Nonlinear Theory of Void Formation in Colloidal Plasmas

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A nonlinear time-dependent model for void formation in colloidal plasmas is proposed. For experimentally relevant initial conditions, the model describes the nonlinear evolution of a zero-frequency linear instability that grows rapidly in the nonlinear regime and subsequently saturates to form a void. A number of features of the model are consistent with experimental observations under laboratory and microgravity conditions.

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A colloidal (or dusty) plasma is an electron-ion plasma containing a dispersed phase of micron-size dust particles. In typical plasma conditions, these particles usually acquire a large negative charge. As a result of the strong Coulomb coupling between the dust particles, a colloidal plasma may undergo phase transitions and exist in a liquid or a crystalline state.

Recently, a number of colloidal plasma experiments, in laboratory as well as under microgravity conditions, have shown the spontaneous development of voids [1–5]. A void is typically a small and stable centimeter-size region (within the plasma) that is completely free of dust particles and characterized by sharp boundaries. In the microgravity experiment [3], condensed states are produced with a liquidlike phase of dust adjacent to crystalline regions. The dynamics of void formation is not well understood and their spontaneous formation presents a significant impediment to the development of three-dimensional colloidal plasma crystals [3].

Voids develop not only in colloidal plasmas but also in colloidal polymer dispersions [6]. There also, the precise theoretical mechanism is not well understood although some researchers have suggested that there exists an attractive component in the potential between colloidal particles. This continues to be a subject of considerable research as well as controversy in the colloidal polymer community [7–10].

In the laboratory experiments involving colloidal plasmas [2], voids are seen to develop from a uniform dust cloud as a consequence of an instability when the dust particles have grown to a sufficiently large size. The instability is first seen as a filamentary mode, which exhibits a sudden onset. The spectrum of the filamentary mode is observed to be broadband, with a peak at about 100 Hz. After onset, the filaments are seen to evolve rapidly (in about 10 ms) to a nonlinear saturated state containing a void.

In order to account for their experimental results, Samsonov and Goree [2] (hereafter, SG) suggested that the ion drag force plays a crucial role in causing the initial instability, which can be described as follows. Imagine a local depletion of negatively charged dust particles within a spatially uniform dusty plasma. The depletion will produce a positive space charge with respect to the surrounding plasma, and, hence, an electric field that points outward from the region of reduced dust density. This electric field will cause an inward electrical force, F_e, on negatively charged dust particles that tends to restore the dust density to its equilibrium value, and an outward force, F_d, due to the ion drag (in the direction of the ion flow) that tends to expel dust particles from the region of depletion. If $F_d > F_e$, which occurs when dust particles have grown to a sufficient size, an instability grows, deepening the initial density depletion.

Theoretical analyses stimulated by SG fall into two types: linear stability analyses that include the effect of ion drag and other additional effects [11–16], and nonlinear but steady-state analyses that yield void solutions [17–20]. As yet, there is no nonlinear time-dependent model that describes the spontaneous development of the linear instability as well as its subsequent saturation to produce a void. A recent two-dimensional numerical simulation [21] attempts to make progress towards this objective but concludes that the simulation results cannot explain the appearance of the void in the microgravity experiment [3].

In this paper, we propose a basic, time-dependent, self-consistent nonlinear model for void formation in a dusty plasma. To the best of our knowledge, this is the first nonlinear time-dependent model that accounts for most significant observed features of the void instability caused by the ion drag, including the sudden onset of the nonlinear instability and the broadband frequency spectrum. An interesting feature of this model is that it does not invoke explicitly an attractive potential between dust particles which is sometimes used to explain voids in colloidal suspensions and plasma crystals [5].

Our basic fluid model for voids contains three elements: (a) an initial instability caused by the ion drag, (b) a nonlinear saturation mechanism for the instability, and (c) the void as one of the possible nonlinearly saturated states, dynamically accessible from the initially

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unstable equilibrium. For the initial instability, we choose a simple variant of the zero-frequency mode described by D'Angelo [11] which grows when $F_d > F_e$. The saturation mechanism we adopt is relevant for collisional voids where ions achieve near-thermal velocities in the void region. In this regime, Fokker-Planck theory shows that F_d initially increases with ion velocity v_i, attains a maximum for $v_i = v_{\rm thi}$, where $v_{\rm thi}$ is the ion thermal velocity, and decreases for $v_i > v_{\rm thi}$ [17]. As the linear instability grows, the ions are initially accelerated in the growing electric field, and F_d initially increases. Eventually, as the ions are accelerated to speeds larger than the ion thermal speed, F_d decreases to balance F_e and thus saturate the instability. (The reduction in F_d can also be brought about by nonlinearity in the ion mobility; however, we will not consider this mechanism here.) We demonstrate by analysis and numerical simulation that, in the saturated state, a stable void is formed.

We now describe the approximations and simplifications made in reducing the fluid equations for dust, electrons, and ions [11,13] to our model equations. The one-dimensional continuity and momentum equations for dust are given, respectively, by

$$\partial_t n_d = -\partial_x(n_d v_d) + D\partial_x^2 n_d, \tag{1}$$

and

$$m_d(\partial_t + v_d\partial_x)v_d = -ZeE + F_d - \nu_{dn}m_d v_d - (T_d/n_d)\partial_x n_d. \tag{2}$$

Here, n_d is the number density of dust particles of charge $-Ze$, mass m_d, and temperature T_d, v_d is the fluid velocity of dust, D is a particle diffusion coefficient for dust, E is the electric field, and the term $\nu_{dn}m_d v_d$ represents the frictional drag on dust grains by neutral atoms, where ν_{dn} is the dust-neutral collision frequency. We approximate the nonlinear ion drag force, F_d, by the expression $F_d = m_d\nu_{di}v_{\rm thi}u/(b+u^3)$, where $u \equiv v_i/v_{\rm thi}$, ν_{di} is the ion-dust collision frequency, and b is a positive constant. This nonlinear expression, with $b = 1.6$, fits well [22] the numerically calculated F_d from Fokker-Planck theory in the range $0.1 \le u \le 5$ (see Fig. 3 of [17]).

For electrons, each of charge $-e$, we neglect all inertial effects in their momentum equation and write $(T_e/n_e)\partial_x n_e = -eE$, where $n_e(T_e)$ is the electron density (temperature) [11,13]. The electric field E is determined by the self-consistent Poisson's equation $\partial_x E = 4\pi e(n_i - n_e - Zn_d)$, where $n_i(T_i)$ is the ion density (temperature). We neglect ion inertia and take the ion motion to be mobility limited, that is, $v_i = eE/(m_i\nu_{in})$, where ν_{in} is the ion-neutral collision frequency [2]. Under the conditions of void formation, ions respond on a time scale given by $x_v/v_{\rm thi} \sim 10\ \mu$s, where $x_v \sim 1$ cm while the dust dynamics occurs on the typical time scale $\omega_{pd}^{-1} \sim 1$ ms, where ω_{pd} is the dust plasma frequency. On the slow dust time scale, we assume that the ion density adjusts quickly and attains a quasisteady constant value. This assumption of constant n_i ensures that a constant supply of ions is maintained at all times to sustain a continuous ion wind in the void. In our model, an ionization mechanism is thus assumed to be present implicitly, but does not enter the governing equations explicitly.

With the approximations discussed above, our model equations (in dimensionless form) are

$$\partial_x E = 1 - \varepsilon_d - \varepsilon, \qquad (1/\varepsilon)\partial_x\varepsilon = -\tau_i^{-1}E, \tag{3}$$

$$\partial_t v_d = [-1 + a/(b+u^3)]E - \alpha_0 v_d - (\delta/\varepsilon_d)\partial_x\varepsilon_d, \tag{4}$$

$$u = \mu E, \tag{5}$$

$$\partial_t\varepsilon_d = -\partial_x(\varepsilon_d v_d) + D_0\partial_x^2\varepsilon_d, \tag{6}$$

where we have neglected the convective nonlinearity in the dust equation and defined $u = |\mathbf{u}|$, $\varepsilon_d \equiv Zn_d/n_{i0}$, $\varepsilon \equiv n_e/n_{io}$, $a \equiv m_d\nu_{di}/(m_i Z\nu_{in})$, $\alpha_0 \equiv \nu_{dn}/\omega_{pd}$, $\tau_{d,i} = T_{d,i}/T_e$, $\delta = \tau_d/Z$, $\mu = \omega_{pi}/(\nu_{in}\tau_i)$, and $D_0 = D\tau_i^2\omega_{pd}/\lambda_{di}^2$. In these equations, the distance x is normalized by λ_{di}/τ_i, where λ_{di} is the ion Debye length, time t is normalized by ω_{pd}^{-1}, the densities of dust and ions are normalized by the initial ion density n_{i0}, and the electric field E is normalized by the quantity $(T_e/e\lambda_{di})$. The last terms in (4) and (6) are due to dust pressure and diffusion, respectively.

We first carry out a simple equilibrium and stability analysis of Eqs. (3)–(6) without electrons. (This is done for analytical simplicity and physical insight, but is followed up by a numerical calculation that includes electrons.) Note that these equations are satisfied identically for a homogeneous field-free equilibrium with $E = 0$, $\varepsilon_d = 1$, and $v_d = 0$. Linearizing (3)–(6) about this equilibrium and assuming for simplicity that $D_0 = \alpha_0 = 0$, it is straightforward to show that this equilibrium is unstable to a zero-frequency (that is, purely growing) mode of wave number k when $a/b > 1 + \delta k^2$. The mode with the largest growth rate has $k = 0$. Since $a/b = F_d/F_e$, the instability condition for the fastest growing mode then reduces to $F_d > F_e$, which is exactly the condition discussed earlier in this paper. The linear instability saturates when F_d is reduced via the cubic nonlinearity in (4) to balance F_e. To investigate whether there is a steady-state solution containing a void that the unstable equilibrium may evolve to, we set $\partial/\partial t = 0$ in (3)–(6), and assume that a void extends from $x = 0$ to $x = x_v$, where $\varepsilon_d = 0$. Consequently, from (1), we obtain $E = x$ in the range $0 \le x < x_v$. In this region, we obtain the condition $F_d > F_e$, which causes the complete expulsion of dust particles. At the boundary $x = x_v$, determined by the relation $-1 + a/(b + \mu^3 x_v^3) = 0$, we obtain $F_d = F_e$ which continues to holds between $x = x_v$ and the dust cloud boundary $x = x_c$. In the range $x_v < x \le x_c$, we obtain $\varepsilon_d = 1$ and $E = (a-b)^{1/3}/\mu$. Although there are no dust particles in the void, by (4) there is a steady velocity profile given by $v_d = \alpha_0^{-1} x[-1 + a/(b + \mu^3 x^3)]$ which yields $v_d = 0$ at the two end points ($x = 0$ and $x = x_v$) of the void. Thus, we have obtained a steady nonlinear solution containing a void that the initially unstable equilibrium can evolve into. It is easy to show that this solution is linearly stable.

We now integrate (3)–(6) numerically in the range $0 \le x \le x_c$ to demonstrate the growth of the linear instability from an initially unstable equilibrium and its evolution to form a saturated void with the attributes discussed above. Experimental observations show that, during the formation of the void, particles continue to escape from the cloud, so we assume that the dust cloud boundary $x = x_c$ is open (that is, $\partial_x n_d = 0$ at $x = x_c$). It is also assumed, as in the microgravity experiment [3], that the solutions are symmetric around $x = 0$, where $v_d(0, t) = E(0, t) = 0$ for all t. The equations are evolved from a homogeneous field-free equilibrium with $E = 0$, $\varepsilon_d = 0.001$, $\varepsilon = 0.999$, and $v_d = 0$, with the parameters $m_d/m_i = 5 \times 10^8$, $\tau_i = 0.125$, $\nu_{in} = 5 \times 10^6\ \mathrm{s}^{-1}$, $\nu_{dn} = 6 \times 10^3\ \mathrm{s}^{-1}$, and $\omega_{pd} = 3 \times 10^3\ \mathrm{s}^{-1}$. For these parameters, which are relevant for the microgravity experiment [3],

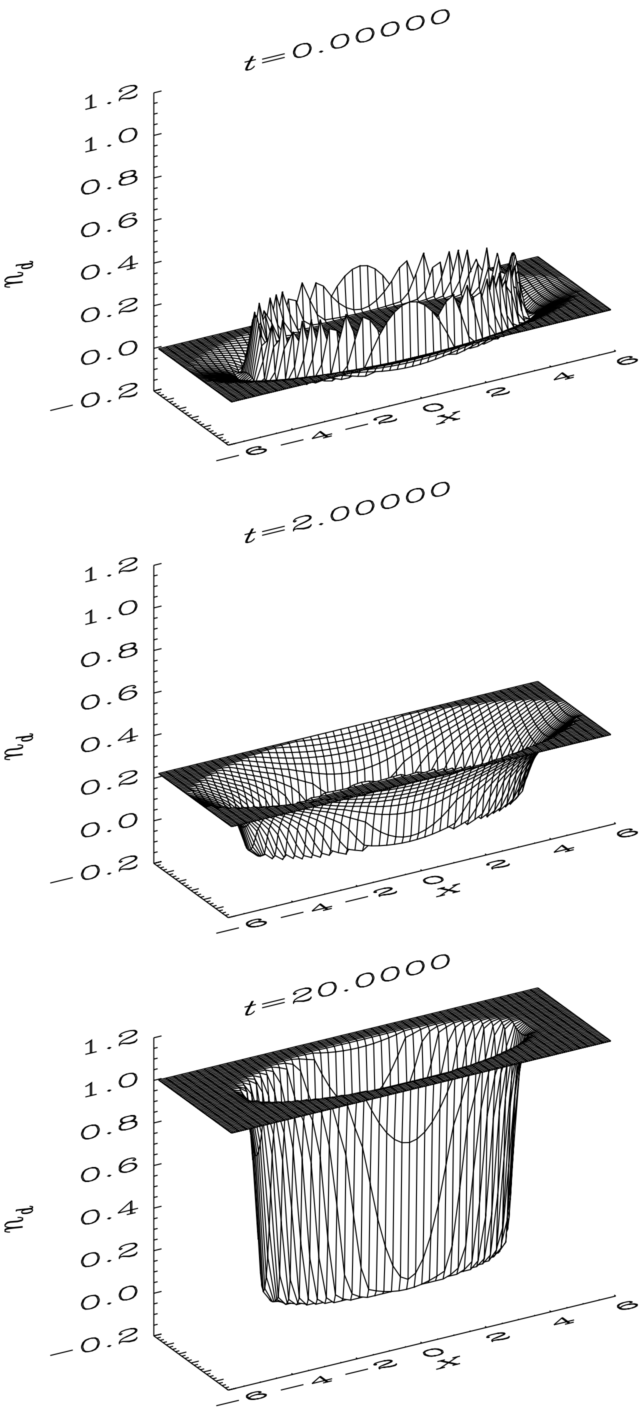

FIG. 1. The time evolution of the dust density at $t = 0$, 2, and 20. The three-dimensional solutions are obtained from the one-dimensional solution $n_d(x)$ by assuming rotational symmetry about the vertical axis. Note the sharp boundary of the void region at $t = 20$. The physical parameters are given in the text.

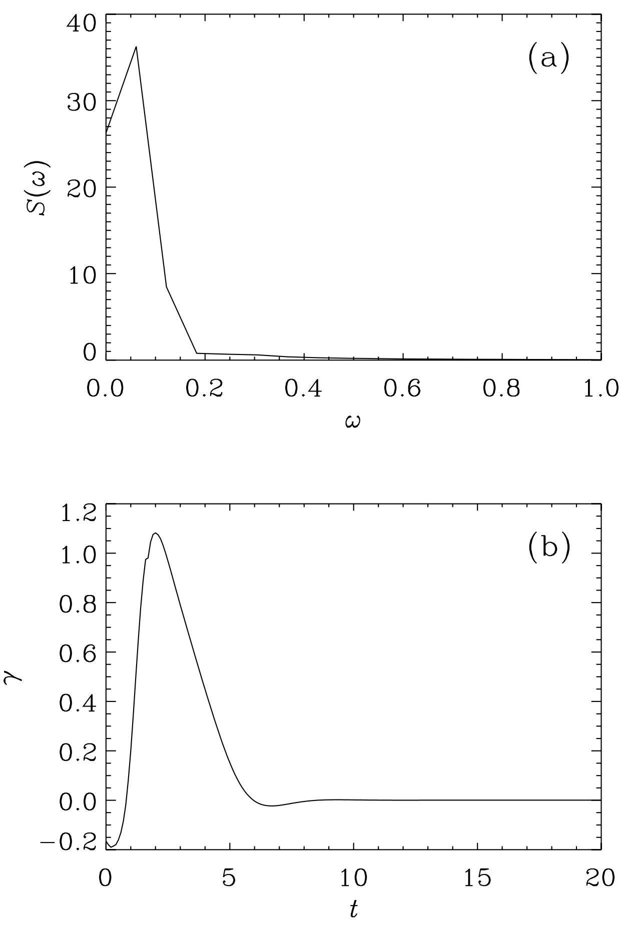

FIG. 2. (a) A typical frequency spectrum $S(\omega) \equiv |n_d(\omega)|^2$ of the dust density during the temporal evolution. (b) The nonlinear growth rate $\gamma \equiv |n_d|^{-2} \partial |n_d|^2 / \partial t$ of the instability, showing a short-lived linear growth phase, a much faster nonlinear growth phase, and a saturation phase. The physical parameters are the same as for Fig. 1.

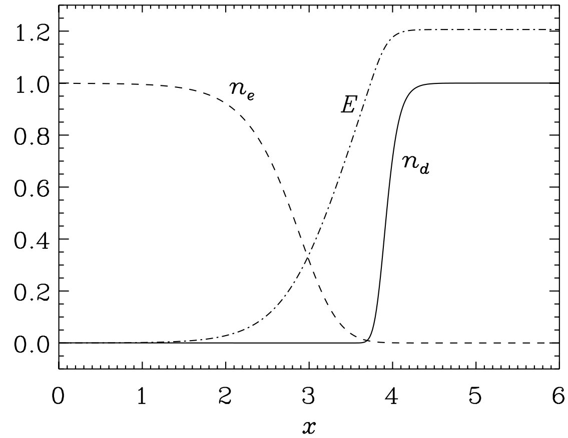

FIG. 3. Plots of the electron density (n_e), electric field (E), and dust density (n_d) in the saturated state of the instability. The physical parameters are the same as for Fig. 1.

we obtain $a = 7.5$, $b = 1.6$, $\alpha_0 = 2$, $\delta \approx 0.001$, $\mu = 1.5$, and $D_0 \approx 10^{-2}$. In Fig. 1, we show various stages of the evolution of the void density when the initial state consists of a uniform equilibrium dust density and a superposition of fluctuations. The three-dimensional plots of the dust density at $t = 0$, 2, and 20 are obtained from the one-dimensional solution by assuming rotational symmetry about the vertical axis. Note the sharp boundary of the void region at $t = 20$ when the system attains a steady-state. The particles originally contained in the void region are expelled in the early stages of the dynamics and escape from the cloud boundary at $x = x_c = 4$. In Fig. 2(a), we show a typical frequency spectrum $S(\omega) \equiv |n_d(\omega)|^2$ of the dust density during the temporal evolution of the instability. [Here $n_d(\omega)$ is the fast Fourier transform of $n_d(t)$.] The spectrum shows that the initially unstable zero-frequency modes evolve nonlinearly into a broadband of finite frequencies with a peak of around a few tens of Hz. This is roughly consistent with experimental observations of a broadband of modes with a maximum around 100 Hz [3]. The nonlinear growth rate of the instability, defined by the relation, $\gamma \equiv |n_d|^{-2}\partial|n_d|^2/\partial t$ and shown in Fig. 2(b), increases very rapidly by nearly an order of magnitude of its linear value before it reduces in the final stages of saturation. This is consistent with the near-explosive growth of the instability seen in the laboratory experiments [2]. We also find that the size of the void increases with ν_{in}, which is also consistent with the experimental observation that the void size increases with neutral gas pressure. In Fig. 3, we show the steady-state profiles of the electric field, the electron density, and the dust density. The electric field is low in the central region, acquires ambipolar values in the middle region, and rises to large values near the void boundary where it balances the large ion drag force.

In summary, we have presented a basic nonlinear time-dependent model that describes the evolution of a zero-frequency linear instability that grows faster than exponential in the nonlinear regime and subsequently saturates to form a void. This model can provide the foundation for more complete analyses in two dimensions, including a more detailed account of ion dynamics, ionization physics, and effects such as charge variation and the thermophoretic force.

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