Lever-Assisted Two-Noise Stochastic Resonance

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A critical interplay of two correlated noises in a nonlinear symmetrical two-well potential system is experimentally demonstrated. One state can become completely noise free, leading to an infinite Kramers time. If an independent lever breaks the potential symmetry, stochastic resonance is recovered. In this new regime, we obtain a *plateau*, i.e., a high signal-to-noise ratio even for vanishing forcing signals.

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Stochastic resonance has become a subject of considerable interest due to its potential technological and biological applications for optimizing the transmission of information through nonlinear dynamical systems [1–3]. Considering a two-state system submitted to both a noise and a modulation signal below threshold, the signature of stochastic resonance is the existence of a maximum in the signal-to-noise ratio for a nonzero value of noise, leading to the famous bell-shaped curve. For a given symmetric barrier, the optimal response to additive noise is obtained when the noise intensity is equal to half the barrier height. Unfortunately, the amplitude of the adiabatic forcing signal must remain close to the barrier height. Indeed, when the signal amplitude is lowered far below threshold, the signal-to-noise ratio collapses [4]. For this reason, the range of application of this interesting phenomenon in real systems remains somewhat limited. Recent theoretical work on the conjunction of two or more correlated or uncorrelated noises in nonlinear dynamical systems has predicted various effects such as the suppression of noise by noise [5], the appearance of noise-induced currents due to noise correlation [6], and of so-called reentrance phenomena due to noise-induced changes in the potential shape [7]. It has also been shown that time-modulated correlated noises could widen the bell-shape response [8]. The effect of symmetry-breaking on the hypersensitivity of stochastic resonance to weak signals has also been discussed, e.g., in [9,10]. One may wonder whether it is possible to take advantage of different noises in a two-well symmetric or asymmetric system in order to preserve a high signal-to-noise ratio for deeply subthreshold signals, i.e., to overcome the fundamental limitation of stochastic resonance. The aim of this Letter is consequently to address this question and to investigate experimentally the interplay of correlated noises in a two-well system driven by vanishingly small signals. In this respect, we need a two-state system which fulfills the following three conditions: (i) The system can be simultaneously subjected to two noises of different nature; (ii) it should be possible to correlate these noises; and (iii) there should be an independent lever for adjusting the asymmetry of the potential.

Let us consider a nonlinear rotator model, which can be applied to mechanical, molecular, or optical systems such as the one sketched in Fig. 1(a). Here, in a quasi-isotropic laser where the light vector **E** can *rotate* between two stable states, the angle θ made by **E** obeys the following nonlinear Langevin equation [11], modified here to include two noises:

$$\frac{d\theta}{dt} = -M_\phi \sin 4\theta + [A_0 \cos(\Omega t) + \zeta(t)]\sin 2\theta + \xi(t) + M_\ell \sin 2\theta, \quad (1)$$

where M_ϕ, A_0, and M_ℓ are constant rates. On the right-hand side of Eq. (1), the first term defines the two wells

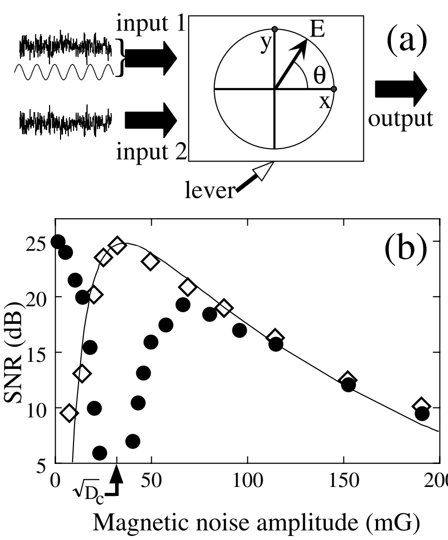

FIG. 1. (a) Sketch of the lever-assisted two-noise stochastic resonance system. **E** rotates between the two stable states x and y. (b) Experimental signal-to-noise ratio (SNR) vs rms magnetic noise amplitude with magnetic noise only (diamonds), and with correlated optical and magnetic noises (circles). Solid line: curve fit using standard $\mathrm{SNR} = (C/D^2)\exp(-U_0/D)$, with D: noise intensity, $C = 1.45 \times 10^{10}$ mG^4, and $U_0 = 5000$ mG^2. Horizontal offset of 14 mG has been added to account for residual noises. $\sqrt{D_c}$: critical magnetic noise amplitude.

located at $\theta \equiv 0 \pmod{\pi}$ and $\theta \equiv \pi/2 \pmod{\pi}$, which correspond to linearly polarized eigenstates along the x and y directions, respectively. The second term models a time-dependent differential perturbation on the two states, schematized as *input* 1 in Fig. 1(a), including the modulation signal ($\Omega/2\pi$ is the modulation frequency) and a first optical noise $\zeta(t)$. The third term is the second magnetic noise $\xi(t)$ coming at *input* 2, which can be correlated to the first noise. The last term introduced here models an independent lever, which allows us to introduce an asymmetry between the two states. Finally, the output signal corresponds to the flipping from one potential well to the other, i.e., from one stable state to the other. Consider the rotator sketched in Fig. 1(a), i.e., a bistable laser oscillating on either of the two x or y orthogonally polarized eigenstates [12], with a linear phase anisotropy of 0.5°. The vectorial optical modulation and noise are applied to the laser by means of a weak reinjection of the emitted laser light (about 1%). This reinjected light is linearly polarized and its amplitude is controlled by a $LiNbO_3$ modulator. If we apply only a subthreshold optical modulation and an axial magnetic noise [two terms in Eq. (1)], the response exhibits a stochastic resonance, as shown in Fig. 1(b). This is the typical bell shape of a one-noise stochastic resonance. It compares well with a fit derived from standard stochastic resonance theory [13] , in which a single Kramers rate of the form $r_k \propto \exp(-U_0/D)$ has been introduced [14,15]. In this expression, U_0 is the height of the potential barriers located at $\theta \equiv \pi/4 \pmod{\pi}$ or at $\theta \equiv 3\pi/4 \pmod{\pi}$, and D is the noise intensity. It is worth mentioning here that the mean residence times [2] in both states are equal.

In order to explore the interplay of two noises in such a stochastic resonance response, the output of our noise generator (100 kHz bandwidth) is divided into two parts: one is the axial magnetic rotation noise $\xi(t)$, while the other is an optical noise $\zeta(t)$ superimposed on the signal. The correlation between these two-noise sources allows us to modify Eq. (1) by defining a Langevin term which takes into account both noises [16,17]. If $\zeta(t)$ and $\xi(t)$ are Gaussian white noises with intensities Q and D, respectively, defined by $\langle\zeta(t)\rangle = \langle\xi(t)\rangle = 0$, $\langle\zeta(t)\zeta(t')\rangle = 2Q\delta(t-t')$, and $\langle\xi(t)\xi(t')\rangle = 2D\delta(t-t')$, and if $\langle\zeta(t)\xi(t')\rangle = \langle\xi(t)\zeta(t')\rangle = 2\lambda\sqrt{DQ}\delta(t-t')$, where λ is the strength of correlation between the two noises, then the vector rotation is described by

$$\frac{d\theta}{dt} = f(\theta, t) + (\sqrt{Q}\sin 2\theta + \lambda\sqrt{D})\chi(t), \tag{2}$$

with $f(\theta, t) = -M_\phi \sin 4\theta + [A_0 \cos(\Omega t) + M_\ell]\sin 2\theta$, $\langle\chi(t)\rangle = 0$, and $\langle\chi(t)\chi(t')\rangle = \delta(t-t')$. $\lambda = \pm 1$ can be chosen experimentally via the sense of the noisy magnetic field. Let us take $\lambda = +1$ in the following. With these two noises, one can wonder how the average residence times in one state or the other evolve. To evaluate these average residence times, we derive the Fokker-Planck equation associated with Eq. (2) and calculate the noise-modified effective potential of the system. We then use a standard formula [18] for the mean first passage times T_+ and T_- to make the transition from x to y, and from y to x, respectively. In order to isolate only the interplay of the two noises, we choose $M_\ell = 0$. Then, in the weak noise limit and with $A_0 \ll M_\phi$, the expressions for a rotator subjected to two noises are given by

$$T_\pm = T_0 \exp[\Delta\Phi_\pm(Q, D)], \tag{3}$$

where T_0 is a constant, and

$$\Delta\Phi_\pm(Q, D) = \frac{2M_\phi}{Q}\left\{\ln\left(\frac{|\sqrt{Q} \pm \sqrt{D}|}{\sqrt{D}}\right) + \frac{\sqrt{Q}}{|\sqrt{Q} \pm \sqrt{D}|}\right\} - 2\ln|\sqrt{Q} \pm \sqrt{D}|. \tag{4}$$

We are thus left with a symmetry-breaking effect which results from the correlation between the two noises. Namely, if $\sqrt{D} = \sqrt{Q}$, T_- diverges to infinity. In the case where $\lambda = -1$, T_+ diverges at the same noise value $\sqrt{D} = \sqrt{Q}$. To verify this prediction on our rotator, we prepare the system at the top of the one-noise stochastic resonance curve: The laser modulation (at a chosen angular frequency $\Omega = 2\pi \times 1$ kHz) is kept just below threshold and the optical noise amplitude $\sqrt{Q}$ is tuned to the optimal point. The average residence times in the two states are equal in this case, as shown by the inset on the lower left-hand side of Fig. 2. We then increase the correlated magnetic noise amplitude $\sqrt{D}$. The measured value of the correlation strength is $\lambda = 0.995$. The experimental response, as depicted in Fig. 2, is in good agreement with the predictions of Eqs. (3) and (4): We observe that the symmetry of the optical gates is broken, as shown in the insets of Fig. 2. In particular, a critical

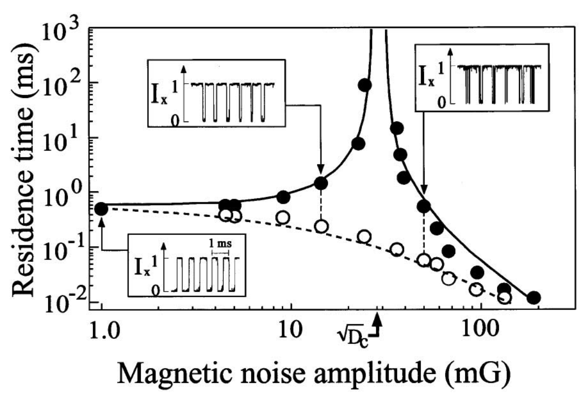

FIG. 2. Experimental average residence time in state x (filled circles) and in state y (open circles) vs magnetic noise amplitude. The optical noise amplitude is chosen to optimize stochastic resonance. Lines are curve fits using Eqs. (3) and (4) with $T_0 = 314$ ms, $M_\phi = 50$ mG^2, and $Q = 29.5$ mG^2. Insets are temporal recordings of the gates at the output of the system.

value of magnetic noise is found, for which the residence time in one state becomes infinite. The corresponding signal-to-noise ratio is shown in Fig. 1(b). Unfortunately, at the critical value (called $\sqrt{D_c}$), the signal-to-noise ratio collapses. Physically, the system does not respond any longer because one state becomes completely noise free. One can, hence, wonder how a good signal-to-noise ratio may be recovered, while keeping a noise-free state.

We know from Eq. (1) that the lever term may skew the system. Experimentally, indeed, M_ℓ can be modified by tuning the laser frequency slightly off resonance, hence inducing a differential gain on the two stable states. By adjusting this internal degree of freedom, one can recover the optical gates at the output of the system as shown in Fig. 3. Note that the gates are symmetrical again, in contrast to the gates obtained without the lever that are depicted in the insets of Fig. 2. Remarkably, once the noise-induced asymmetry has been compensated for by the lever, the destructive interference of the two noises in one well is preserved. The noise correlation is fully exploited here: The fluctuations provoked in two different physical parameters interfere constructively in one well, and destructively in the other. Note that a small residual noise appears due to spurious uncorrelated noises in the system. Moreover, by changing the sign of the magnetic rotation noise (hence, the sign of λ), the noise-free well can be specified [compare Figs. 3(a) and 3(b)]. We have also verified that the system responds to signal frequencies up to 40 kHz, including aperiodic signals. These features can be understood by the fact that, when the system is skewed, flipping from the noise-free well to the other well is achieved through the modulation only, whereas the reverse flip is due to the enhanced noise. In this novel situation, one may now ask if the system could respond to modulation amplitudes well below threshold.

In the usual stochastic resonance, the signal-to-noise ratio decreases linearly with the modulation intensity [4]. Indeed, in our system, this behavior is observed experimentally when the optical noise is fixed at the maximum of the bell-shaped curve of Fig. 1, and when the modulation amplitude is decreased below threshold. This yields the squares depicted in Fig. 4. Here, the signal-to-noise ratio decreases from 25 to 5 dB when the normalized modulation amplitude is reduced from 1 to 0.15. At this lowest modulation amplitude, if we now add the correlated magnetic noise, the response is completely different. Indeed, setting this second noise to its critical value $\sqrt{D_c}$ [corresponding to $\sqrt{D} = \sqrt{Q}$ in Eqs. (3) and (4)] makes one state noise free. Adjustment of the compensating lever then allows us to recover the 25 dB value for the signal-to-noise ratio. In this case, the second correlated noise and the lever are optimized for the lowest possible signal amplitude, and are then left at their optimal values. For all higher values of the input modulation, the signal-to-noise ratio stays at the same 25 dB level. Remarkably, the signal-to-noise ratio now exhibits a *plateau* over all this signal amplitude range: The system response becomes independent of the forcing signal level, widening the potentialities of stochastic resonance.

In conclusion, we have shown that the critical interplay of two correlated noises of different nature in a two-well system leads to new dynamical behaviors. The noise correlation allows the noise from one well to be completely washed out, at the expense of enhanced noise in the other well, leading to different Kramers times. In such a critical regime, with the help of an independent lever, we can adjust the asymmetry of the potential in order to restore the response of the system. The striking and novel consequence of this is that the system remains sensitive even to vanishing input signals. Indeed, we have obtained a plateau in the signal-to-noise ratio when the modulation amplitude is decreased far below threshold.

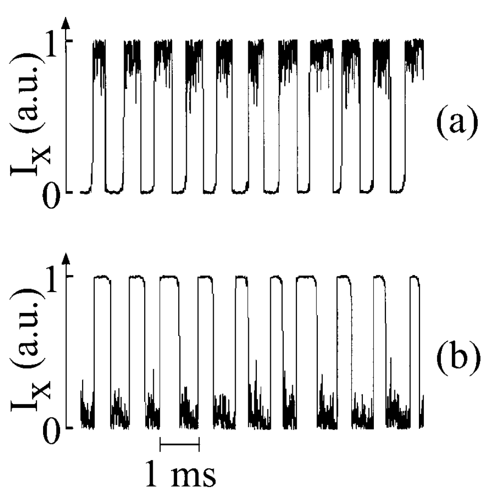

FIG. 3. Experimental recovery of symmetric optical gates observed through a linear polarizer aligned along x after careful adjustment of the lever. The magnetic noise is tuned to the critical value with (a) $\lambda = +1$ and (b) $\lambda = -1$. Note that in each case one state is noise free.

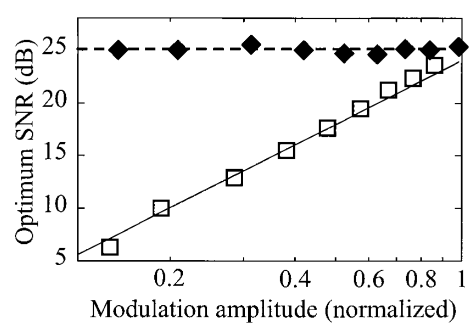

FIG. 4. Optimum signal-to-noise ratio in the usual one-noise stochastic resonance (squares) and in the lever-assisted two-noise stochastic resonance, i.e., the plateau (diamonds). Solid line: curve fit using SNR $= 20 \log A_0 + B$, with A_0: modulation amplitude normalized to the threshold value, and $B = 23.6$ dB.

Moreover, the fact that the output gates are noise free in a chosen state could modify the usual extinction ratio and may be useful for telecommunication systems [19]. Furthermore, this behavior may appear in other two-well systems subjected to small periodic or aperiodic forcing signals. Finally, this lever-assisted two-noise stochastic resonance may be of use in multiple-well systems such as rf SQUID loops [20], one- or two-dimensional ratchets [21,22], or optical lattices [23].

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