Interaction-Induced Magnetoresistance: From the Diffusive to the Ballistic Regime

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We study interaction-induced quantum correction $\delta\sigma_{\alpha\beta}$ to the conductivity tensor of electrons in two dimensions for arbitrary $T\tau$, where T is the temperature and τ the transport mean free time. A general formula is derived, expressing $\delta\sigma_{\alpha\beta}$ in terms of classical propagators ("ballistic diffusons"). The formalism is used to calculate the interaction contribution to the magnetoresistance in a classically strong transverse field and smooth disorder in the whole range of temperatures from the diffusive ($T\tau \ll 1$) to the ballistic ($T\tau \gtrsim 1$) regime.

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The magnetoresistance (MR) in a transverse field B is one of the most frequently studied characteristics of the two-dimensional (2D) electron gas [1,2]. Within the Drude-Boltzmann theory, the longitudinal resistivity of an isotropic degenerate system is B independent, $\rho_{xx}(B) = \rho_0 = (e^2\nu v_F^2\tau)^{-1}$, where ν is the density of states per spin direction, v_F the Fermi velocity, and τ the transport scattering time. There are several distinct sources of a nontrivial MR, which reflect the rich physics of 2D systems. First, quasiclassical memory effects may lead to a MR [3], which shows no T dependence at low temperatures. Second, weak localization [1] induces a negative MR restricted to the range of very weak magnetic fields. Finally, another quantum correction to MR is generated by the electron-electron interaction. This effect is the subject of the present paper.

It was discovered by Altshuler and Aronov [1] that the Coulomb interaction enhanced by the diffusive motion of electrons gives rise to a quantum correction to conductivity, which has in 2D the form (we set $k_B = \hbar = 1$)

$$\delta\sigma_{xx} \simeq (e^2/2\pi^2)\ln T\tau, \qquad T\tau \ll 1. \tag{1}$$

It is assumed here for simplicity that $\kappa \ll k_F$, where $\kappa = 4\pi e^2\nu$ is the inverse screening length. The condition $T\tau \ll 1$ under which Eq. (1) is derived [1] implies that electrons move diffusively on the time scale $1/T$ and is termed the "diffusive regime." Subsequent works [4] showed that Eq. (1) remains valid in a strong magnetic field, leading (in combination with $\delta\sigma_{xy} = 0$) to a parabolic interaction-induced quantum MR,

$$\frac{\delta\rho_{xx}(B)}{\rho_0} \simeq \frac{(\omega_c\tau)^2 - 1}{\pi k_F l}\ln T\tau, \qquad T\tau \ll 1, \tag{2}$$

where $\omega_c = eB/mc$ is the cyclotron frequency and $l = v_F\tau$ the transport mean free path. Indeed, a T-dependent negative MR was observed in experiments [5] and attributed to the interaction effect. However, the experiments [5] cannot be directly compared with the theory [1,4] since they were performed at higher temperatures, $T\tau \gtrsim 1$. (In high-mobility GaAs heterostructures conventionally used in MR experiments, $1/\tau$ is typically $\sim$100 mK and becomes even smaller with improving quality of samples.) There is thus a clear need for a theory of the MR in the ballistic regime, $T \gtrsim 1/\tau$.

In fact, the effect of interaction on the conductivity at $T \gtrsim 1/\tau$ has attracted a great deal of interest in a context of low-density 2D systems showing a seemingly metallic behavior, $d\rho/dT > 0$ [6]. Recently, Zala, Narozhny, and Aleiner [7] developed a systematic theory of the interaction corrections valid for arbitrary $T\tau$. In the ballistic range of temperatures, this theory (improving earlier calculation of temperature-dependent screening [8]), predicts a linear-in-T correction to conductivity σ_{xx} and a $1/T$ correction to the Hall coefficient ρ_{xy}/B at $B \to 0$, and describes the MR in a *parallel* field.

The consideration of [7] is restricted, however, to *classically weak* transverse fields, $\omega_c\tau \ll 1$, and to the *white-noise* disorder. The latter assumption is believed to be justified for Si-based and some p-GaAs structures, and the results of [7] have been by and large confirmed by most recent experiments [9] on such systems. On the other hand, the random potential in n-GaAs heterostructures is, as a rule, due to remote donors and has a long-range character. Thus, the impurity scattering is predominantly of a small-angle nature and is characterized by two relaxation times, the transport time τ and the single-particle (quantum) time τ_s, with $\tau \gg \tau_s$.

We present here a general theory of the interaction-induced corrections to the conductivity of 2D electrons valid for arbitrary temperatures, transverse magnetic fields, and disorder range. We further apply it to the problem of magnetotransport in a smooth disorder at $\omega_c\tau \gg 1$ [10]. In the ballistic limit, $T\tau \gg 1$ (where the character of disorder is crucially important), we show that while the correction to ρ_{xx} is exponentially suppressed for $\omega_c \ll T$, a MR arises at stronger B where it scales as $B^2T^{-1/2}$.

To find $\delta\sigma_{\alpha\beta}$, we make use of the "ballistic" generalization of the Matsubara diffuson diagram technique of Ref. [1]. We consider the exchange contribution first and will discuss the Hartree term later on. The relevant diagrams are shown in Fig. 1. The shaded blocks in Fig. 1

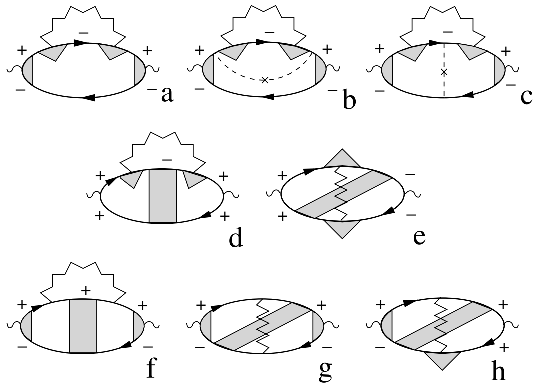

FIG. 1. Diagrams for the interaction correction to $\sigma_{\alpha\beta}$. The wavy (dashed) lines denote the interaction (impurity scattering), the shaded blocks are impurity ladders, and the $+/-$ symbols denote the signs of the Matsubara frequencies. The diagrams obtained by a flip and/or by an exchange $+ \leftrightarrow -$ should also be included.

denote the impurity-line ladders, which we term "ballistic diffusons." The temperature range of main interest in the present Letter is restricted by $T\tau_s \ll 1$, since at higher T the MR will be small in the whole range of the quasiclassical transport $\omega_c\tau_s \ll 1$ (see below). In this case the ladders are dominated by contributions with many ($\gg 1$) impurity lines. Our general formula below is, however, valid irrespective of the value of $T\tau_s$.

After the Wigner transformation, the ballistic diffuson takes the form $\mathcal{D}(\omega; \mathbf{r}, \mathbf{n}; \mathbf{r}', \mathbf{n}')$ and describes the quasiclassical propagation of an electron in the phase space [11] ($\mathbf{n}$ is the direction of velocity on the Fermi surface). In contrast to the diffusive regime, where $\mathcal{D}$ has a universal and simple structure $\mathcal{D}(\omega, \mathbf{q}) = 1/(Dq^2 - i\omega)$ determined by the diffusion constant D only, its form in the ballistic regime is much more complicated. We are able, however, to get a general expression for $\delta\sigma_{\alpha\beta}$ in terms of the propagator $\mathcal{D}(\omega, \mathbf{q}; \mathbf{n}, \mathbf{n}')$. The result reads

$$\delta\sigma_{\alpha\beta} = -2e^2 v_{\mathrm{F}}^2 \nu \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\partial}{\partial\omega} \left\{ \omega \coth \frac{\omega}{2T} \right\} \times \int \frac{d^2\mathbf{q}}{(2\pi)^2} \mathrm{Im}[U(\omega, \mathbf{q}) B_{\alpha\beta}(\omega, \mathbf{q})], \qquad (3)$$

where $U(\omega, \mathbf{q})$ is the interaction potential equal to a constant V_0 for pointlike interaction and to

$$U(\omega, \mathbf{q}) = \frac{1}{2\nu} \frac{\kappa}{q + \kappa[1 + i\omega\langle\mathcal{D}(\omega, q)\rangle]} \qquad (4)$$

for screened Coulomb interaction. For small-angle impurity scattering the tensor $B_{\alpha\beta}(\omega, \mathbf{q})$ in (3) is given by

$$B_{\alpha\beta}(\omega, \mathbf{q}) = \frac{T_{\alpha\beta}}{2} \langle\mathcal{D}\mathcal{D}\rangle + T_{\alpha\gamma} \left(\frac{\delta_{\gamma\delta}}{2} \langle\mathcal{D}\rangle - \langle n_\gamma \mathcal{D} n_\delta \rangle \right) T_{\delta\beta} - 2T_{\alpha\gamma} \langle n_\gamma \mathcal{D} n_\beta \mathcal{D} \rangle - \langle \mathcal{D} n_\alpha \mathcal{D} n_\beta \mathcal{D} \rangle, \qquad (5)$$

where $T_{\alpha\beta} = \langle n_\alpha \mathcal{D} n_\beta \rangle|_{q=0, \omega\to 0} = \sigma_{\alpha\beta}/e^2 v_F^2 \nu$. The angular brackets $\langle \ldots \rangle$ in (4) and (5) denote averaging over velocity directions, e.g., $\langle n_x \mathcal{D} n_x \rangle = (2\pi)^{-2} \int d\phi_1 d\phi_2 \times \cos\phi_1 \mathcal{D}(\omega, \mathbf{q}; \phi_1, \phi_2) \cos\phi_2$, where ϕ is the polar angle of $\mathbf{n}$. The first term in (5) originates from the diagrams (a), (b), and (c) in Fig. 1 (forming together the Hikami box), the second term from (a), (f), and (g) [12], the third term from (h), and the last one from (d) and (e).

In the more general situation, when the scattering is at least partly of the large-angle character, the first term in (5) acquires a slightly more complicated form,

$$\pi\nu T_{\alpha\alpha'} [\langle \mathcal{D} S_{\alpha'\beta'} \mathcal{D} \rangle - 2\langle \mathcal{D} n_{\alpha'} W n_{\beta'} \mathcal{D} \rangle] T_{\beta'\beta}, \qquad (6)$$

where $W(\mathbf{n}, \mathbf{n}')$ is the scattering cross section and $S_{xx} = S_{yy} = W(\mathbf{n}, \mathbf{n}')$, $S_{xy} = -S_{yx} = \omega_c/2\pi\nu$. In particular, for the case of purely white-noise disorder [when $\tau = \tau_s$ and $W(\mathbf{n}, \mathbf{n}') = 1/2\pi\nu\tau$] and $B = 0$ we then recover (using the explicit form of the ballistic propagator for this case) the result for $\delta\sigma$ obtained in a different way in [7]. Needless to say, in the diffusive limit, we reproduce (for arbitrary B and disorder range) the logarithmic correction (1) and (2) determined by the diagrams (a)–(e).

Before turning to the analysis of the results for the strong B regime, we consider briefly the $B = 0$ case assuming the ballistic temperature range $T\tau \gg 1$. The structures of Eqs. (3), (5), and (6) imply that the interaction correction is governed by returns of a particle to the original point in a time $t \lesssim T^{-1} \ll \tau$. Such a quick return may be induced by a single back-scattering process, yielding the contribution $\delta\sigma_{xx} \sim e^2 \nu\tau W(2k_F) T\tau$. For the case of white-noise disorder this reduces to $\delta\sigma_{xx} \sim e^2 T\tau$, in agreement with [7,8]. However, in a smooth disorder with a correlation length $d \gg k_F^{-1}$ this contribution is suppressed by the factor $2\pi\nu\tau W(2k_F) \sim e^{-k_F d}$. The probability to return after many small-angle scattering events is also exponentially suppressed for $t \ll \tau$, yielding a contribution $\delta\sigma_{xx} \sim \exp[-\mathrm{const}(T\tau)^{1/2}]$. Thus, the interaction correction in the ballistic regime is exponentially small at $B = 0$ for the case of smooth disorder. Moreover, the same argument applies to the case of a nonzero B, as long as $\omega_c \ll T$.

The situation changes qualitatively in a strong magnetic field, $\omega_c\tau \gg 1$ and $\omega_c \gg T$. The particle experiences then within the time $t \sim T^{-1}$ multiple cyclotron returns to the region close to the starting point. The corresponding ballistic propagator satisfies the equation

$$\left[-i\omega + iv_{\mathrm{F}} q \cos\phi + \omega_c \frac{\partial}{\partial\phi} - \frac{1}{\tau} \frac{\partial^2}{\partial\phi^2} \right] \mathcal{D}(\omega, q; \phi, \phi') = 2\pi\delta(\phi - \phi'). \qquad (7)$$

The approximate solution of (7) at $\omega_c\tau \gg 1$ has the form

$$\mathcal{D}(\omega, q; \phi, \phi') = \exp\{-iqR_c(\sin\phi - \sin\phi')\}\left[\frac{\chi(\phi)\chi(\phi')}{Dq^2 - i\omega} + \sum_{n\neq 0}\frac{e^{in(\phi-\phi')}}{Dq^2 - i(\omega - n\omega_c) + n^2/\tau}\right], \tag{8}$$

where $\chi(\phi) = 1 - iqR_c\cos\phi/\omega_c\tau$ and $D \simeq R_c^2/2\tau$ in strong B. Since characteristic frequencies in (3) are $\omega \sim T \ll \omega_c$, it is sufficient to keep only the first term in square brackets in (8) to obtain the leading contribution. Then $\langle\mathcal{D}\rangle$ in (4) is given by

$$\langle\mathcal{D}\rangle = J_0^2(qR_c)/(Dq^2 - i\omega), \tag{9}$$

where $J_0(x)$ is the Bessel function. Furthermore, combining all four terms in (5), we get

$$B_{xx}(\omega, q) = \frac{J_0^2(qR_c)}{(\omega_c\tau)^2}\frac{D\tau q^2}{(Dq^2 - i\omega)^3}. \tag{10}$$

Note that Eqs. (9) and (10) differ from those obtained in the diffusive regime by the factor $J_0^2(qR_c)$ only. This is related to the fact that the motion of the guiding center is diffusive even on the ballistic time scale $t \ll \tau$ (provided $t \gg \omega_c^{-1}$), while the additional factor corresponds to the averaging over the cyclotron orbit.

Substituting (10) into (3), and rescaling the momentum $q \to qR_c \equiv z$, we see that all the B dependence drops out from $\delta\sigma_{xx}$, and the exchange contribution in the case of pointlike interaction reads

$$\delta\sigma_{xx} = -(e^2/2\pi^2)\nu V_0 G_0(T\tau), \tag{11}$$

$$G_0(x) = \pi^2 x^2 \int_0^\infty \frac{du\,\exp(-u)}{u^3\sinh^2(\pi x/u)}[I_0(u)(1-u) + uI_1(u)].$$

The Hartree term in this case is of the opposite sign and twice larger due to the spin summation (we neglect the Zeeman splitting). Since the relative correction to the Hall conductivity turns out to be smaller by the factor $\sim(\omega_c\tau)^{-2}$ compared to (11), $\delta\sigma_{xy}/\sigma_{xy} \ll \delta\sigma_{xx}/\sigma_{xx}$, the MR is given by $\delta\rho_{xx}/\rho_0 = (\omega_c\tau)^2\delta\sigma_{xx}/\sigma_0$. The MR is thus quadratic in ω_c, with the temperature dependence determined by the function $G_0(T\tau)$, which is shown in Fig. 2(a). It has the asymptotics $G_0(x) \simeq -\ln x + \text{const}$ at $x \ll 1$ (diffusive regime) and $G_0(x) \simeq c_0 x^{-1/2}$ with $c_0 = 3\zeta(3/2)/16\sqrt{\pi} \simeq 0.276$ at $x \gg 1$ (ballistic regime). Let us note that the crossover between the two limits takes place at numerically small values $T\tau \sim 0.1$.

For the case of the Coulomb interaction the result turns out to be qualitatively similar. Substituting (4), (9), and (10) into (3), we get the exchange (Fock) contribution

$$\begin{aligned}\frac{\delta\rho_{xx}^{\mathrm{F}}(B)}{\rho_0} &= -\frac{(\omega_c\tau)^2}{\pi k_F l}G_{\mathrm{F}}(T\tau),\\ G_{\mathrm{F}}(x) &= 32\pi^2x^2\int_0^\infty dz z^3 J_0^2(z)\mathcal{G}_{1,3,2}(z),\\ \mathcal{G}_{jkl}(z) &= \sum_{n=1}^\infty \frac{n\{12\pi x n[1-J_0^2(z)] + [3 - jJ_0^2(z)]z^2\}}{(4\pi x n + z^2)^k\{4\pi x n[1-J_0^2(z)] + z^2\}^l},\end{aligned} \tag{12}$$

with $G_{\mathrm{F}}(x \ll 1) \simeq -\ln x + \text{const}$ and $G_{\mathrm{F}}(x \gg 1) \simeq (c_0/2)x^{-1/2}$; see Fig. 2(b).

We turn now to the Hartree term, assuming first $\kappa \ll k_F$. The expression for its triplet part is analogous to (3) with the replacement of $U(\omega, \mathbf{q})$ by $-\frac{3}{2}U(0, 2k_F\sin\frac{\phi-\phi'}{2})$, where ϕ and ϕ' are starting and final angles of the electron velocity. As to the singlet part, it is renormalized by mixing with the exchange term, yielding

$$U(\omega, q) \to \frac{\langle U(0, 2k_F\sin\frac{\phi-\phi'}{2})\rangle - U(0, 2k_F\sin\frac{\phi-\phi'}{2})}{2[1 + i\omega\langle\mathcal{D}(\omega, q)\rangle]^2}.$$

After the angle integration, $J_0^2(z)$ in (10) is replaced by $-(3y/2\pi)\int_0^\pi d\phi J_0(2z\sin\phi)/(y + 2\sin\phi)$ for the triplet, and by $\mathcal{J}(y, z) = -(y/2\pi)\int_0^\pi d\phi[J_0(2z\sin\phi) - J_0^2(z)]/(y + 2\sin\phi)$ for the singlet term ($y = \kappa/k_F$). This yields for the total Hartree contribution

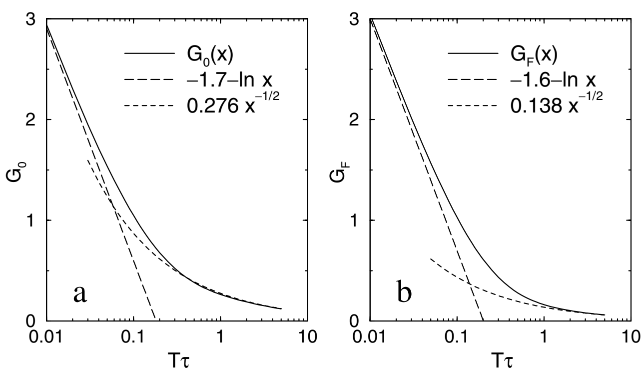

FIG. 2. Functions $G_0(T\tau)$ (a) and $G_F(T\tau)$ (b) determining the T dependence of the exchange term for pointlike, Eq. (11), and Coulomb, Eq. (12), interaction, respectively.

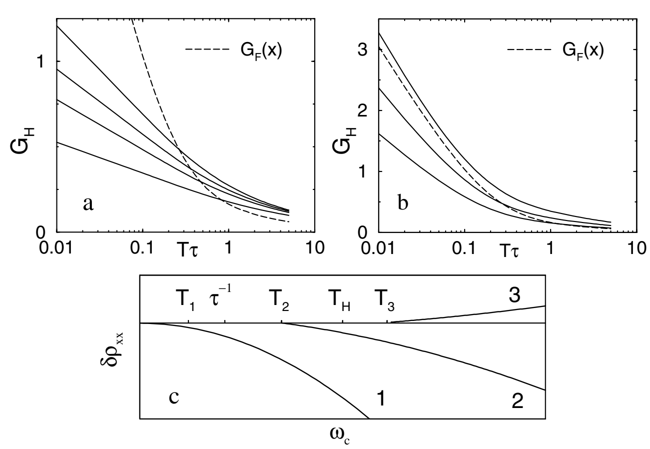

FIG. 3. Hartree contribution, $G_{\mathrm{H}}(T\tau)$, for (a) weak interaction, $\kappa/k_F = 0.1, 0.2, 0.3, 0.5$, and (b) strong interaction, $F_0 = -0.3, -0.4, -0.5$ (from bottom to top); (c) schematic plot of MR $\delta\rho_{xx}(B)$ in different temperature regimes: (1) $T_1 \ll \tau^{-1}$; (2) $\tau^{-1} \ll T_2 \ll T_{\mathrm{H}}$; (3) $T_3 \gg T_{\mathrm{H}}$.

$$\frac{\delta\rho_{xx}^{\mathrm{H}}(B)}{\rho_0} = \frac{(\omega_c\tau)^2}{\pi k_F l}[G_{\mathrm{H}}^{\mathrm{s}}(T\tau, y) + 3G_{\mathrm{H}}^{\mathrm{t}}(T\tau, y)] \simeq \frac{(\omega_c\tau)^2}{\pi^2 k_F l}\begin{cases} y\ln y[\frac{3}{4}\ln(T\tau) + \ln y], & T\tau \ll 1, \\ y\ln^2[y(T\tau)^{1/2}], & 1 \ll T\tau \ll y^{-2}, \\ \pi c_0(T\tau)^{-1/2}, & T\tau \gg y^{-2}, \end{cases}$$
$$G_{\mathrm{H}}^{\mathrm{s}}(x, y) = 32\pi^2 x^2 \int_0^\infty dz z^3 J(y, z)\mathcal{G}_{2,2,3}(z), \tag{13}$$
$$G_{\mathrm{H}}^{\mathrm{t}}(x, y) = \frac{\pi x^2}{4}\int_0^\infty \frac{du}{u^3 \sinh^2(\pi x/u)}\int_0^\pi d\phi \frac{y}{y + 2\sin\phi}\exp[-2u\sin^2\phi](1 - 2u\sin^2\phi).$$

We see that at $\kappa/k_F \ll 1$ a new energy scale $T_{\mathrm{H}} \sim \tau^{-1}(k_F/\kappa)^2$ arises where the MR changes sign. Specifically, at $T \ll T_{\mathrm{H}}$ the MR, $\delta\rho_{xx} = \delta\rho_{xx}^{\mathrm{F}} + \delta\rho_{xx}^{\mathrm{H}}$, is dominated by the exchange term and is therefore negative, while at $T \gg T_{\mathrm{H}}$ the interaction becomes effectively pointlike and the Hartree term wins, $\delta\rho_{xx}^{\mathrm{H}} = -2\delta\rho_{xx}^{\mathrm{F}}$, leading to a positive MR with the same $(T\tau)^{-1/2}$ temperature dependence; see Figs. 3(a) and 3(c).

If κ/k_F is not small, the exchange contribution (12) remains unchanged, while the Hartree term is subject to strong Fermi-liquid renormalization [1,7] and is determined by angular harmonics $F_m^{\sigma,\rho}$ of the Fermi-liquid interaction $F^{\sigma,\rho}(\theta)$. The formula for arbitrary $T\tau$ becomes then rather cumbersome [13]; here we restrict ourselves to a discussion of limiting cases. In the diffusive regime, $T \ll 1/\tau$, we reproduce the known result [1,7] $G_{\mathrm{H}}(T\tau) = 3[1 - \ln(1 + F_0^\sigma)/F_0^\sigma]\ln T\tau$. In the ballistic limit, $T \gg 1/\tau$, we find for the Hartree contribution

$$G_{\mathrm{H}}(T\tau) = -\frac{c_0}{2}\left[\sum_{m\neq 0}\frac{F_m^\rho}{1 + F_m^\rho} + 3\sum_m \frac{F_m^\sigma}{1 + F_m^\sigma}\right]\frac{1}{\sqrt{T\tau}}.$$

Finally, within a frequently used approximation neglecting all F_m with $m \neq 0$, the Hartree term takes the form of Eq. (12) with an additional overall factor of 3 and with $J_0^2(z)$ multiplied by $F_0^\sigma/(1 + F_0^\sigma)$ everywhere; the result is shown in Fig. 3(b) for several values of F_0^σ.

In summary, we have derived a general formula for the interaction-induced quantum correction $\delta\sigma_{\alpha\beta}$ to the conductivity tensor of 2D electrons valid for arbitrary temperature, magnetic field, and disorder range. It expresses $\delta\sigma_{\alpha\beta}$ in terms of classical propagators in random potential (ballistic diffusons). Applying this formalism, we have calculated the interaction contribution to the MR in strong B in a system with smooth disorder. We have shown that the parabolic MR found earlier in the diffusive limit $T\tau \ll 1$ persists in the ballistic regime $T\tau \gtrsim 1$, where it scales as $T^{-1/2}$. Further applications of our formalism [13] include the model of mixed disorder (smooth random potential with rare short-range scatterers) [3], periodically modulated systems, and frequency-dependent MR.

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