Nonequilibrium Transport through a Kondo Dot in a Magnetic Field: Perturbation Theory and Poor Man's Scaling

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We consider electron transport through a quantum dot described by the Kondo model in the regime of large transport voltage V in the presence of a magnetic field B with $\max(V, B) \gg T_K$. The electric current I and the local magnetization M are found to be universal functions of V/T_K and B/T_K, where T_K is the equilibrium Kondo temperature. We present a generalization of the perturbative renormalization group to frequency dependent coupling functions, as necessitated by the structure of bare perturbation theory. We calculate I and M within a poor man's scaling approach and find excellent agreement with experiment.

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The transport of electrons through quantum dots in the Coulomb blockade regime is strongly suppressed. In the case of an odd number of electrons on the dot, however, when the dot carries a spin $\mathbf{S}$, exchange coupling of $\mathbf{S}$ to the spin of the conduction electrons in the two leads (L, R) gives rise to a Kondo resonance at low temperature $T \ll T_K$ (T_K is the Kondo temperature). Electrons may then cross the dot via resonance tunneling, thus circumventing the Coulomb blockade, as seen in a number of recent experiments [1,2]. While there is a wealth of methods such as Bethe ansatz, conformal field theory, numerical renormalization group (RG), and self-consistent perturbation theory available to treat the Kondo model in equilibrium, most of these methods fail in the presence of a finite bias voltage V, for $V \geq T_K$.

The first theoretical treatment of this problem dates back to the 1960s [3] when the current through a Kondo impurity, in the presence of finite V and magnetic field B, was calculated in perturbation theory (PT) including leading logarithmic corrections. These early works also attempted a resummation of the logarithms within Nagaoka's or Abrikosov's approximations. All of these works, however, neglected salient nonequilibrium physics of this problem (see below).

A crucial difference between a Kondo dot in the regime of large voltage $V \gg T_K$ and the equilibrium Kondo problem is the presence of inelastic processes, associated with the finite current through the dot, down to the lowest temperatures [4,5]. These processes destroy coherence on an energy scale Γ and prevent the full formation of the Kondo singlet resonance state. As discussed in [5], for a conventional Kondo dot described by an Anderson model, $\Gamma \sim V/\ln^2(V/T_K) \gg T_K$ for $V \gg T_K$. In this regime, which we consider here, a perturbative treatment is sufficient to capture the developing Kondo correlations. As is well known from the usual Kondo effect, even in the perturbative regime it is necessary to resum the leading logarithmic terms in order to recover the scaling behavior. For the equilibrium case, it is known how to achieve this in a systematic and controlled way by employing the perturbative renormalization group method. For the nonequilibrium Kondo problem, a comparable treatment has not been developed thus far. A certain resummation of PT has been considered [6] within an Anderson model for the differential conductance $G(V, B)$. However, the summation of leading logarithms remained incomplete.

In this Letter, we present the first systematic study of renormalized PT for the nonequilibrium Kondo problem, considering, in particular, the influence of a magnetic field B. We focus on the regime $V \gg T_K$, for any B. Our results are also applicable for large $B \gg T_K$, and arbitrary V. In both cases, the full formation of the Kondo resonance is inhibited, and a perturbative treatment in the exchange coupling J is possible. At first sight, one might expect the nonequilibrium Kondo effect to be similar to the high temperature regime $T \gg T_K$ of the usual Kondo model. It differs, however, in three qualitative aspects from the latter: (i) The occupation probabilities of the local spin are not thermal and have to be determined by solving a quantum Boltzmann equation. This leads to an unusual dependence, e.g., of the spin susceptibility on V and to a novel structure of the logarithmic corrections in PT. (ii) Electrons in a wide range of energies $\sim V$ contribute to the low-T properties. In contrast to the case of large T, where all features are smeared on the scale T, we find that it is therefore essential to keep track of the frequency dependence of the renormalized coupling constants. (iii) Lifetime effects play a crucial role in nonequilibrium, as the corresponding rates are larger than T but smaller than V [5].

As a first step, we present and discuss the results of bare PT for the current $I(V, B)$ and the magnetization $M(V, B)$ in leading logarithmic order. Next, we turn to the derivation of renormalization group equations for the running coupling constants. The structure of bare PT forces one to keep the frequency dependence of the coupling functions. Employing simplifications in the spirit of Anderson's poor man's scaling, we derive a set of RG equations in one-loop order. These have to be supplemented by a self-consistent determination of the relaxation rate Γ, providing a cutoff of the RG flow. Finally, the physical quantities current I, magnetization M, and differential conductance G are calculated to leading order in $1/\ln[\max(V, B)/T_K]$ by inserting the renormalized frequency dependent coupling functions into the lowest order PT expressions. In this way, all physical quantities show scaling behavior in V/T_K and B/T_K, where T_K is the equilibrium Kondo temperature.

We consider the Kondo Hamiltonian,

$$H = \sum_{\alpha=L,R,\mathbf{k},\sigma} (\varepsilon_{\mathbf{k}} - \mu_\alpha) c^\dagger_{\alpha\mathbf{k}\sigma} c_{\alpha\mathbf{k}\sigma} - BS_z + \frac{1}{2} \sum_{\alpha,\alpha'=L,R,\mathbf{k},\mathbf{k}',\sigma,\sigma'} J_{\alpha'\alpha} \mathbf{S} \cdot (c^\dagger_{\alpha'\mathbf{k}'\sigma'} \boldsymbol{\tau}_{\sigma'\sigma} c_{\alpha\mathbf{k}\sigma}), \quad (1)$$

where $\mu_{L,R} = \pm V/2$. $\mathbf{S}$ is the spin 1/2 on the dot and $\boldsymbol{\tau}$ are the Pauli matrices. We shall use the dimensionless coupling constants $g_d = N_0 J_{LL} = N_0 J_{RR}$ and $g_{LR} = N_0 J_{LR} = N_0 J_{RL}$ (assuming a symmetric dot), where N_0 is the local density of states. The local spin $\mathbf{S} = \frac{1}{2}\sum_{\gamma\gamma'} f^\dagger_\gamma \boldsymbol{\tau}_{\gamma\gamma'} f_{\gamma'}$ is represented by pseudofermions (PF) in the sector of the Hilbert space with $\sum_\gamma f^\dagger_\gamma f_\gamma = 1$.

The electric current $I = (e/\hbar) J_{LR} \,\mathrm{Im}[D^K_{LR}(t,t)]$ through the quantum dot may be expressed through the Keldysh component of the contour-ordered correlation function $D_{LR}(t,t') = (-i)^2 \sum_{\mathbf{k},\mathbf{k}'} \langle T_c\{c^\dagger_{L\mathbf{k}'\sigma'}(t)(\boldsymbol{\tau}_{\sigma'\sigma}/2) c_{R\mathbf{k}\sigma}(t) \times \mathbf{S}(t')\}\rangle$. To obtain the leading logarithmic corrections, we evaluate the diagrams shown in Fig. 1(a), which for T, V, B much smaller than the band cutoff D yield

$$I = \frac{2e}{2\pi\hbar}\left(\frac{\pi}{2} g_{LR}\right)^2 \{u_2(V) + u_2(V+B) + u_2(V-B) - M[c(B) - c(-B)]\}, \quad (2)$$

with $u_p(x) = x(1 + 2p g_d \ln\frac{D}{|x|})$, $(p = 1, 2)$, and $c(B) = \coth\frac{V+B}{2T}[u_1(V+B) + 2V g_d \ln\frac{D}{|V|} + 2B g_d \ln\frac{D}{|B|}]$, where all logarithms are implicitly cut off by T. Appelbaum's expression for the current [3] differs from (2) in that he assumed M to be given by its thermal equilibrium value. In the zero-field limit M vanishes and (2) reduces to $I = \frac{2e}{2\pi\hbar}(\frac{\pi}{2} g_{LR})^2 3V(1 + 4 g_d \ln\frac{D}{|V|})$ derived earlier in Refs. [3,7]. To calculate the current for finite B, one needs to know the local magnetization $M = n_\uparrow - n_\downarrow$, and at finite voltage this necessarily involves solving a quantum Boltzmann equation. In diagrammatic language, this is nothing but the Dyson equation of the lesser PF Green function $G^<_\gamma(\omega)$ in steady state: $G^<_\gamma(\omega)\Gamma_\gamma(\omega) = \Sigma^<_\gamma(\omega) A_\gamma(\omega)$, where Γ_γ is the imaginary part of the self-energy, and A_γ the PF spectral function. The leading logarithmic corrections are obtained from the diagrams in Fig. 1(b), yielding

$$M = \frac{\mathcal{T}}{\coth\frac{B}{2T}[\frac{\mathcal{T}}{2} + g_-^2 u_1(B)] + g_{LR}^2[c(B) + c(-B)]}, \quad (3)$$

with $\mathcal{T} = 2g_+^2 u_1(B) + 2g_{LR}^2[u_1(V+B) - u_1(V-B) - 2B]$

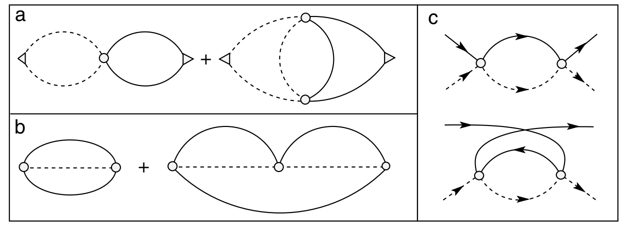

FIG. 1. Feynman diagrams for (a) D_{LR}, (b) PF self-energies, and (c) vertices entering the one-loop RG equation. PF (electron) propagators are displayed as dashed (full) lines.

and $g_\pm^2 = (g_d^2 \pm g_{LR}^2)$. Note that the magnetization differs from the equilibrium result even to order g^0, as has been derived independently in [8] [see also Eq. (8) below] in disagreement with [9]. Most notably, the magnetic susceptibility changes from the usual $1/T$ Curie law to a $1/V$ behavior for $V \gg T$. Corrections of order $g^3 \ln D$ to the collision integrals in the quantum Boltzmann equation result in corrections of order $g \ln D$ to M. Thus, they are much larger than the usual $g^2 \ln D$ corrections obtained in equilibrium. In the limit $V \to 0$, Eq. (3) simplifies to the noninteracting result $M = \tanh(B/2T)$, as we have neglected the subleading $g^2 \ln D$ corrections which can be obtained by including the $\mathrm{Re}\,\Sigma$ shift of the PF energy levels. The effects of $\mathrm{Im}\,\Sigma$ are more important and are discussed below.

In the scaling regime, $V, B \ll D$, bare PT is not valid and has to be resummed even for small couplings g and in the weak-coupling regime $V, B \gg T_K$ [see Fig. 3(b)]. The method of choice for such a resummation is the perturbative RG, using the basic idea that a change of the cutoff D can be absorbed in a redefinition of the coupling constants g [11].

A close inspection of Eqs. (2) and (3) indeed reveals that, in the regime where, under renormalization, D gets smaller than V, it is *not* possible to absorb a change of D in a redefinition of the couplings. For example, logarithmic corrections to g_{LR} in the denominator of Eq. (3) are proportional to $2\ln(D/V)$ for $B \to 0$, while the analogous correction in the numerator takes the form $\ln(D/V) + \ln(D/T)$. This apparent breakdown of scaling is due to the fact that electrons in a finite energy window $\mu_R \lesssim \omega \lesssim \mu_L$ contribute to low-energy properties. How much their scattering is renormalized will depend on their respective positions within this window: Upon renormalization, the coupling "constants" acquire *frequency dependences.* Taking this properly into account, one recovers scaling. One way to derive perturbative RG equations in such a situation is to start from so-called "exact" RG equations for (ω-dependent) one-particle irreducible Green functions by a straightforward generalization of equilibrium RG methods [12]. We will not follow this route here, but suggest a substantially simpler (but less systematic) approach. A considerably more involved real-time RG scheme has been proposed by Schoeller and König [13], but has not yet been applied to the present problem.

We start from the observation that all logarithmic terms in next-to-leading order PT stem from the simple vertex renormalizations shown in Fig. 1(c) when the real part $\approx 1/(\omega \pm B/2)$ of the PF Green function is convoluted with the Keldysh component of the electron line $-2\pi i N_0 \tanh[(\omega - \mu_\alpha)/2T]$. Using cutoffs symmetric with respect to $\mu_{L,R}$, respectively, one obtains at $T = 0$,

$$\frac{\partial}{\partial \ln D} \int_{-D}^{D} d\omega \frac{\text{sign}\omega}{\omega - \Delta\omega} \approx 2\Theta(D - |\Delta\omega|), \quad (4)$$

where $\Delta\omega$ depends on μ_α, B, and the incoming and outgoing frequencies. Now we include the effect of frequency and spin-dependent coupling constants denoted by $g^{\alpha\sigma\omega_c;\alpha'\sigma'\omega'_c}_{\gamma\omega_f;\gamma'\omega'_f}$ for an incoming electron in lead $\alpha = L, R$ with energy ω_c and spin $\sigma = \uparrow, \downarrow$ interacting with a PF of spin γ and frequency ω_f describing the local spin. Primed quantities refer to outgoing particles. Generally, vertices are complex and have Keldysh indices, but we will keep track only of the real parts on one Keldysh contour, which we believe to be justified to leading order in $1/\ln[V/T_K]$. The pseudofermion spectral functions are strongly peaked at $\omega = \pm B/2$, which allows setting $\omega_f(\omega'_f)$ to $-\gamma\frac{B}{2}(-\gamma'\frac{B}{2})$. Furthermore, neglecting the remaining frequency dependence of the coupling functions in the frequency window $|\omega| < D$ [14], one is led [15] to two RG equations for the coupling functions $\tilde{g}_{z\sigma}(\omega)$ and $\tilde{g}_\perp(\omega)$ (assuming $J_{LL} = J_{RR} = J_{LR} = J_{RL} = J$):

$$\frac{\partial \tilde{g}_\perp(\omega)}{\partial \ln D} = -\sum_{\beta,\sigma=-1,1} \frac{\tilde{g}_\perp(\frac{\sigma B + \beta V}{2})\tilde{g}_{z\sigma}(\frac{\beta V}{2})}{2} \Theta_{\omega + (\sigma B + \beta V)/2},$$

$$\frac{\partial \tilde{g}_{z\sigma}(\omega)}{\partial \ln D} = -\sum_{\beta=-1,1} \tilde{g}_\perp\Big(\frac{B + \beta V}{2}\Big)^2 \Theta_{\omega + \sigma[B + \beta(V/2)]}, \quad (5)$$

with $\tilde{g}_\perp(\omega) = \tilde{g}_\perp(-\omega)$, $\tilde{g}_{z\uparrow}(\omega) = \tilde{g}_{z\downarrow}(-\omega)$, and

$$g^{\alpha\sigma,\omega;\alpha'\bar{\sigma},\omega-\gamma B}_{\gamma,-\gamma B/2;\bar{\gamma},-\bar{\gamma}B/2} = (\tau^x_{\gamma\bar{\gamma}}\tau^x_{\sigma\bar{\sigma}} + \tau^y_{\gamma\bar{\gamma}}\tau^y_{\sigma\bar{\sigma}})\, \tilde{g}_\perp(\omega - \gamma B/2),$$

$$g^{\alpha\sigma,\omega;\alpha'\sigma,\omega}_{\gamma,-\gamma B/2;\gamma,-\gamma B/2} = \tau^z_{\gamma\gamma}\tau^z_{\sigma\sigma}\tilde{g}_{z\sigma}(\omega), \quad (6)$$

with the initial conditions $\tilde{g}_{z\sigma}(\omega) = \tilde{g}_\perp(\omega) = JN_0$ at $D = D_0$, the bare cutoff, and $\Theta_{\Delta\omega} = \Theta(D - |\Delta\omega|)$. Despite the fact that we use the same functions for both diagonal (LL, RR) and off-diagonal (LR, RL) processes, their renormalization is drastically different as different frequency ranges are probed in the two cases (see below). In various limits, analytic solutions are possible but, as the formulas turn out to be rather lengthy, we will here restrict ourselves mainly to numerical results.

The RG Eqs. (5) are valid in the regime of cutoff values D where the effect of spin-flip processes destroying the quantum coherence of the Kondo bound state is negligible, or $D > \Gamma$. Because of the finite current, relaxation processes contributing to Γ are present even at $T = 0$, and lead to an imaginary part of the self-energy of the pseudofermions [e.g., in the PF propagator in the diagrams of Fig. 1(c)] and to vertex corrections. Although slightly different relaxation rates may enter in the various coupling constants, with logarithmic accuracy we may use a single rate Γ. As all relevant processes involve at least one spin flip, we identify Γ with the transverse spin relaxation rate $1/T_2$, which is given in terms of the renormalized couplings as

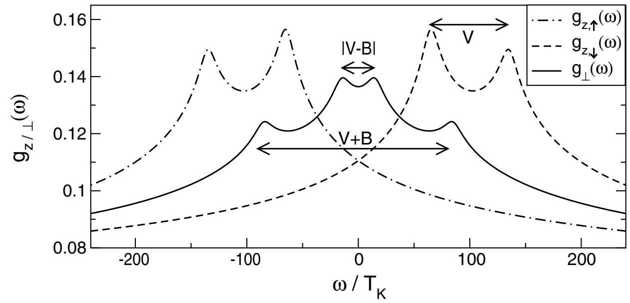

FIG. 2. Renormalized coupling constants $\tilde{g}_{z,\sigma}(\omega)$ (dashed lines) and $\tilde{g}_\perp(\omega)$ (solid line) for $B = 100T_K$ and $V = 70T_K$. For these values, Eqs. (5) and (7) yield $\Gamma = 6.47T_K$.

$$\Gamma = \frac{\pi}{4\hbar} \sum_{\substack{\alpha,\alpha'=L,R\\ \gamma=-1,1}} \int d\omega [\tilde{g}_{z\gamma}(\omega)^2 f_{\omega-\mu_\alpha}(1 - f_{\omega-\mu_{\alpha'}}) + \tilde{g}_\perp(\omega - \gamma B/2)^2 \times f_{\omega-\mu_\alpha}(1 - f_{\omega-\mu_{\alpha'}-\gamma B})]. \quad (7)$$

Note the missing factor 2 in front of the spin-flip term $\tilde{g}_\perp(\omega - \gamma B/2)^2$ in Γ [compared to the corresponding term in (9), below], which reflects the fact that $1/T_2$ [16] is *not* given by a pseudofermion self-energy and arises from vertex corrections in a linear-response calculation of the transverse susceptibility. The effect of T_2 is to provide an upper time cutoff for the coherent scattering processes entering Eq. (5). Hence, it is modeled phenomenologically by replacing Θ_ω with $\Theta(D - \sqrt{\omega^2 + \Gamma^2})$ in Eq. (5).

A self-consistent solution of Eqs. (5) and (7) can now be obtained with little numerical effort (formally, effects of self-consistency for Γ are subleading for $V, B \gg T_K$). The resulting couplings $\tilde{g}_{z/\perp}(\omega)$ are shown in Fig. 2. In the renormalization process, the $\tilde{g}_{z/\perp}(\omega)$ develop peaks at frequencies $\omega = -\sigma B \pm \frac{V}{2}$ for $\tilde{g}_{z\sigma}$ and $\omega = \pm\frac{1}{2}(B \pm V)$ for $\tilde{g}_\perp$, at which resonant scattering from one Fermi surface to another becomes possible. Only these resonant processes survive for small running cutoff D, as can be seen from the structure of the functions $\Theta_\omega = \Theta(D - |\omega|)$ in (5). This Kondo-type resonant enhancement is eventually cutoff by Γ. Note that Γ remains larger than T_K for $V, B > T_K$ prohibiting the flow towards strong-coupling contrary to the suggestion of [9] (Ref. [5] discusses conditions for strong-coupling physics in the regime $V \gg T_K$).

Knowing the renormalized $\tilde{g}_{z/\perp}(\omega)$ at $D = 0$, the magnetization M can be found from solving the Boltzmann equation $\partial_t n_\sigma = 0$ or

$$n_\uparrow \sum_{\alpha,\alpha'=L,R} \int d\omega \tilde{g}_\perp(\omega - B/2)^2 f_{\omega-\mu_\alpha}(1 - f_{\omega-\mu_{\alpha'}-B}) = n_\downarrow \sum_{\alpha,\alpha'=L,R} \int d\omega \tilde{g}_\perp(\omega + B/2)^2 f_{\omega-\mu_\alpha}(1 - f_{\omega-\mu_{\alpha'}+B}), \quad (8)$$

the solution of which in turn determines the current as

$$I = \frac{\pi e}{16\hbar}\int d\omega \sum_{\sigma=-1,1} [\tilde{g}_{z\sigma}(\omega)^2 f_{\omega-\mu_L}(1 - f_{\omega-\mu_R}) + 4\tilde{g}_\perp(\omega - \sigma B/2)^2 \times f_{\omega-\mu_L}(1 - f_{\omega-\mu_R-\sigma B}) n_\sigma] - (L \leftrightarrow R), \quad (9)$$

In Eqs. (7)–(9), there are further lifetime effects, which become relevant for $|V - B|, T \lesssim \Gamma$. This broadening we model phenomenologically by smearing the Fermi functions over a width Γ [15]. Note that expanding these results in bare PT, one recovers Eqs. (2) and (3).

In Fig. 3, we show $M(V/B, B/T_K)$ and $G(V/B, B/T_K)$. Within PT, G shows threshold behavior at $|V| = B$, due to the opening of another transport channel involving spin flip, requiring the Zeeman energy B. Higher order processes enhance the steps into peaks [Fig. 3(b)]. The agreement with experiment [2] is excellent [Fig. 3(c)], considering that there is no free parameter and our result is valid up to terms of order $1/\ln[\max(V, B)/T_K]$ only. We emphasize that neither perturbation theory [Fig. 3(b)] nor Appelbaum's result (as shown in [2]), nor the result of [6] can describe the experiment.

In conclusion, we propose a simple method to generalize poor man's scaling (one-loop order) to nonequilibrium. Compared to the equilibrium situation, it is necessary to include nonequilibrium distribution functions, decay rates, and the ω dependence of coupling constants. The different structures arising even in the perturbative regime may be best exemplified by considering the local magnetic susceptibility for $V \gg T_K, B \to 0$,

$$\chi(V) = \frac{2}{V} \frac{1 + 2\alpha \ln\frac{V}{T_K}(1 + \alpha \ln\frac{V}{T_K})}{\left(1 - \frac{(1+2\alpha\ln\frac{V}{T_K})\ln[\ln\frac{V}{T_K}(1+\alpha\ln\frac{V}{T_K})]}{(1+\alpha\ln\frac{V}{T_K})\ln\frac{V}{T_K}}\right)^2}, \quad (10)$$

with $\alpha = (g_d^2 - g_{LR}^2)/(2g_{LR})$ and $T_K = De^{-1/(g_d+g_{LR})}$. To the same order of approximation, one obtains in equilibrium for $T \gg T_K$ just $\chi = 1/T$.

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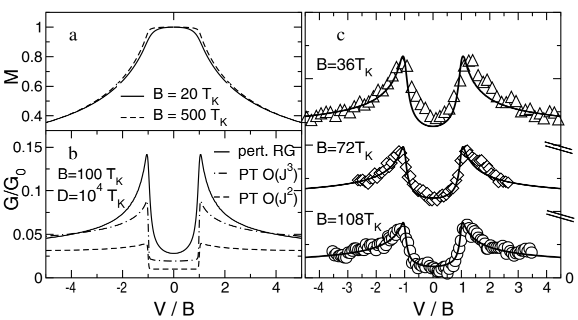

FIG. 3. (a) Local magnetization $M(V/B, B/T_K)$ of a symmetric dot for fixed magnetic field B. (b) Differential conductance in units of $G_0 = e^2/(\pi\hbar)$. Comparison of perturbative RG and bare PT to $O(J^2)$ and $O(J^3)$. (c) Conductance measurements of Ref. [2] (symbols) on metallic point contacts in magnetic fields 0.85, 1.7, 2.55 T ($B = 36T_K$, $72T_K$, and $104T_K$ with $T_K \approx 30$ mK [2]). Assuming that the corresponding point contact is described by a single-channel ($J_{LR} = \sqrt{J_{LL}J_{RR}}$) Kondo model, $(J_{RR}/J_{LL}) \approx 4.2$ is determined from $G(V = 0, B = 0, T = 50\,\text{mK}) = [(4J_{LL}J_{RR})/(J_{LL} + J_{RR})^2]G_{\text{sym}}(T/T_K)$, where G_{sym} is known exactly from numerical RG calculations [10]. This fixes *all* parameters for our RG calculation (solid lines, $T = 0$) which uses a straightforward generalization of (5)–(8) for $J_{LL} \neq J_{RR}$. As the (B-dependent) background is not known experimentally, we subtract $\Delta G = G_B - 5.2 \times 10^{-5} G_0(V/T_K)$, where G_B is fitted to our results at large V.

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