Model-Independent Dark Energy Differentiation with the Integrated Sachs-Wolfe Effect

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We study the integrated Sachs-Wolfe effect using a model-independent parametrization of the dark energy equation of state, $w(z)$. Cosmic variance severely restricts the class of models distinguishable from one based on cold dark matter and a cosmological constant unless $w(z)$ currently satisfies $w_Q^o > -0.8$, or exhibits a rapid, late-time, transition at redshifts $z < 3$. Because of the degeneracy with other cosmological parameters, models with a slowly varying $w(z)$ cannot be differentiated from each other or from a cosmological constant. This may place a fundamental limit on our understanding of the origin of the currently observed acceleration.

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Introduction.—Cosmological distance measurements of high redshift type Ia supernova, combined with measurements of the cosmological volume via galaxy cluster surveys, weak lensing tomography, and quasar clustering, can provide a new insight into the nature of the dark energy sourcing the present acceleration of the Universe. In spite of their potential for breaking degeneracies between cosmological parameters, such tests are limited by our ignorance of effects such as possible supernova evolution or nonlinear galaxy cluster physics. In this sense the cosmic microwave background (CMB) observations are uniquely pure, limited only by systematic experimental effects and cosmic variance (an unavoidable theoretical error). This purity motivates us to ask the following question: how much model-independent information can the CMB give us about the dark energy? We will show that if the dark energy clusters only on very large scales, then only models with equations of state, $w(z)$, which vary rapidly at low redshifts $z < 3$, can be distinguished from the "concordance" ΛCDM (cold dark matter and a cosmological constant) model using the CMB alone. The importance of differentiating between Λ and dynamical models of dark energy such as one (or more) evolving scalar fields [quintessence (Q)] [1–3] can hardly be overstated. Λ is essentially antigravity; the other is a new long-range force. We tackle this differentiation issue using a very general parametrization of $w(z) \equiv p/\rho$ described in [4] and used in simplified form in [5]. This form accurately encompasses most quintessence models, independent of the number of fields involved, and many other models for the acceleration. In principle, cosmological distance measurements can distinguish between ΛCDM and QCDM models by constraining the present value of the dark energy equation of state w_Q^o. This does not work, however, since current CMB data suggest that w_Q^o is close to the cosmological constant value of -1 [5–9]. However, the situation is not hopeless since dark energy clusters on very large scales and hence can leave a distinctive contribution in the anisotropies of the CMB radiation. This is particularly clear in the case of a rapid late-time transition in $w(z)$. Indeed. current data do (weakly) prefer exactly such a situation [5].

In this Letter we focus on the relation between the dark energy properties and the integrated Sachs-Wolfe (ISW) effect in the CMB power spectrum. Although this is model dependent we have recently shown that an appropriate parametrization of the equation of state, $w(z)$, accurately describes a large class of models given by Eq. (4) in Ref. [4]. Assuming that the dark energy contribution during the radiation era is negligible, then different dark energy models are specified by the vector $\overline{W} = (w_Q^o, w_Q^m, a_c^m, \Delta)$ which, respectively, specifies the equation of state today (o) and during the matter era (m), the value of the scale factor where the equation of state changes from w_Q^m to w_Q^o, and the width of the transition. We now discuss how these parameters affect the CMB and show that only a small range of $\overline{W}$s leave an imprint on the CMB distinguishable from ΛCDM.

ISW effect in dark energy models.—A late-time mechanism to generate anisotropies is due to CMB photons climbing in and out of evolving gravitational potentials [10]. During the matter dominated era the gravitational potential Φ associated with the density perturbations is constant and there is no ISW effect. However, in ΛCDM models Φ starts decaying at redshifts when Λ starts to dominate, producing large angular scale anisotropies [11]. In dark energy scenarios the cosmic acceleration is not the only contribution to the decay of Φ: on large scales the clustering properties alter the growth rate of matter perturbations [12,13]. It is the signal of this clustering [14] that we are hunting in as model-independent a way as possible. We assume a flat spatial geometry and fix the value of the Hubble constant $H_o = 70\ \mathrm{K\,ms^{-1}\,Mpc^{-1}}$, the scalar spectral index $n = 1$, the baryon density $\Omega_b h^2 = 0.021$, and the amount of matter (CDM) $\Omega_m = 0.3$. We can usefully distinguish two classes of models: (1) those with a slowly varying equation of state for which $0 < a_c^m/\Delta < 1$, as in the case of the inverse power law potential [15], and (2) a rapidly varying $w(z)$, such as the "Albrecht-Skordis" model [16] and the two-exponential potential [17], with $a_c^m/\Delta > 1$. This class also includes many interesting radical models such as vacuum metamorphosis [18], late-time phase transitions [19], and backreaction-induced acceleration [20]. We show these two classes in Fig. 1. The solid line corresponds to dark energy that tracks the dust during the matter era ($w_Q^m = 0.0$) and evolves slowly toward $w_Q^o = -1$, and the dotted line corresponds to a model with a rapid transition in its equation of state at $a_c^m = 0.1$ ($z = 9$). Given current data it is worth studying the case with $w_Q^o = -1$ (since it is also the most difficult to distinguish from ΛCDM), while allowing the other parameters w_Q^m and a_c^m to vary.

Figure 2 shows the anisotropy power spectrum, $C_l^{\rm ISW}$, produced through the integrated Sachs-Wolfe effect by a rapidly evolving (top panels) and a slowly evolving (bottom panels) equation of state; the solid line corresponds to the ΛCDM model. As we can see in Fig. 2(a), varying a_c^m can produce a strong ISW. The effect is larger if the transition in the equation of state occurs at redshifts $z < 3$. On the other hand, the $C_l^{\rm ISW}$ is the same as in the cosmological constant regime if $a_c^m < 0.2$ ($z > 4$). In Fig. 2(b) we plot the ISW for two different values of w_Q^m, corresponding to $w_Q^m = 0.0$ (dashed line) and $w_Q^m = -0.1$ (dot-dashed line). We note that the signal is larger if the quintessence field is perfectly tracking the background component. But as w_Q^m diverges from the dust value, the ISW effect becomes the same as in ΛCDM. This means that even for rapidly varying $w(z)$ (small Δ), the ISW is distinguishable from that in the ΛCDM scenario only if $w(z)$ during matter domination closely mimics the dust value and the transition occurs at low redshifts, $z < 3$. We can see that the amplitude of the integrated Sachs-Wolfe effect is smaller in slowly varying models (bottom panels). As we expect, the $C_l^{\rm ISW}$ is independent of a_c^m [Fig. 2(c)], since for these models a different value in the transition redshift does not produce a large effect on the evolution of the dark energy density. In Fig. 2(d), the ISW power spectrum is large for $w_Q^m = 0.0$ (dashed line) and becomes smaller than the cosmological constant on horizon scales as w_Q^m has negative values (dot-dashed line), and increases toward Λ for w_Q^m approaching -1. This class of models is then more difficult to distinguish from the ΛCDM if the equation of state today is close to $w_\Lambda = -1$. This can be qualitatively explained noting that perfect tracking between dark energy and CDM causes a delay in the time when the gravitational potential starts to decay, compared to the case of ΛCDM. This effect is stronger for models with rapidly varying equations of state since the rapid change in w_Q produces a stronger variation in the gravitational potential.

CMB power spectrum.—The imprint of the ISW effect in the CMB spectrum results in a boost of power at low multipoles. This affects the position of the first acoustic peak and the COBE (Cosmic Background Explorer) normalization. In particular, since some of the anisotropies at large scales are produced at late times, as we have seen in the previous section, normalizing the spectrum relative to the COBE measurements produces a suppression of power at smaller scales. A simple way to characterize the amplitude of the CMB spectrum is to consider the height of the first three acoustic peaks relative to the power at $l = 10$, i.e., $H_i = C_{l_i}/C_{10}$, and their multipole positions ℓ_i, $i = 1, 2, 3$ [21]. These numbers allow us to quantify the discrepancy between the dark energy models and ΛCDM. In particular, since H_1 depends on the amplitude of the

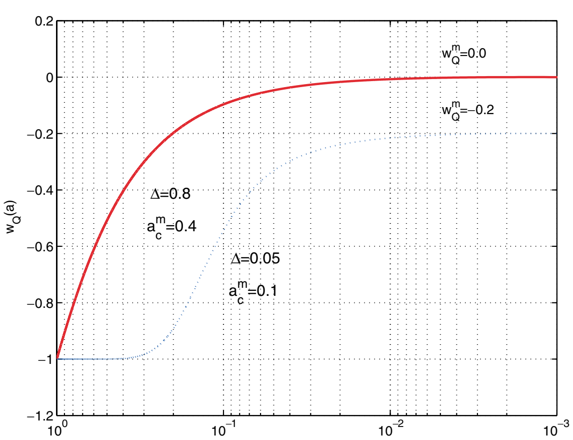

FIG. 1 (color online). Time evolution of the equation of state for two classes of models, with slow (solid line) and rapid transition (dotted line). The dark energy parameters specify the features of $w_Q(a)$.

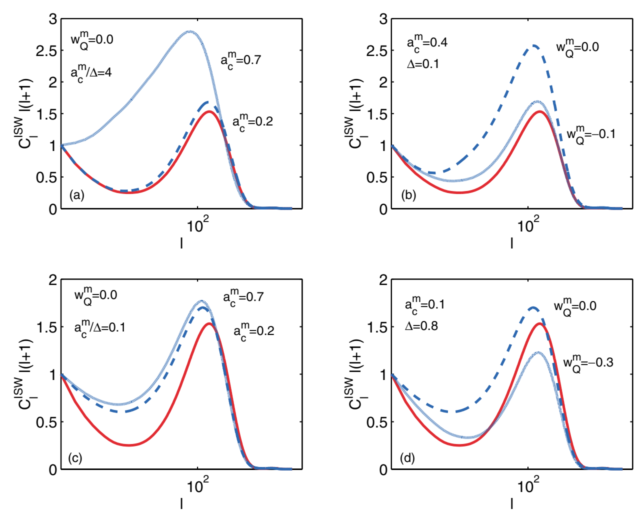

FIG. 2 (color online). Power spectrum of the ISW for rapidly varying models (top panels) and slowly varying ones (bottom panels). The solid line shows the ISW effects produced in the cosmological constant case. A detailed explanation is in the text.

power spectrum at the multipoles characteristic of the ISW effect, we expect H_1 to be more sensitive to quintessence signatures. We have computed the CMB spectra for the class of models with $w_Q^o = -1$ described in the previous section, determined the parameters, H_i (plotted in Fig. 3), and compared them with those of the ΛCDM spectrum. The rapidly varying models are shown in the top panels. We can see the strong ISW effects produced by changing a_c^m are now evident in the large discrepancy between H_1 and H_1^Λ (solid line) [Fig. 3(a)]: it can be larger than 20% for $a_c^m > 0.6$. The effect on H_2, H_3 is smaller. However, varying w_Q^m [Fig. 3(b)] produces a discrepancy of only order 4% on H_1, while H_2 and H_3 remain the same as in ΛCDM. For a slowly varying equation of state, H_1, H_2, and H_3 are independent of a_c^m [Fig. 3(c)]. The dark energy imprint is only on H_1 for which the discrepancy to the Λ case is about 10%. Such discrepancy decreases when changing the value of the quintessence equation of state during matter from $w_Q^m = 0.0$ to $w_Q^m = -1$ [Fig. 3(d)]. Values of the equation of state today $w_Q^o > -1$ imply a stronger ISW effect. Consequently, the curves of Fig. 3 are shifted upwards. For instance, in Fig. 4 we plot the class of models previously analyzed, with $w_Q^o = -0.88$. We note the same behavior as we vary the dark energy parameters, but the discrepancy with the ΛCDM model is now larger. In Fig. 4(d) it is worth noticing the case $w_Q^m = -1$ that corresponds to a model very similar to a "k essence" model [22]. We can see that the relative difference with the ΛCDM case is of the order of a few percent, in agreement with [23] for the same value of $w_Q^o = -0.88$. At this point we ask the key question whether such differences are observable. We have shown that H_1 is a good estimator of the ISW effect, and that it is a tracer of the dark energy imprint on the CMB. However, its estimation from the data will be affected by cosmic variance at $l = 10$. Hence, with even perfect measurements of the first acoustic peak, the uncertainty on H_1 will be dominated by the 30% uncertainty due to cosmic variance. With the plots of Figs. 3 and 4 in mind, this means that, if the present value of the equation of state is close to -1, slowly varying dark energy models are hardly distinguishable from ΛCDM, while rapidly varying ones can produce a detectable signature only if the transition in the equation occurred at $a_c^m > 0.7$; in any case it will be difficult to constrain w_Q^m. The degeneracy of H_1 with the baryon density is marginal since varying $\Omega_b h^2$ mainly affects H_2 which is insensitive to dark energy effects. Hence, only an accurate determination of the angular diameter distance, inferred from the location of the acoustic peaks, would allow detection of such deviations from the cosmological constant model. The relation between the position of the CMB peaks and the dark energy has been widely discussed ([24] and references therein). The shift of the multipole positions (ℓ_i) of the acoustic peaks caused by the evolution of the dark energy in the class of models analyzed in Fig. 3 can be seen in Fig. 5, where we plot the relative difference of l_1, l_2, and l_3 to the Λ case. We note that due to the additional shift induced on the first acoustic peak by the ISW effect the difference with the ΛCDM model for the first peak is generally larger than for the second and third peaks. As with the comparison of the amplitude of the CMB spectrum, the largest effect is produced by models with a rapid transition occurring at small redshifts. However, the degeneracy of the angular diameter distance, in particular, with the value of the Hubble constant and the amount of dark energy density, will limit our ability to put tight constraints on the dark energy parameters. Fortunately, there are alternative ways in which these

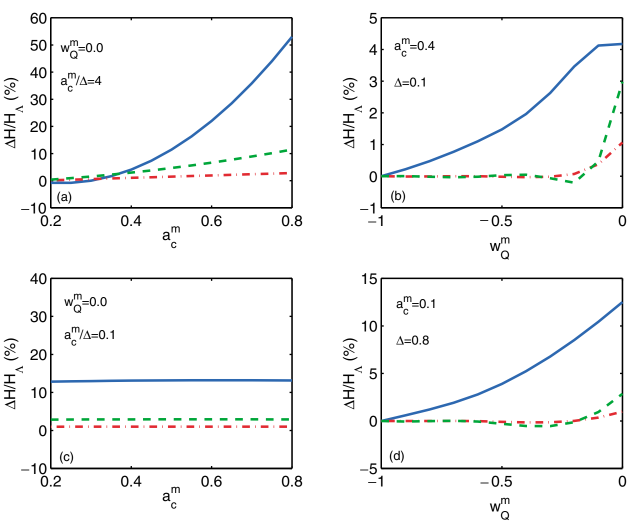

FIG. 3 (color online). Relative difference of H_1 (solid line), H_2 (dashed line), and H_3 (dash-dotted line) to the ΛCDM model, for rapidly varying models (top panels), and with slow transition (bottom panels). For these models the present value of the equation of state is $w_Q^o = -1$.

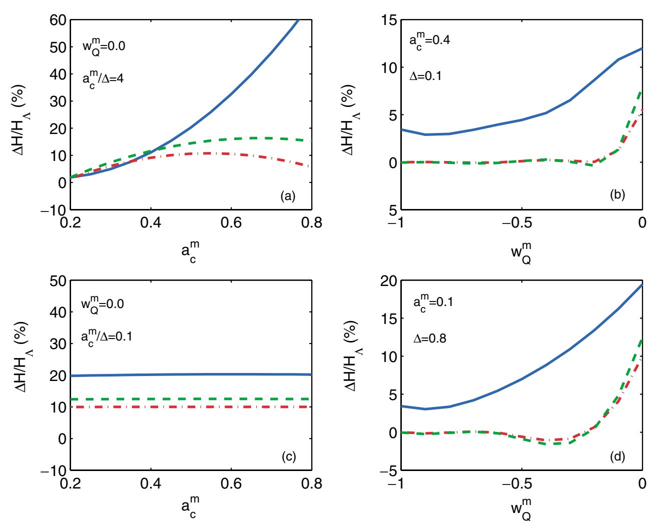

FIG. 4 (color online). As in Fig. 3 for $w_Q^o = -0.88$.

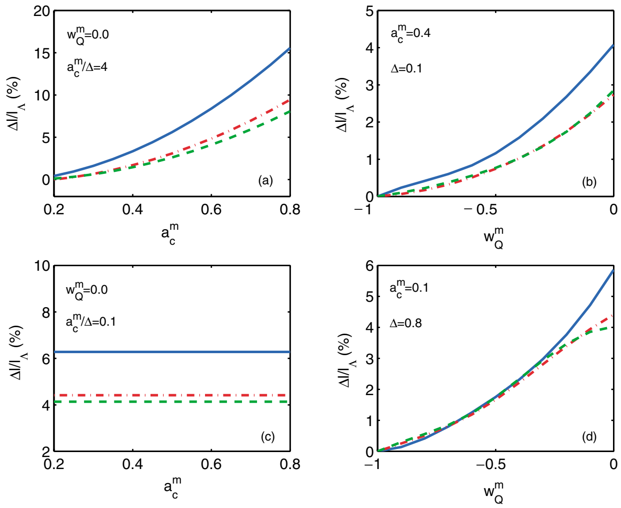

FIG. 5 (color online). Relative difference of l_1 (solid line), l_2 (dashed line), and l_3 (dot-dashed line) to the ΛCDM model, for rapidly varying models (top panels), and with slow transition (bottom panels). For these models the present value of the equation of state is $w_Q^o = -1$.

problems can be alleviated, for instance, cross correlating the ISW effect with the large scale structure of the local universe [25–27]. An efficient approach would also be to combine different observations in order to break the degeneracies with the cosmological parameters [28,29].

Conclusions.—The next generation of high resolution CMB experiments will measure the anisotropy power spectrum with an accuracy close to theoretical limits. It is therefore of particular interest to study the sensitivity of CMB observations to the effects produced by a dark energy component in the CMB. A quintessential contribution leaves a distinctive signature in the ISW effect. On the other hand, such an imprint occurs at low multipoles; consequently, cosmic variance strongly limits the possibility of differentiating dark energy models from a cosmological constant. In particular, we find that for values of $-1 \leq w_Q^o < -0.8$ the only distinguishable cases are those with a rapidly varying equation of state. In fact, for the slowly varying models, the ISW is the same as in ΛCDM and a deviation from $w_Q^o = -1$ can be inferred only from an accurate determination of the location of the acoustic peaks. Such measurements will be affected by the degeneracy of the angular diameter distance with the value of the Hubble constant and the amount of dark energy density. Therefore CMB observations are insensitive to this class of models, and we will be left with a fundamental uncertainty as to whether the cosmic acceleration is due to an evolving field or a cosmological constant, an issue of great theoretical importance for our understanding of the foundations of quantum gravity and string theory.

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