Dynamic Localization in Quantum Dots: Analytical Theory

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We analyze the response of a complex quantum-mechanical system (e.g., a quantum dot) to a time-dependent perturbation $\phi(t)$. Assuming the dot to be described by random-matrix theory for the Gaussian orthogonal ensemble, we find the quantum correction to the energy absorption rate as a function of the dephasing time t_φ. If $\phi(t)$ is a sum of d harmonics with *incommensurate* frequencies, the correction behaves similarly to that for the conductivity $\delta\sigma_d(t_\varphi)$ in the d-dimensional Anderson model of the orthogonal symmetry class. For a *generic* periodic perturbation, the leading quantum correction is absent as in the systems of the unitary symmetry class, unless $\phi(-t+\tau) = \phi(t+\tau)$ for some τ, which falls into the quasi-1D orthogonal universality class.

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The process of energy absorption by a quantum system with a time-dependent Hamiltonian underlies a large part of modern physics, both fundamental and applied. A generic Hamiltonian can be written in the form

$$\hat{H}(t) = \hat{H}_0 + \hat{V}\phi(t), \tag{1}$$

where we explicitly separated the time-independent part $\hat{H}_0$ and the external perturbation $\hat{V}$ with the time dependence specified by a given function $\phi(t)$. Most often the relevant case is that of the classical Ohmic Joule absorption. The simplest nonlinear effects that restrict the absorption rate are the saturation effects originating from an upper bound on the spectrum of $\hat{H}_0$.

In the past two decades, attention of the scientific community was drawn to a different and much less trivial example of saturation when the spectrum of H_0 is essentially unlimited in the energy space, yet after a certain time the absorption stops. This so-called dynamic localization (DL) in the energy space was observed in numerical simulations of the kicked quantum rotor (KQR)—particle on a circle with $\hat{H}_0 = -\partial^2/\partial\theta^2$ and $\phi(t)$ being a periodic sequence of δ pulses [1], as well as in an actual experimental realization of the KQR—trapped ultracold atoms in the field of a modulated laser standing wave [2]. Mapping of the KQR to the quasirandom 1D Anderson model has been done in Ref. [3], and a similar analogy was exploited in Ref. [4] to demonstrate the DL in a mesoscopic disordered ring. In Ref. [5] an analogy between the KQR and band random matrices was pointed out; the latter have been reduced to a 1D nonlinear σ model [6]. In Ref. [7] the direct correspondence between the KQR and a 1D nonlinear σ model was demonstrated.

However, numerical simulations for a δ-kicked particle in an infinite potential well differing from the KQR only by the boundary conditions revealed no DL [8]. This example shows that DL is not a consequence of the one-dimensional character of the energy space but it depends on the details of both the unperturbed system and the perturbation. Therefore, the results on DL [3] for the standard map [1] cannot be automatically applied to a generic quantum-mechanical system under arbitrary time-dependent perturbation. Even less clear is the status of a peculiar KQR model with the time-dependent perturbation with three incommensurate periods where the phenomenon similar to the 3D Anderson localization-delocalization transition has been found numerically [9].

The most general assumption about a complex quantum system would be the randomness of the Hamiltonian. It is well known [10] that at energies smaller than the Thouless energy E_c description of complex quantum systems falls into one of the three universality classes, each corresponding to a Gaussian ensemble of random matrices. Considering a nonmagnetic electron system with the spin-rotation symmetry, we arrive at Eq. (1) with $\hat{H}_0$ and $\hat{V}$ from the Gaussian orthogonal ensemble (GOE). An analogous Hamiltonian was considered in connection with laser pumping of complex organic molecules [11], but DL was not a target at that stage. The problem of DL in systems described by the Hamiltonian (1) has been addressed in Ref. [12]. However, most results of this study were qualitative in character. The DL found in Ref. [12] numerically for a harmonic perturbation in the GOE was unstable with respect to adding even a small amount of noise in the time dependence.

In this Letter, we develop an *analytical* theory of DL for a closed system described by the Hamiltonian (1) with $\hat{H}_0$ and $\hat{V}$ from the GOE of $N \times N$ random matrices (RMT) and a generic time dependence $\phi(t)$. In the limit $N \to \infty$ the existence of DL and even of the classical Ohmic Joule regime with constant absorption rate depends strongly on the character of $\phi(t)$. We calculate the first (one-loop) quantum interference correction δW to the absorption rate W, which determines the *weak* DL.

For a *periodic* $\phi(t) = \sum_n A_n e^{-in\omega t}$ with $|A_n|^2$ decreasing as $1/n^3$ or slower, W diverges as $N \to \infty$ and the

classical Ohmic regime does not exist. For A_n decreasing rapidly, δW could be significant only if the condition

$$\phi(-t+\tau) = \phi(t+\tau) \tag{2}$$

is fulfilled for some τ. In this case, $\delta W(t)$ is *negative* and grows as $\sqrt{t}$ with the time t of the action of the time-dependent perturbation, such as in the quasi-1D Anderson localization (AL) in the *orthogonal* symmetry class. δW becomes comparable with the Joule absorption rate W_0 at some time t_*, indicating a crossover from the weak to the strong localization in the energy space. If the system dephasing time $t_\varphi < t_*$ the dissipation remains Ohmic, with the rate smaller than W_0 by the value of $\delta W(t_\varphi)$. A periodic $\phi(t)$ with several harmonics obeys Eq. (2) only for a special choice of relative phases. However, Eq. (2) is satisfied by *any* single-harmonic function, making the latter rather an *exception* than a paradigm of a periodic perturbation. For periodic functions not obeying Eq. (2) the time-dependent quantum correction emerges only in the two-loop approximation, as in systems of the *unitary* symmetry class. Finally, we find that for $\phi(t)$ being a sum of d harmonic functions with *incommensurate* frequencies the first quantum correction to the absorption rate is similar to the weak localization correction to conductivity of a d-dimensional disordered system of the *orthogonal* symmetry class.

Qualitative picture.—The similarity and difference between DL and AL, as well as between KQR and RMT, can be seen from the exact correspondence between a quantum system under a (multi)periodic time-dependent perturbation and a tight-binding lattice model with the time-independent Hamiltonian. Consider a system with energy levels ("orbitals") E_l under a harmonic perturbation $V_{ll'}e^{-i\omega t} + V^*_{l'l}e^{i\omega t}$. As follows directly from the Schrödinger equation, its time evolution can be expressed in terms of eigenfunctions and eigenvalues of another system (see Fig. 1), obtained from the original one by replicating it into a one-dimensional lattice and shifting the levels of each consequent site by $\hbar\omega$ so that the energy of the orbital l on the site s is given by $E_{l,s} = E_l - s\hbar\omega$, and introducing the coupling between the lth orbital of the site s and the l'th orbital of the site $s+1$ by the *stationary* perturbation matrix element $V_{ll'}$. In the same way one can show that higher harmonics in the perturbation ($2\omega, 3\omega \ldots$) correspond to the coupling to next neighboring sites ($s+2, s+3, \ldots$), while the presence of several incommensurate frequencies $\omega_1, \ldots, \omega_d$ requires a d-dimensional lattice with sites $\mathbf{s} = (s_1, \ldots s_d)$, on-site energies $E_l - s_1\omega_1 - \cdots - s_d\omega_d$, and the matrix element $V^{(i)}_{ll'}$ at the ith frequency corresponding to the coupling along the ith dimension.

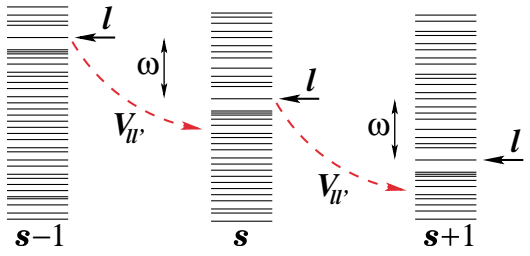

FIG. 1 (color online). Lattice analogy for a system under a monochromatic perturbation.

From this picture it is clear why DL does not exist in RMT for slowly decreasing A_n. It happens because of the possibility of long-range hops over many sites. In the KQR case long-range hops are not so dangerous for localization because in the standard KQR model, the time-dependent perturbation corresponding to $A_n = \text{const}$ is coupled to $\cos\theta$, which in the basis of eigenfunctions $e^{il\theta}$ of $\hat{H}_0$ corresponds to matrix elements $V_{l,l\pm1}$ that may connect only neighboring orbitals. For a long enough distance between sites the coupled orbitals are out of resonance and the hopping is suppressed. For an infinite potential well [8] the basis is $\sin(l\theta)$, $V_{l,l'} \propto 1/|l-l'|$ are long-range in the orbitals' space, so that the resonance hopping between remote sites may still occur, though only between remote orbitals. For the RMT $V_{l,l'}$ do not decrease with the distance $|l-l'|$, but only *neighboring* sites are coupled for a *harmonic* $\phi(t)$. This shows that the KQR with δ kicks and the time-dependent RMT with *harmonic* $\phi(t)$ are in fact two *complementary* models.

Description of the formalism.—Consider a closed system of noninteracting fermions with the single-particle Hamiltonian (1), $\hat{H}_0$ and $\hat{V}$ being real symmetric Gaussian random $N \times N$ matrices with correlators $\langle (H_0)_{mn}(H_0)_{m'n'}\rangle = N(\delta/\pi)^2[\delta_{mn'}\delta_{nm'} + \delta_{mm'}\delta_{nn'}]$, $\langle V_{mn}V_{m'n'}\rangle = (\Gamma\delta/\pi)[\delta_{mn'}\delta_{nm'} + \delta_{mm'}\delta_{nn'}]$, δ being the mean level spacing at the band center, and Γ measuring the sensitivity of the energy levels E_l to variations of ϕ. Such a form of time-dependent random-matrix theory corresponds to a quantum dot under a perturbation with characteristic frequencies $\omega \ll E_c$ [13].

To treat the dynamical problem (1) we employ the nonlinear σ model recently developed in Ref. [14] on the basis of Keldysh nonequilibrium formalism [15,16]. In the limit $N \to \infty$ the effective action of the σ model reads:

$$S[Q] = \int\left[\frac{\pi i}{2\delta}\mathrm{tr}\{i\tau_3\sigma_0\delta_{tt'}\partial_t Q_{tt'}\} + \frac{\pi\Gamma}{8\delta}[\phi(t)-\phi(t')]^2\mathrm{tr}\{Q_{tt'}Q_{t't}\}\right]dt\,dt'. \tag{3}$$

Here $Q_{tt'}$ is a 4×4 matrix in the direct product of the 2×2 particle-hole and Keldysh spaces. Pauli matrices in these spaces are denoted by τ_i and σ_i, respectively (τ_0, σ_0 being the unit matrices). The first term in S is the standard random-matrix action [17] responsible for the whole spectral statistics, while the second, kinetic, term accounts for the effects of the time-dependent perturbation. The matrix Q is subject to the constraints $(Q^2)_{tt'} = \tau_0\sigma_0\delta_{tt'}$ and $\tau_2\sigma_1 Q\sigma_1\tau_2 = Q^T$, where the product and the transpose involve the time arguments, too.

The saddle point of the action (3) is

$$\Lambda_{tt'} = \begin{pmatrix} \delta_{tt'} & 2F^{(0)}_{tt'} \\ 0 & -\delta_{tt'} \end{pmatrix} \otimes \tau_3, \tag{4}$$

where the function $F^{(0)}_{tt'}$ satisfies the kinetic equation

$$\{\partial_t + \partial_{t'} + \Gamma[\phi(t) - \phi(t')]^2\} F^{(0)}_{tt'} = 0. \tag{5}$$

$F_{tt'}$ is the electron distribution function in the time representation; a more familiar quantity is its Wigner transform $F_E(t) = \int d\tau e^{iE\tau} F_{t+\tau/2, t-\tau/2}$. In equilibrium with temperature T it is $F_E = \tanh(E/2T)$. Out of equilibrium it satisfies the Wigner-transformed Eq. (5), which after averaging over fast oscillations in t reduces to the diffusion equation in the energy space:

$$[\partial_t - D\partial_E^2] F^{(0)}_E(t) = 0, \qquad D = \Gamma\overline{(\partial_t \phi)^2}, \tag{6}$$

the overline meaning the average over time. Equation (6) gives Ohmic Joule absorption rate

$$W_0 = D/\delta. \tag{7}$$

The saddle-point expression (7) is valid provided that (i) the perturbation is sufficiently fast, $|\partial_t \phi| \gg v_K \equiv \delta^{3/2}\Gamma^{-1/2}$, which is the antiadiabaticity condition, and (ii) interference effects responsible for DL are neglected.

The perturbative correction to Eq. (7) at finite values of $|\partial_t \phi|/v_K$ was calculated in Ref. [14] for the case of a *monotonic,* linear bias $\phi(t) = vt$. There the interference leading to DL is ineffective and the correction is *positive* which is a remnant of the Landau-Zener regime of dissipation [18]. Here we focus on the limit where Landau-Zener corrections are weak and concentrate on the case of *re-entrant* $\phi(t)$, when the saddle-point approximation is invalidated by the DL interference effects.

Quantum corrections to the mean-field absorption rate (7) can be obtained in the regular way by expanding over Gaussian fluctuations near the saddle point. The deviation of the Q matrix from Λ is parametrized [14] by the diffuson and cooperon modes $b_{tt'}$ and $a_{tt'}$ with the bare propagators: $\langle b_{t_+t_-} b^*_{t'_+t'_-} \rangle = (2\delta/\pi)\delta(\eta - \eta')\mathcal{D}_\eta(t, t')$ and $\langle a_{t_+t_-} a^*_{t'_+t'_-} \rangle = (\delta/\pi)\delta(t - t')C_t(\eta, \eta')$, where [13]

$$\mathcal{D}_\eta(t, t') = \theta(t - t') \exp\left[- \int_{t'}^{t} \gamma(\tau, \eta) d\tau \right], \tag{8}$$

$$C_t(\eta, \eta') = \theta(\eta - \eta') \exp\left[-\frac{1}{2} \int_{\eta'}^{\eta} \gamma(t, \tau) d\tau \right], \tag{9}$$

and we have denoted $t_\pm = t \pm \eta/2$, $t'_\pm = t' \pm \eta'/2$, and $\gamma(t, \eta) \equiv \Gamma[\phi(t_+) - \phi(t_-)]^2$. In the presence of fluctuations, the average matrix $\langle Q \rangle$ still has the form (4) but with $F^{(0)}$ substituted by the renormalized electron distribution F which determines the energy absorption rate:

$$W(t) \equiv \partial_t \langle E(t) \rangle = -\frac{i\pi}{\delta} \lim_{\eta \to 0} \partial_t \partial_\eta F_{t+\eta/2, t-\eta/2}. \tag{10}$$

The first quantum correction to $\langle Q \rangle$ contains a *loose diffuson* [13] with a cooperon loop at the end. We evaluate this diagram for a generic perturbation $\phi(t)$ switched on at $t = 0$. Using the asymptotics $F_{t_+t_-} \sim (i\pi\eta)^{-1}$ following from the property $\lim_{E \to \pm\infty} F_E(t) = \pm 1$, we obtain for the absorption rate:

$$W(t) = W_0 + \frac{\Gamma}{\pi} \int_0^t \partial_t \phi(t) \partial_t \phi(t - \xi) C_{t-\xi/2}(\xi, -\xi) d\xi. \tag{11}$$

Expression (11) is the main result of this part of the Letter and will be the base for the subsequent considerations. It can also be obtained from the conventional diagrammatic technique [13,19].

Results.—First, we consider a *periodic* perturbation: $\phi(t) = \sum_n A_n \cos(n\omega t - \varphi_n)$. To study the long-time, period-averaged dynamics at $t, \xi \gg 1/\omega$ we can approximate $C_{t-\xi/2}(\xi, -\xi) \approx e^{-\xi\, \gamma(t-\xi/2)}$, where

$$\gamma(t) = 2\Gamma \sum_n A_n^2 \sin^2[n\omega t - \varphi_n]. \tag{12}$$

For a particular choice of phases, $\varphi_n = n\varphi$, there exists a set of points $\xi_k = 2t - 2(\varphi + \pi k)/\omega$ with integer k where the dephasing rate (12) is equal to zero [Fig. 2(a)]. Existence of such *no-dephasing points* [20] is equivalent to the generalized time-reversal symmetry (2) of the perturbation. At large $\Gamma\xi$ only $\xi \approx \xi_k$ contribute to the integral (11) (otherwise the cooperon is exponentially small) which then can be calculated with the steepest descent method. Performing summation over the no-dephasing points ξ_k, we obtain a quantum interference correction to the Ohmic absorption rate (7):

$$\frac{W(t)}{W_0} = 1 - \sqrt{\frac{t}{t_*}}, \qquad t_* = \frac{\pi^3 \Gamma \overline{n^2}}{2\delta^2} = \frac{\pi^3 W_0}{\omega^2 \delta}, \tag{13}$$

where $\overline{n^2} \equiv \sum_n n^2 A_n^2$ and the limit $t \gg 1/\omega, 1/\Gamma$ is implied. Note that both the diffusion coefficient $D = \omega^2 \Gamma \overline{n^2}/2$ and t_* are finite only if A_n decrease as $n^{-3/2}$ or faster. If it is not the case (δ kicks), the diffusion coefficient D, the Joule absorption rate W_0, and the time t_* diverge. In real systems this divergence is limited by the condition of validity of RMT $n\omega \ll E_c$.

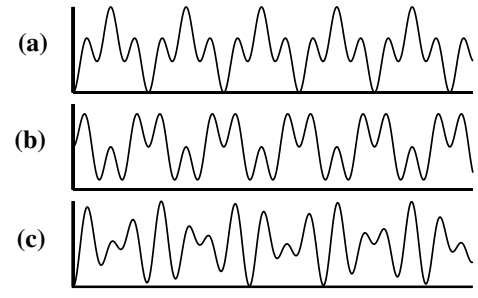

FIG. 2. Dephasing rate $\gamma(t) = \overline{\gamma(t, \eta)}$ averaged over η vs t: (a) periodic $\phi(t)$ obeying Eq. (2): a regular array of zeros; (b) generic periodic $\phi(t)$: a gap; (c) quasiperiodic $\phi(t)$ with two incommensurate frequencies: a pseudogap.

The $\sqrt{t}$ dependence is remarkably similar to the $\sqrt{t_\varphi}$ dependence of the first quantum correction to the conductivity of a particle in a quasi-one-dimensional disordered sample with the phase relaxation time t_φ [21]. In our case the relative quantum correction becomes comparable to unity at time t_*. This is an indication of DL at the characteristic energy scale $E_* \sim \sqrt{Dt_*} \sim \omega\Gamma n^2/\delta$. Taking $\delta = 3\ \mu$eV, $\hbar\omega = 40\ \mu$eV ($\omega/2\pi \approx 10$ GHz), $\Gamma n^2 = 10\ \mu$eV (corresponding to the microwave electric field of a few V/m over the dot size of 1 μm), we obtain $t_* \sim 10$ ns, $E_* \sim 400\ \mu\text{eV} \sim 5$ K, while typical values of the Thouless energy are $E_c \sim 100$–$1000\ \mu$eV.

When the generalized time-reversal symmetry (2) is absent for $\phi(t)$, the dephasing rate $\gamma(t)$ is positive and separated from zero by a finite gap [Fig. 2(b)]. In this case the integral in Eq. (11) converges exponentially, so the first quantum correction stays small even at $t \to \infty$, as in the systems of the unitary symmetry class.

Finally, we consider the case of *d incommensurate* frequencies ω_n: $\phi(t) = \sum_{n=1}^{d} A_n \cos(\omega_n t - \varphi_n)$. Here the relationship between the phases φ_n does not matter. The reason is that in the incommensurate case the dephasing rate can become arbitrarily small in an infinite number of points regardless of phases [Fig. 2(c)], and the integral is dominated by these nearly no-dephasing points.

The cooperon with the dephasing rate (12), where $n\omega$ is substituted by ω_n, can be expanded in a d-dimensional Fourier series involving the modified Bessel functions $I_\nu(z)$. Averaging it over t is significantly simplified due to incommensurability, but the analytical expression for arbitrary amplitudes A_n is still bulky. We write the resulting expression valid at $t \gg 1/\omega, 1/\Gamma$ when all $A_n = 1$:

$$\frac{\overline{W(t)}}{W_0} = 1 - \frac{\delta}{\pi\Gamma}\int_0^{\Gamma t} dz\, e^{-zd}[I_0(z)]^{d-1}\frac{dI_0(z)}{dz}. \qquad (14)$$

Using the asymptotic form $I_0(z) \approx dI_0(z)/dz \approx e^z/\sqrt{2\pi z}$ at $z \gg 1$, we recover Eq. (13) for a harmonic perturbation. For $d = 2$, the relative correction is $(\delta/2\pi^2\Gamma)\ln\Gamma t$, whereas for $d > 2$ it saturates $\propto -\text{const} + t^{1-d/2}$ in the limit of large t, in complete analogy with the behavior of the quantum correction of Ref. [21] in d dimensions. Equation (14) is relevant also for rational ratios $\omega_n/\omega_m = P/Q$, $P \sim Q \gg 1$ provided that $t \ll Q^2/\Gamma$.

In conclusion, we have developed an analytical approach based on the *zero-dimensional, time-dependent* nonlinear sigma model and obtained the *weak dynamic localization* in complex quantum systems under time-dependent perturbation described by the *random matrix theory*. The character of energy absorption in such systems is determined entirely by the frequency spectrum of a time-dependent perturbation. In particular, we obtained no DL for the time-periodic δ-function perturbation, and the dynamical localization corrections similar to the d-dimensional weak localization corrections to conductivity if the perturbation is a sum of d incommensurate harmonic functions.

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