Universal Features of the Time Evolution of Evanescent Modes in a Left-Handed Perfect Lens

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The time evolution of evanescent modes in Pendry's perfect lens proposal for ideally lossless and homogeneous, left-handed materials is analyzed. We show that time development of subwavelength resolution exhibits universal features, independent of model details. This is due to the unavoidable near degeneracy of surface electromagnetic modes in the deep subwavelength region. By means of a mechanical analog, it is shown that an intrinsic time scale (missed in stationary studies) has to be associated with any desired lateral resolution. A time-dependent cutoff length emerges, removing the problem of divergences claimed to invalidate Pendry's proposal.

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Long ago, Veselago [1] pointed out that very unusual properties, such as negative refraction, would be exhibited by materials with negative refraction index. Recently, Pendry [2] has claimed that those so-called left-handed (LH) materials with $\epsilon(\omega_0) = \mu(\omega_0) = -1$, can act as perfect lenses with, ideally, arbitrary subwavelength resolution. In addition to its genuine conceptual importance, this proposal has attracted much attention due to the practical realization of man-made materials expected to be left handed [3], where negative refraction has been claimed to be observed [4]. The challenge to conventional ideas conveyed in Pendry's work has fueled a heated debate that has contributed to sharpen the issue, if not to settle it [5–8].

The criticism expressed by García and Nieto-Vesperinas [7] (see also Refs. [5,9]) seems of particular importance, for it would imply that some sort of fundamental violation of physical laws is unavoidable in Pendry's perfect lens. The idea is that the necessary amplification of evanescent modes would turn a square-integrable incoming wave into a non-normalizable signal, something deemed unacceptable by the authors of Refs. [5,7,9]. Pendry [8] has replied that losses, unavoidable in the real world, would provide a natural cutoff, forbidding the amplification of large wave vectors (though degrading the perfect resolution). Haldane [10] has put the blame of divergences on the role played by surface modes (polaritons) [11] localized at the interfaces of the LH material: amplification of evanescent modes is a gift of those modes [2]. For a homogeneous material, those surface modes exist and become dispersionless for increasing wave vectors, being the unphysical absence of an intrinsic cutoff length which causes the pathologies [10]. Any realization of a LH material (think of the composite nature of proposed LH materials [4] or photonic crystals [12]) must have a natural length below which the homogeneous description fails, providing the necessary cure to divergences at the price of lower resolution.

Although Pendry's and Haldane's escapes from divergences are certainly safe, they seem to suggest that the textbook idealization of lossless and homogeneous media, that works so well otherwise [13], is fundamentally flawed when applied to a material that happens to satisfy $\epsilon(\omega_0) = \mu(\omega_0) = -1$, at some frequency ω_0. Apparently, only after the inclusion on the real world constraints (losses and/or small-distance structure) could the proposal be made physically acceptable. I find this state of affairs very unsatisfactory, and it is the purpose of this Letter to show that, *even within the self-imposed idealizations of a lossless (for $\omega = \omega_0$) and purely homogeneous, left-handed material, Pendry's perfect lens proposal is correct.*

To this end, we will consider the time evolution [14,15] of subwavelength features. Time development requires the study of the frequency dispersion of a particular model (or material), apparently preventing us from drawing general conclusions. However, it will be shown that the structure of the relevant magnitudes in the immediate vicinity of the target frequency is dominated by the surface polariton modes [10,11]. These modes show universal features for any LH material at long wave vectors, therefore allowing us to extract general conclusions. A key point will be the identification of a time scale for any intended length resolution. This time scale, linked to the near degeneracy of surface modes and missed in stationary studies, will provide the clue for overcoming the problem of divergences.

Consider a transverse current oscillating at frequency ω, located in a source plane at $x = x_0$ (see Fig. 1). The Fourier component (k) of the current leads to an electric field given by (S polarization, for simplicity):

$$E = [0, E_k(x, t), 0]\, e^{ikz}. \tag{1}$$

In vacuum and for evanescent modes, $E_k(x, t) \sim e^{-\rho|x-x_0|} e^{-i\omega t}$, where $\rho = \sqrt{k^2 - \omega^2}$, in units with light velocity $c = 1$ (the connection between current and field need not concern us here [16]). In the presence of a slab of

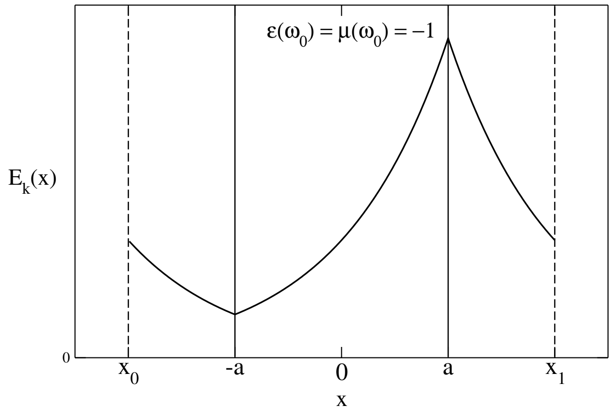

FIG. 1. Geometry of the problem: slab of LH material between planes $x = \pm a$, with source plane at x_0, and image plane at x_1. The curve of $E_k(x)$ illustrates perfect restoration of evanescent fields when $x_0 = -2a$ and $x_1 = 2a$.

LH material between $x = \pm a$, the field acquires the usual reflected and transmitted components (only the vacuum part explicit and for arbitrary normalization of the incoming wave):

$$E_k(x, t) = [e^{-\rho|x-x_0|} + r_k(\omega)\, e^{\rho(x+x_0)}]\, e^{-i\omega t}, \qquad x < -a,$$
$$E_k(x, t) = t_k(\omega)\, e^{-\rho(x-x_0)}\, e^{-i\omega t}, \qquad x > a. \tag{2}$$

Pendry [2] has shown that, for a frequency ω_0 such that $\epsilon(\omega_0) = \mu(\omega_0) = -1$, the reflection and transmission coefficients are

$$r_k(\omega_0) = 0, \qquad t_k(\omega_0) = \exp(4\rho_0 a), \tag{3}$$

with

$$\rho_0 = \sqrt{k^2 - \omega_0^2}. \tag{4}$$

This leads to a perfect restoration at the focal plane $x_1 = 2a$ of a source field at $x_0 = -2a$, as shown in Fig. 1.

The exponential amplification inside the LH material, necessary for image restoration, is at the root of the divergences pointed out before [7,9]. Notice that a square-integrable field with Fourier components $\mathcal{E}_k$ at the source plane $x_0 = -2a$, will emerge at the plane $x = a$ with norm:

$$\mathcal{N} = \int d^2 r_\| |E(x = a, r_\|)|^2 \propto \sum_k |\mathcal{E}_k e^{\rho_0 a}|^2. \tag{5}$$

This norm certainly diverges for large wave vectors, unless unjustified constraints are put on the source field.

So far we have described the stationary solution, but our aim is the time development of such a situation. Therefore, the frequency behavior of $r(\omega)$ and $t(\omega)$ is needed. In the vicinity of $\omega = \omega_0$, and in the deep subwavelength region, the reflection and transmission coefficients exhibit a highly singular behavior controlled by the appearance of simple poles. This singular behavior can be described by

$$t_k(\omega) = e^{4\rho_0 a}\, \frac{(\omega_0^2 - \omega_{k,+}^2)(\omega_0^2 - \omega_{k,-}^2)}{(\omega^2 - \omega_{k,+}^2)(\omega^2 - \omega_{k,-}^2)},$$
$$r_k(\omega) = 2\, t_k(\omega)\, \frac{(\omega^2 - \omega_0^2)}{(\omega_{k,+}^2 - \omega_{k,-}^2)}, \tag{6}$$

and is depicted in Fig. 2 (left panel). One can see that, for instance, $t_k(\omega)$ is basically the sum of four simple poles located at frequencies $\pm\omega_{k,\pm}$. Notice that the target frequency is sandwiched between two such poles $\omega_{k,\pm}$, corresponding to the surface modes (right panel of Fig. 2). These are the even and odd combinations of the fundamental mode of an isolated interface, that would take place exactly at $\omega = \omega_0$. The frequencies $\omega_{k,\pm}$ are solutions of the equations [11]: $[\tanh(\rho_m a)]^{\pm 1} = -\mu(\omega)\rho/\rho_m$, with $\rho_m = \sqrt{k^2 - \epsilon(\omega)\mu(\omega)\omega^2}$. This gives:

$$\omega_{k,\pm}^2 = \omega_0^2 \pm \eta_k^2, \tag{7}$$

with an exponentially small coupling [10] for large wave vectors

$$\eta_k^2 \simeq 8\, C\, \omega_0^2\, e^{-2\rho_0 a}, \tag{8}$$

where $(C\omega_0)^{-1} = 2\mu_0' + (\epsilon_0' + \mu_0')(k^2/\omega_0^2 - 1)^{-1}$, with $\mu_0' = d\mu(\omega_0)/d\omega$ and $\epsilon_0' = d\epsilon(\omega_0)/d\omega$. It is important to realize that the expressions of Eq. (6), though only valid in the neighborhood of ω_0, do contain the singular structure associated with the surface modes. Therefore, the relevant dynamics of these modes is expected to be well reproduced. Ruppin [11] has studied these modes thoroughly, and their importance for the amplifying process has been noticed before [2,10,15].

We can now study, for instance, the time evolution of the transmitted field at the right interface $E_{\rm trans}(x = a, t)$, in terms of the incident field at the left interface $E_{\rm inc}(x = -a, t)$. This is enough to understand the amplification

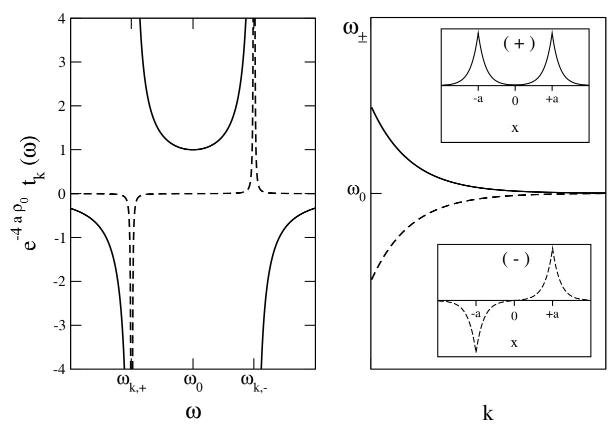

FIG. 2. Left panel: Real (continuous line) and imaginary (dashed line, artificially enlarging delta functions) parts of the transmission coefficient $t_k(\omega)$ in the vicinity of ω_0. Right panel: dispersion relation of surface modes $\omega_{k,\pm}$, illustrating near degeneracy for large wave vectors. Insets show the spatial form of both modes in the weak-coupling regime.

issue. We will assume a pure sinusoidal wave at the frequency ω_0, but with a well-defined time origin, chosen to coincide with the signal arrival at the left interface:

$$E_{\rm inc}(x = -a, t) = \theta(t)(E_0 e^{-i\omega_0 t} + {\rm c.c.}), \quad (9)$$

$\theta(t)$ being Heaviside's unit step function.

The evaluation of $E_{\rm trans}(x = a, t)$ is now trivial but, given the unusual and controversial nature of this subject, we choose to change the language in the hope of bringing the results to a far more familiar situation. Let us say that we have two identical (left and right) oscillators, (x_l, x_r), with natural frequency ω_o, and a weak coupling η_k^2 that splits the degeneracy $\omega_{k,\pm}^2 = \omega_0^2 \pm \eta_k^2$. Now we force the left oscillator with the external force $f(t)$ and watch the dynamics of the right oscillator. The equations of motion are

$$\begin{aligned} \ddot{x}_l + \gamma \dot{x}_l + \omega_0^2 x_l + \eta_k^2 x_r &= f(t), \\ \ddot{x}_r + \gamma \dot{x}_r + \omega_0^2 x_r + \eta_k^2 x_l &= 0, \end{aligned} \quad (10)$$

where a damping γ has been added for later convenience but, in accordance with our idealization, is supposed to be $\gamma = 0$, for the moment. Upon identifying the external force, left, and right oscillators with the incident, reflected ($E_{\rm ref}$), and transmitted fields in the following manner:

$$\begin{aligned} f(t) &\longleftrightarrow \eta_k^2\, e^{2\rho_0 a}\, E_{\rm inc}(x = -a, t), \\ x_l &\longleftrightarrow E_{\rm ref}(x = -a, t), \\ x_r &\longleftrightarrow E_{\rm trans}(x = +a, t), \end{aligned} \quad (11)$$

this simple problem is entirely equivalent to our original one. The interpretation of Eq. (11) is direct: the incoming wave plays the role of an external force hitting the left interface, and exciting a reflected (left oscillator) and a transmitted (right oscillator) wave. This mapping allows us to understand the physics of the original situation in simpler terms. For instance, forcing the system (left oscillator) with frequency ω_0, the stationary solution tells us that, surprisingly, only the right oscillator moves. This corresponds [through Eq. (11)] to the absence of a reflected wave.

The time development of the field corresponding to the incoming perturbation of Eq. (9) is now easily obtained, with the following result:

$$E_{\rm trans}(x = +a, t) = A(t)\, e^{2\rho_0 a} \theta(t) E_0 e^{-i\omega_0 t} + {\rm c.c.}, \quad (12)$$

where a characteristic time-dependent modulation amplitude $A(t)$ appears:

$$A(t) = 1 - (e^{-i\Delta\omega_+ t} + e^{-i\Delta\omega_- t}) \,/\, 2, \quad (13)$$

where $\Delta\omega_\pm = \omega_{k,\pm} - \omega_0 \simeq \pm\Delta\omega_k/2$ for $|\Delta\omega_k| \ll \omega_0$, with the splitting of polariton modes $\Delta\omega_k$ given by

$$\Delta\omega_k = \omega_{k,+} - \omega_{k,-} \simeq 8\, C\, \omega_0\, e^{-2\rho_0 a} \quad (14)$$

in the deep subwavelength region. This beating of modes, familiar from the mechanical analog, is sketched in Fig. 3.

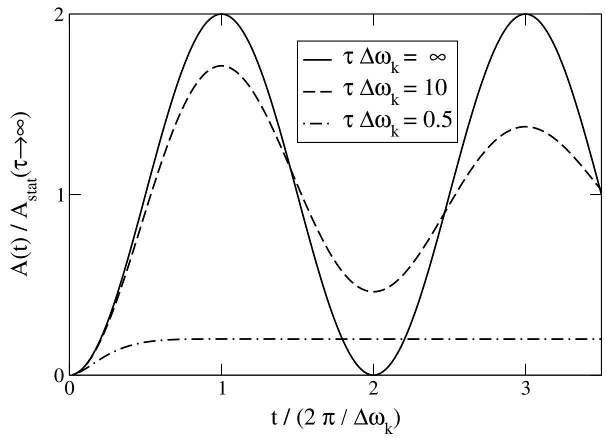

FIG. 3. Time evolution of the modulation amplitude (in units of the stationary, nearly lossless, limit) for three values of surface lifetime corresponding to zero ($\tau\,\Delta\omega_k = \infty$), weak ($\tau\,\Delta\omega_k = 10$), and strong ($\tau\,\Delta\omega_k = 0.5$) damping.

Notice that the total response is the superposition of the stationary solution (ω_0), which is Pendry's solution, with the normal modes ($\omega_\pm$). The latter are unavoidably excited and do not decay in time owing to the absence of losses around ω_0.

The relevance of this behavior for the problem of divergences should be clear by now. For a fixed lapse of time t (large in units of the bare period: $\omega_0 t \gg 1$, but otherwise arbitrary), short wavelengths corresponding to splittings smaller than $\Delta\omega_k \sim t^{-1}$, have barely begun to emerge at the right interface. Therefore, we can identify a time-dependent crossover wave vector $k^{\rm eff}(t)$, satisfying the condition:

$$t\,\Delta\omega_{k^{\rm eff}} = 1, \quad (15)$$

such that, for $k \gg k^{\rm eff}$, then

$$A(t) \sim (\Delta\omega_k t)^2 \sim (C\omega_0 t)^2 e^{-4\rho_0 a}. \quad (16)$$

The norm of Eq. (5) is now replaced by

$$\mathcal{N} \propto (C\omega_0 t)^4 \sum_k |\mathcal{E}_k e^{-3\rho_0 a}|^2, \quad (17)$$

for $k \gg k^{\rm eff}$. Therefore, amplification has been replaced by decay, solving the problem of divergences. Of course, progressively shorter details take longer and longer time (and more energy from the current source) to develop [14,15], but nothing pathological affects the physics of the system. I emphasize again that this picture is unavoidable in the deep subwavelength limit, irrespective of the model details of $\epsilon(\omega)$ and $\mu(\omega)$. It amounts to recognizing that, for a given wave vector k, the system's dynamics has a characteristic time scale given by the (inverse) of the frequency splitting between the two surface polaritons corresponding to that wave vector [Eq. (14)]. The relevance of this time scale would remain hidden if only stationary solutions were analyzed. It is a curious though rewarding fact that, in a system that has been idealized to be homogeneous down to arbitrary small scales (absence of intrinsic length scale), time takes up the responsibility of providing the necessary cutoff length in order to prevent divergent catastrophes.

It could be argued that the solution provided here to the problem of divergences is an artifact of the ideal lossless nature of the problem. Real systems would show losses that will eventually kill any transient regime, perhaps leaving again the divergent stationary solution. This is not the case, as can be seen by the explicit inclusion of a finite lifetime ($\tau = \gamma^{-1}$) in Eq. (10). A stationary solution is indeed established, but with amplitude A_{stat} greatly reduced from the lossless limit $A_{\mathrm{stat}}(\infty)$ at large wave vectors:

$$\frac{A_{\mathrm{stat}}(\tau)}{A_{\mathrm{stat}}(\infty)} = \frac{1}{1 + (\tau\Delta\omega_k)^{-2}}. \tag{18}$$

Notice that, for small values of $\tau\Delta\omega_k$, the stationary amplitude is what would have been expected if the lossless evolution were suddenly stopped at a time of the order of τ. Therefore, we see exactly the same physics as in the lossless case, but now the role of the observation lapse of time is taken by the polariton lifetime. A resolution wave number (k^{eff}) can be defined again [compare Eq. (15)]:

$$\tau\,\Delta\omega_{k^{\mathrm{eff}}} = 1 \tag{19}$$

such that, for wave numbers larger than this cutoff, exponential decay rather than amplification is observed at the exit of the slab. The norm of field at the exit is now as in Eq. (17), with the exchange $t \leftrightarrow \tau$.

The transient behavior for the two regimes $\tau \gg (\Delta\omega_k)^{-1}$ and $\tau \leq (\Delta\omega_k)^{-1}$ is illustrated in Fig. 3. The picture is simple: when the lifetime of surface polaritons is much longer that the characteristic time scale (inverse splitting of modes) for the wave vector k, the stationary limit approaches the lossless case. This means that surface modes have had enough time to build the final response before dying away. On the other hand, if the surface modes do not live long enough, no amplification is possible.

In the deep subwavelength limit, Eq. (19) allows us to provide an explicit expression for the minimum lifetime (τ_{min}) required to get a resolution $l_{\mathrm{res}} = 2\pi/k^{\mathrm{eff}}$, with radiation of wavelength $\lambda_0 = 2\pi/\omega_0$:

$$\tau_{\mathrm{min}} \sim \frac{1}{8\,C\,\omega_0}\,\exp\left(4\pi a\sqrt{l_{\mathrm{res}}^{-2} - \lambda_0^{-2}}\right). \tag{20}$$

Similar results have been obtained before for the resolution [15], further reinforcing the correctness of our restriction to the polariton dynamics. Notice the characteristic exponential dependence on resolution. Although mainly concerned with matters of principle in this Letter, this demanding result clearly shows that the polariton lifetime may well be a major limiting factor for a practical realization of perfect lenses. I believe this vulnerability [15] of the ideal situation, a fingerprint of the near degeneracy of surface modes, is at the root of similar results found before [17,18].

In spite of potential difficulties in the road to a practical left-handed amplifier, the essential physics described in this Letter may have been observed in a different system. The role of coherence between surface modes (and the required time scale associated with it) in the explanation of the extraordinary transmission through hole arrays [19] is strikingly similar to our treatment. This reinforces the view [2] that both problems (left-handed amplification and extraordinary transmission through hole arrays) are probably different manifestations of the same physical behavior of surface modes.

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