as a power β of the force, with β in the range 1.5 to 2.2 [3,4,11,12]. In contrast to the critical-state model of elastic depinning [8], where scaling occurs only very close to threshold, the power law is observed to hold for forces of several times the threshold force.

In this work we demonstrate power-law collective transport with a *single* driven particle in a disordered glassy matrix of other nondriven particles in two dimensions, realized via a molecular-dynamics simulation. We find a power law with $\beta = 1.5$ over two decades of force that is insensitive to the system size or the density of the surrounding medium and appears when the driven particle is larger than the surrounding particles. This contrasts with a single particle driven over a substrate with *quenched* disorder where a scaling of $\beta = 1/2$ is expected [8]. Our result suggests that the origin of the $\beta > 1$ scaling in a variety of systems may be simpler than previously supposed. A single driven particle drags other particles with it, thus slowing it down. The faster it moves, the fewer particles it drags, and hence the exponent β becomes larger than 1.0. We explicitly show that an analysis in terms of fracture in front of the particle gives $\beta = 1.5$. The fracture leads to a one-dimensional (1D) plastic zone, which appears as a riverlike flow of particles. While in other cases such rivers are attributed to easy paths through a background of quenched disorder, in our system the only disorder is due to the glassiness of the medium. The plastic zone appears due to the softness of the system; for stiffer interactions, the plastic zone disappears and $\beta = 1$ is the observed scaling.

In addition to offering a simpler model of depinning, our results are relevant to systems in which two species of particles move in opposite directions with respect to each other, as in certain electrophoresis experiments, pedestrian motion, self-driven particles, and molecular motors [13]. Our model should apply to systems where the particles interact with a screened Coulomb interaction, such as driving a single particle through a disordered colloidal medium or driving a single dust particle in a disordered dusty plasma [14].

Further, driving a single particle through a soft matter system can be used as a powerful experimental probe of dynamics of the medium far from equilibrium. For example, in recent experiments a magnetic particle is dragged through a colloidal system near the glass transition [5]. The absence of momentum conservation in our system leads to very different physics from the transfer of momentum through hydrodynamic and hard-core interactions. Long-range hydrodynamic interactions are not fully understood in these systems. However, such experiments on colloidal systems and emulsions in confined geometries between parallel walls, where the fluid can give up momentum to the wall, may reduce the effect of hydrodynamic interactions leading to a system like ours.

Simulations and dissipation balance.—We drive a single particle with a charge q_D at constant force through a two-dimensional (2D) disordered system with periodic boundary conditions in the x and y directions. To create a glassy medium and prevent formation of a triangular lattice, we simulate a mixture of two species of particles with different charges $q_A = q$ and $q_B = 2q$, for a given fixed charge $q = 0.086$, with equal numbers of both particles. The particles were rapidly quenched from

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high temperature to zero temperature to produce a glassy state. For the driven particle, we consider a range of values from $q_D = 0.15q$ to $q_D = 60q$, so that we consider both $q_D < q$ and $q_D > q$. The overdamped equation of motion for particle i is $\eta \mathbf{v} = \mathbf{f}_i = -\sum_j \nabla U(r_{ij}) + F_d \hat{\mathbf{x}}$, where $\mathbf{v}$ is the particle velocity, the damping coefficient $\eta = 1$, and $F_d = 0$ on all particles except the one with charge q_D. We use a screened Coulomb interaction, given by

$$U(r_{ij}) = q_i q_j \frac{e^{-2r_{ij}}}{r_{ij}}, \qquad (1)$$

between particles i and j separated by a distance r_{ij}. We have considered a variety of system sizes for $N = 480$ and $N = 2150$ particles, as well as separately considering a range of particle densities (this is equivalent to considering a range of screening lengths). Defining a lattice constant a for the system by the lattice constant of a triangular lattice with the same density, we considered lattice constants from $a = 1.085$ to $a = 2.3$.

In Fig. 1 we show the positions of the particles and the trajectories for a fixed number of time steps for a system with $N = 2100$ particles at a drive $F_d = 0.75$. The driven particle, marked as a large dot, has $q_D = 20$. We find that in general there is a finite threshold force F_c for the particle to move. The perturbation of the other particles by the driven particle is anisotropic, with a larger perturbation in the direction of drive than in the transverse direction. Particles more than a few a away from the path of the driven particle move elastically in small nearly closed orbits of radius less than a. Particles in front and behind the driven particle exhibit plastic motion. This plastic zone *decreases* in size for higher velocities.

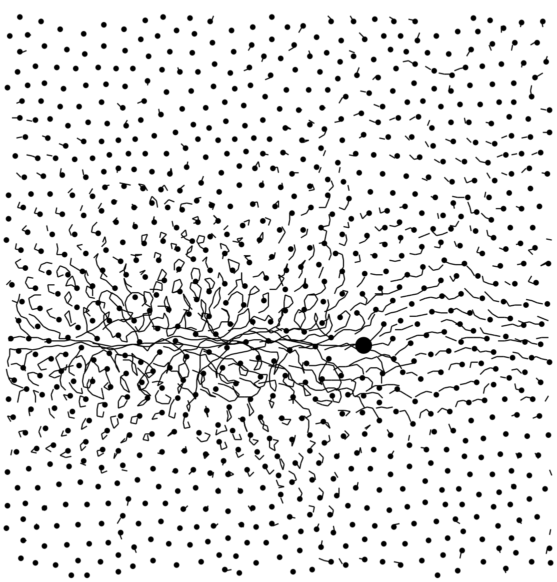

FIG. 1. The particle positions (black dots) and trajectories (black lines). The driven particle (large dot) has charge $q_D = 20$; the remaining particles are a mixture of $q_A = q$ and $q_B = 2q$ with a lattice constant of 1.085. There are a total of 2100 particles. The trajectories are drawn for a fixed number of time steps with a constant applied drive of $F_d = 0.75$.

The medium, while disordered, is a solid. It has a threshold, $F_d = F_c$, for failure, and can support shear stress. Thus, the particle must *fracture* the medium to move through it. From momentum balance, we have that at all times $F_d = \eta V + \eta \sum v_A$, where V is the velocity of the driven particle and the sum ranges over all other particles. Below threshold, where the medium moves with the particle, the momentum balance yields $V = F_d/N$, where N is the number of particles. Above threshold, to obtain power-law scaling, some large number n of other particles, $1 \ll n \ll N$, must move with the driven particle, with some finite size corrections to scaling due to net background motion of the remaining $N - n$ particles. Eventually, as $n \to 1$ far above threshold, the velocity returns to linear scaling.

In Fig. 2 (lower solid curve) we show the log-log plot for the velocity versus applied drive $F_d - F_c$, for the driven particle in Fig. 1, where $F_c = 0.7$ is the threshold for the large particle to move. We find a good scaling with a fit of $\beta = 1.47 \pm 0.03$. We find a similar scaling for a variety of other parameters with the exponents of 1.5 ± 0.05. At sufficiently large drive, β returns to 1, as expected. The size of the scaling region decreases with decreasing q_D/q, until for $q_D/q < 1.0$, no anomalous scaling is observed. In this case the driven particle does not induce plasticity in the other particles but only a smaller perturbation of size less than a. In the upper solid curve in Fig. 2 we show the velocity force curve for $q_D = 0.5$ showing a scaling fit with $\beta = 1.0$. For a fixed value of F_d the velocity decreases with increasing q_D as the driving particle interacts more strongly with the surrounding particles. In Fig. 3(b), V vs q_D for $F_d = 4.0$ shows a $q_D^{-1/2}$ scaling in the regime where there is plastic deformation, and then V flattens out in the elastic regime

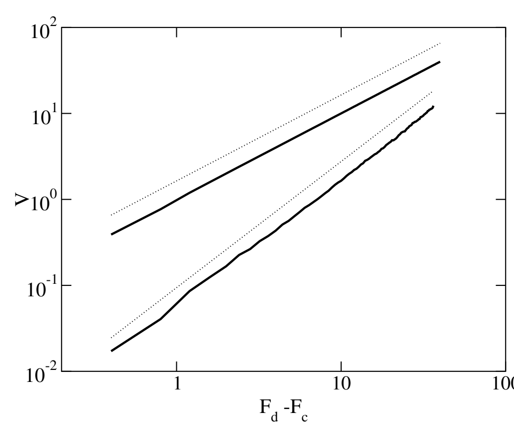

FIG. 2. The velocity V vs $F_D - F_c$ for (upper solid curve) a driven particle with $q_D = 0.5$. The upper dashed curve is a fit with $\beta = 1.0$. The lower solid curve is for $q_D = 20$, with the lower dashed curve a fit with $\beta = 1.47$.

for $q_D < 1$. Additionally in the plastic flow regime the particle motion is highly intermittent as illustrated in Fig. 3(a), where a time trace of V for $q_D/q = 20.0$ shows the motion occurring in bursts separated by quiet periods. As the drive increases, the motion becomes less intermittent, and the noise spectrum of the time trace develops a well-defined rollover frequency ω, with ω scaling approximately linearly in $F_d - F_c$. We find that the rollover frequency corresponds to the time scale between peaks in Fig. 3(a), rather than to the duration of a single peak.

In Fig. 4, we show a contour plot of the average of v^2 for the other particles in the medium, as a function of position relative to the driven particle, for systems with $q_D/q = 20$, $N = 2100$, $a = 1.085$, and $F_d = 0.9$, 1.5, and 40, respectively. This measures energy dissipation, as from energy balance $F_d\overline{V} = \eta(\overline{V}^2 + \sum \overline{v}_A^2)$. These systems are above the depinning threshold of 0.7, but still within the scaling regime. The dissipation is centered strongly around the moving particle and extends anisotropically into the surrounding medium, with a larger region of dissipation in the direction of drive.

Theory: elasticity and fluid flow.—We now consider various scenarios for the particle motion to understand the simulation results. We begin by considering the response of the medium away from the plastic zone, where the particles move less than a and elasticity theory is applicable. The driven particle exerts a force on the elastic medium at a location that is moving at velocity $\overline{V}$. At long wavelengths, elasticity theory gives for a lattice displacement $\vec{u}(\vec{k})$ the equation of motion

$$\dot{\vec{u}}(\vec{k}) = C_1\vec{k}(\vec{k}\cdot\vec{u}) + C_2\vec{k}\times(\vec{u}\times\vec{k}). \qquad (2)$$

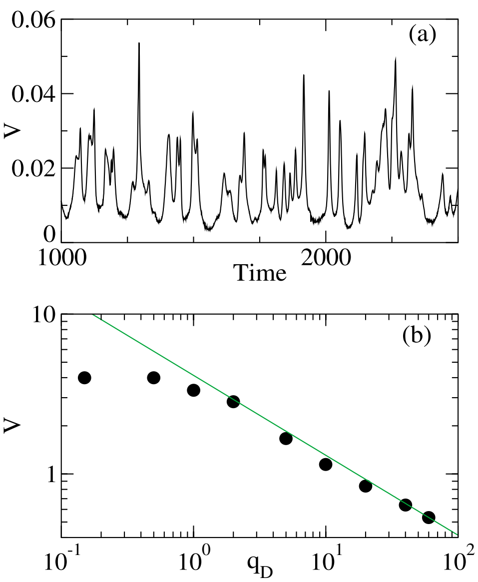

FIG. 3 (color online). (a) The velocity versus time for a fixed drive of $F_D = 0.75$ with $q_D = 20$, showing intermittent bursts of motion. (b) The velocity versus q_D for a fixed drive $F_d = 4.0$. The solid line is a fit of $q_D^{-1/2}$.

To obtain elastic constants C_1, C_2, we consider a triangular lattice of particles, with appropriate lattice constant a, and assume that all particles have the same charge, $q_{A,B} = \sqrt{2}q$. For a lattice constant $a = 1.085$, one finds $C_1 \approx 0.025/\eta$, $C_2 \approx 0.0018/\eta$. The particle exerts a force on this medium, along the direction of motion of the particle. There is also a force normal to this direction, pushing the medium out sideways in opposite directions on opposite sites of the moving fracture. In the comoving frame, the displacement in any given direction decays exponentially at large distance with a characteristic length $l = C_1/\overline{V}$ along that direction and $l = C_2/\overline{V}$ normal to the direction. The energy dissipation rate resulting is of order $\eta\overline{V}^2(l/a)^2 = \eta C_1C_2/a^2$, and is thus independent of $\overline{V}$. This indicates a force on the driven particle of order $1/\overline{V}$, in addition to the drag force, $\eta\overline{V}$, on the driven particle. For small $\overline{V}$, this force becomes arbitrarily large, ultimately exceeding the yield stress of the medium. In such a region the material must yield and the elastic picture becomes invalid.

Thus, we must consider the plastic zone. It is not consistent to have a 2D chunk of solid moving with the particle, with a plastic zone between that solid and the rest of the medium: if the particle force is sufficient to fracture the medium, it would fracture the solid moving with it. At the same time, one cannot have a 2D fluid zone around the particle. The overdamped nature of the dynamics (assuming viscosity and pressure terms like a Newtonian fluid) would concentrate all the vorticity of the fluid within a length a around the particle, and thus this scenario is also inconsistent: more than a length a from the particle, the stress is too small to fracture the medium and produce the fluid. By elimination we turn next to a 1D plastic zone.

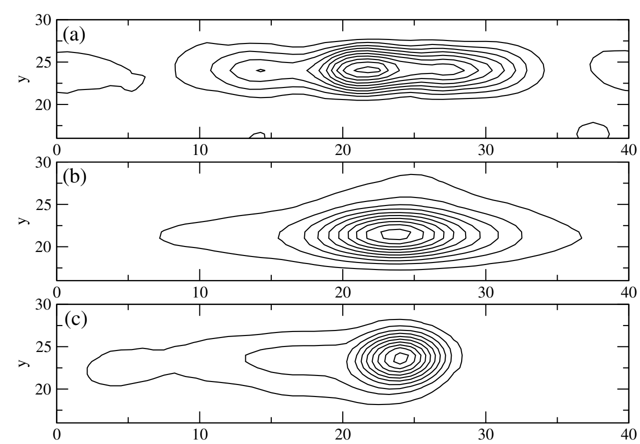

FIG. 4. Contour plot of $\overline{v}^2$ as a function of position relative to the driven particle. The lowest contour is normalized to 5% of the maximum of $\overline{v}^2$, and each contour represents a 10% increase. The system size is 48×48, and the driven particle position is fixed to (24, 24).

Theory: compressed column.—We now propose a scenario based on competition between shear and compressive failure which accounts for all the numerical results and illustrate it using the specific systems of Fig. 4. The driven particle creates a fracture in front of it. Naively the fracture force would be expected to be of the order of the particle interaction force $aU''(a)$, or roughly 0.015 for the given $q_D = 20$. In fact, it is equal to 0.7, nearly 2 orders of magnitude greater, for two reasons: the force $U'(r)$ rises rapidly at short r, while the charge $q_D > q$ further increases the fracture force. After fracture, the particles in front of the driven particle must then move out of its way, either by failing in shear and moving along with the driven particle, or by failing in compression and moving transversely out of its way. Initially, consider just the first possibility so that in front of the driven particle there is a growing 1D column of particles failing in shear. One finds that the rate at which this length increases is determined *not* by the velocity of the driven particle, but by the (faster) velocity at which a compressive front ahead of that particle moves. 1D elasticity theory would imply that in time t, this compression reaches a distance $l \approx a\sqrt{U''t/\eta}$. Since this system is far from equilibrium, we have checked this result by simulating a 1D system with a single driven particle in a fixed, periodic background potential to mimic the medium. We find that this behavior remains valid, even close to the depinning transition, albeit with a greatly increased value for U''.

The 1D column does not grow indefinitely in length due to the possibility of transverse motion of the particles, in which case they squirt out of the column. The time scale for this process would naturally be of order $\eta a/(F_d - F_c)$. Thus, the number of particles in the column is $l/a \approx \sqrt{U''a/(F_d - F_c)}$. By momentum balance, this number is equal to $(F_d - F_c)/\eta\overline{V}$, giving

$$v \propto (F_d - F_c)^{3/2}, \tag{3}$$

as observed, where some fixed force F_c is required to create the fracture. The exit of particles from the column due to compressive failure is naturally highly intermittent, leading to intermittent motion of the driven particle, as seen in Fig. 3(a). This predicts the linear scaling of the rollover frequency with $F_d - F_c$, with the prefactors such that the rollover frequency is increased above the naive expectation: at $F_d = 1.5$, $\omega = 0.05$, larger than $\eta a/(F_d - F_c)$.

The 1D plastic zone is visible in the contours shown in Fig. 4. The aspect ratio of the contour increases for lower drives as the length of the contours increases at constant width, confirming that dissipation arises in a 1D plastic zone, rather than due to the elastic response of a 2D medium which would instead give a constant aspect ratio. Further, the size of the contours is not consistent with 2D elastic response, which would appear in Fig. 4(b) as an exponential decay of the dissipation on a length scale of approximately 0.35 for $\overline{V} = 0.072$, while the actual contours are significantly larger.

As the charge of the driven particle increases, it produces a depletion zone of missing particles around it to compensate its charge; the area of this zone is proportional to q_D/q. The length of the compression was determined above; the width will be proportional to the radius of the depletion zone, $\sqrt{q_D/q}$, accounting for the scaling of velocity with q_D, as seen in Fig. 3(b).

Summary.—We have found a robust power law for the velocity of a single driven particle fracturing a glassy environment of smaller particles, with $\beta = 1.5 \pm 0.05$. We give a theoretical explanation based on a 1D plastic zone, which decreases in size as the particle moves faster leading to $\beta > 1$. This behavior arises due to the softness of the material, as for larger lattice constants a or smaller q_D, the interaction becomes stiffer, and the width of the scaling region decreases until it eventually disappears leaving only linear scaling. While for conventional solids this scaling region is absent with no column observed in front of the particle, the origin of anomalous transport features in many disordered systems may be due to the mechanisms discussed herein. Our results could be tested for driving a single colloid through a glassy colloidal assembly, a disordered dusty plasma, or a vortex lattice past a small density of pinning sites.

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