

# Weak Phase $\gamma$ Using Isospin Analysis and Time-Dependent Asymmetry in $B_d \rightarrow K_S \pi^+ \pi^-$

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We present a method for measuring the weak phase  $\gamma$  using isospin analysis of three body  $B$  decays into  $K\pi\pi$  channels. Differential decay widths and time-dependent asymmetry in  $B_d \rightarrow K_S \pi^+ \pi^-$  mode needs to be measured into even isospin  $\pi\pi$  states. The method can be used to extract  $\gamma$ , as well as the size of the electroweak penguin contributions. The technique is free from assumptions like SU(3) or neglect of any contributions to the decay amplitudes. By studying different regions of the Dalitz plot, it may be possible to reduce the ambiguity in the value of  $\gamma$ .

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Time-dependent measurements of asymmetries of decay modes of  $B_d$  into  $CP$  eigenstates [1] are very useful in determining angles of the unitarity triangle. This technique is particularly significant, since weak phases can be extracted without any theoretical uncertainty from modes whose amplitudes have a single weak phase. The time-dependent  $CP$  asymmetry in the golden mode  $B_d \rightarrow J/\psi K_S$  thus yields information on  $\sin 2\beta$ . This method has proved successful in the measurement of  $\sin 2\beta = 0.78 \pm 0.08$  [2], which is in good agreement [3] with theoretical estimates.

Measurement of other angles using modes such as  $B_d \rightarrow \pi^+ \pi^-$  are beset with theoretical uncertainties because the amplitude gets contributions from tree and penguin diagrams which have different dependence on weak phases. Nevertheless, the theory error can in principle be removed using the method of Gronau and London [4]. This method relies on the assumption of isospin invariance and the fact that the amplitude for  $B^+ \rightarrow \pi^+ \pi^0$  gets contributions only from the tree diagrams (barring a small contribution from the electroweak penguin). This method will lead to a measurement of  $\sin 2\alpha$ .

It is widely believed that  $\gamma$  cannot be measured using the time-dependent techniques developed to measure the phases  $\beta$  and  $\alpha$ . As an alternative, several other methods have been developed [5] to measure this weak phase. While  $\gamma$  can be measured cleanly using some of these techniques at a later date, techniques [6] assuming flavor SU(3) are expected to provide the first estimates of angle  $\gamma$ .

In this Letter, we propose a method to measure  $\gamma$ , which uses time-dependent asymmetry in the three body  $K\pi\pi$  decay mode of the  $B_d$ . Our technique is on almost as good a footing as the Gronau-London method and relies on construction of triangles based on isospin analysis. The extra ingredient that we use is that the tree and the electroweak penguin pieces of the weak

Hamiltonian responsible for  $\Delta I = 1$  transition have the same strong phase because of the operator structure of the interaction in the standard model (SM). However, the method is free from approximations such as SU(3) symmetry, neglect of annihilation or rescattering contributions. Further, our method is sensitive to the relative weak phase between the tree and penguin contribution and as such will probe new physics (NP). Recently, several three body noncharmed decay modes of the  $B$  meson have been observed. In particular the branching ratios of the modes  $B^0 \rightarrow K^0 \pi^+ \pi^-$  and  $B^0 \rightarrow K^+ \pi^- \pi^0$  have been measured [7,8] to be around  $5 \times 10^{-5}$ . In fact, even with limited statistics, a Dalitz plot analysis has been performed and quasi-two-body final states have been identified.

The three body decay modes such as  $B \rightarrow K\pi\pi$  provide valuable information that can pin down the phases in the SM. The importance of these modes was first pointed out by Lipkin, Nir, Quinn, and Snyder [9], however, their analysis did not incorporate the large electroweak penguin effects known to be present in these decays [10]. These decays are described by six independent isospin amplitudes  $A(I_i, I_{\pi\pi}, I_f)$ , where  $I_i$  stands for the transition isospin that describes the transformation of the weak Hamiltonian under isospin and can take the values 0 and 1 in the SM;  $I_{\pi\pi}$  is the isospin of the pion pair and can be 0, 1, and 2;  $I_f$  is the final isospin, which can be 1/2 or 3/2. Even values of  $I_{\pi\pi}$  have the pair of pions in a symmetric state and thus have even angular momenta. Similarly, states with  $I_{\pi\pi}$  odd must be odd under the exchange of two pions. A separation between  $I_{\pi\pi} = \text{even}$  and  $I_{\pi\pi} = \text{odd}$  should be possible through a study of the Dalitz plot.

We shall consider only the  $I_{\pi\pi} = 0$  and 2 channels in this paper, which are described by the three amplitudes  $A(0, 0, \frac{1}{2})$ ,  $A(1, 0, \frac{1}{2})$ , and  $A(1, 2, \frac{3}{2})$ . The amplitudes for the various decay modes with  $I_{\pi\pi} = \text{even}$  obey useful isospin relations. It is straightforward to derive [9]:

$$\begin{aligned}
A[B^{+(0)} \rightarrow K^{0(+)}(\pi^{+(0)}\pi^0)_e] &= \pm X, \\
A[B^{+(0)} \rightarrow K^{+(0)}(\pi^+\pi^-)_e] &= \mp \frac{1}{3}X \mp Y + Z, \\
A[B^{+(0)} \rightarrow K^{+(0)}(\pi^0\pi^0)_e] &= \mp \frac{2}{3}X \pm Y - Z,
\end{aligned} \quad (1)$$

where  $X = \sqrt{\frac{2}{3}}A(1, 2, \frac{3}{2})$ ,  $Y = \frac{1}{3}A(1, 0, \frac{1}{2})$ , and  $Z = \sqrt{\frac{1}{3}}A(0, 0, \frac{1}{2})$ . The subscript  $e$  represents the even isospin of the  $\pi\pi$  system. It is easy to see that Eq. (1) implies the following two isospin triangle relations:

$$\begin{aligned}
A[B^+ \rightarrow K^0(\pi^+\pi^0)_e] &= A[B^0 \rightarrow K^0(\pi^+\pi^-)_e] \\
&\quad + A[B^0 \rightarrow K^0(\pi^0\pi^0)_e], \quad (2)
\end{aligned}$$

$$\begin{aligned}
A[B^0 \rightarrow K^+(\pi^-\pi^0)_e] &= A[B^+ \rightarrow K^+(\pi^+\pi^-)_e] \\
&\quad + A[B^+ \rightarrow K^+(\pi^0\pi^0)_e] \quad (3)
\end{aligned}$$

and also implies the relation,

$$A[B^+ \rightarrow K^0(\pi^+\pi^0)_e] = -A[B^0 \rightarrow K^+(\pi^-\pi^0)_e]. \quad (4)$$

Decays corresponding to conjugate processes will obey similar relations. The isosceles triangle represented by Eq. (2) and its conjugate are the ones that interest us.

The decay  $B(p_B) \rightarrow K(k)\pi(p_1)\pi(p_2)$  (where  $p_B, k, p_1$ , and  $p_2$  are the four momentum of the  $B, K, \pi_1$ , and  $\pi_2$ , respectively) may be described in terms of the usual Mandelstam variables  $s = (p_1 + p_2)^2$ ,  $t = (k + p_1)^2$ , and  $u = (k + p_2)^2$ . States with  $I_{\pi\pi} = \text{even}$  must be symmetric under the exchange  $t \leftrightarrow u$ . In what follows we shall be concerned with differential decay rates  $d^2\Gamma/(dtdu)$ . These can be extracted from the Dalitz plot of the three body decays. A detailed angular analysis will permit extraction of even isospin  $\pi\pi$  events. Note that  $B \rightarrow K_S \pi^0 \pi^0$  mode being symmetric in pions always has pions in the isospin even state.

For simplicity we define the amplitudes  $A^{+-}, A^{00}$ , and  $A^{+0}$  corresponding to the modes  $B^0 \rightarrow K_S(\pi^+\pi^-)_e, B^0 \rightarrow K_S(\pi^0\pi^0)_e$ , and  $B^+ \rightarrow K_S(\pi^+\pi^0)_e$ , respectively. It may be understood that all observables, amplitudes, and strong phases depend on the two independent Mandelstam variables  $t$  and  $u$ , even though we suppress explicitly stating the  $t$  and  $u$  dependences. Using unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [11], we separate these amplitudes into contributions containing the  $V_{ub}$  and  $V_{cb}$  elements respectively:

$$A^{+-} = a^{+-} e^{i\delta_a^{+-}} e^{i\gamma} + b^{+-} e^{i\delta_b^{+-}}, \quad (5)$$

$$A^{00} = a^{00} e^{i\delta_a^{00}} e^{i\gamma} + b^{00} e^{i\delta_b^{00}}, \quad (6)$$

$$A^{+0} = a^{+0} e^{i\delta_a^{+0}} e^{i\gamma} + b^{+0} e^{i\delta_b^{+0}}. \quad (7)$$

Note that the magnitudes  $a^{+-}, b^{+-}, a^{00}, b^{00}, a^{+0}$ , and  $b^{+0}$  actually contain contributions from all possible diagrams (tree, color suppressed, annihilation, W exchange, penguin, penguin annihilation, and electroweak penguin) and include the magnitudes of the CKM elements.

Their explicit composition is irrelevant for this analysis, except for the fact that the isospin 3/2 amplitude  $A^{+0}$  cannot get contributions from gluonic penguins. The amplitudes  $\bar{A}^{+-}, \bar{A}^{00}$ , and  $\bar{A}^{+0}$ , corresponding to the conjugate process  $\bar{B} \rightarrow \bar{K} \pi\pi$  can be written similarly with the weak phase  $\gamma$  replaced by  $-\gamma$ . In the presence of two contributions to the amplitude as described in Eqs. (5) and (6), the direct asymmetry is nonvanishing. The time-dependent  $CP$  asymmetry for  $B^0(t) \rightarrow f$  then has the form,

$$\begin{aligned}
A_{CP}^f(t) &= \frac{\Gamma[\bar{B}^0(t) \rightarrow f] - \Gamma[B^0(t) \rightarrow f]}{\Gamma[\bar{B}^0(t) \rightarrow f] + \Gamma[B^0(t) \rightarrow f]}, \\
&= a_{\text{dir}}^f \cos(\Delta mt) + \frac{2\text{Im}(\lambda^f)}{1 + |\lambda^f|^2} \sin(\Delta mt), \quad (8)
\end{aligned}$$

where

$$a_{\text{dir}}^f = \frac{|\bar{A}^f|^2 - |A^f|^2}{|\bar{A}^f|^2 + |A^f|^2}, \quad \lambda_f = \frac{q\bar{A}^f}{pA^f}, \quad \text{and} \quad \frac{q}{p} = e^{-2i\beta}. \quad (9)$$

Figure 1 depicts the two triangles formed by the amplitudes  $A^{+-}, A^{00}$ , and  $A^{+0}$ , and the corresponding conjugate amplitudes in isospin space, along with the relative orientations.  $\zeta$  ( $\bar{\zeta}$ ) are defined as the angle between  $A^{+-}$  ( $\bar{A}^{+-}$ ) and  $A^{+0}$  ( $\bar{A}^{+0}$ ) and the angle  $2\tilde{\gamma}$  is the angle between  $A^{+0}$  and  $\bar{A}^{+0}$ . The relative phase between  $A^{+-}$  and  $\bar{A}^{+-}$  (i.e.,  $\arg[(A^{+-})^* \bar{A}^{+-}]$ ), defined as  $2\theta^{+-}$ , can be obtained from the coefficient of the  $\sin(\Delta mt)$  piece in the time-dependent  $CP$  asymmetry for the mode  $B^0(t) \rightarrow K_S(\pi^+\pi^-)_e$ :

$$\frac{2\text{Im}(\lambda^{+-})}{1 + |\lambda^{+-}|^2} = y^{+-} \sin(2\theta^{+-} - 2\beta), \quad (10)$$

where  $y^f$  is defined as  $y^f = \sqrt{1 - (a_{\text{dir}}^f)^2}$ . Note that this

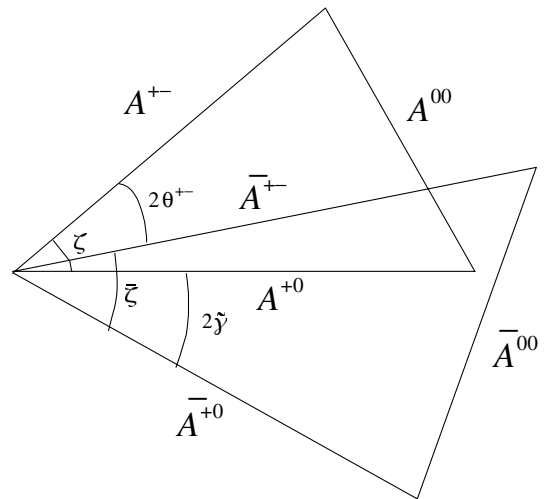


FIG. 1. The isospin triangles formed by the  $B \rightarrow K\pi\pi$  amplitudes, as represented in Eq. (2) and that for the corresponding conjugate processes. Only one orientation of the conjugate triangle is depicted; this triangle could have been flipped around the base  $\bar{A}^{+0}$ .

measurement involves time-dependent asymmetry in the partial decay rate  $d^2\Gamma^{+-}/dtdu$  at a fixed  $t$  and  $u$ . Again, events symmetric in  $t \leftrightarrow u$  need to be selected in the Dalitz plot.

With the knowledge of  $\beta$ , the angle  $2\theta^{+-}$  may be regarded as an observable. In addition, measurement of six partial decay rates  $d^2\Gamma^{+0}/dtdu$ ,  $d^2\Gamma^{+-}/dtdu$ , and  $d^2\Gamma^{00}/dtdu$ , as well as their conjugates at the same  $t$  and  $u$  as used for  $\theta^{+-}$  determination, now allows us to construct the two triangles in Fig. 1 with twofold ambiguity. We see, from Fig. 1, that the angle  $2\tilde{\gamma}$  is related to  $2\theta^{+-}$  as  $\zeta \pm \tilde{\zeta} + 2\tilde{\gamma} = 2\theta^{+-}$ . The ‘‘plus-minus’’ sign ambiguity in the above reflects the possibility of same-side or opposite-side orientation of the triangles. Once  $2\tilde{\gamma}$  is known, it is possible to determine  $\gamma$ . The crucial additional information necessary is the observation of Neubert and Rosner [12] that the electroweak penguin operators  $Q_9$  and  $Q_{10}$  are Fierz equivalent to the operators  $Q_1$  and  $Q_2$ . The isospin 3/2 amplitude  $A^{+0}$  is symmetric in the two pions ( $\pi^+\pi^0$ ). Hence, within the SM only the operator  $(Q_1 + Q_2)$  with coefficient  $\frac{1}{2}[\lambda_u(C_1 + C_2) - \frac{3}{2}\lambda_t(C_9 + C_{10})]$  contributes, while the operator  $(Q_1 - Q_2)$  does not. The amplitude  $A^{+0}$  thus has a common strong phase  $\delta = \delta_a^{+0} = \delta_b^{+0}$  arising from the same quark operator. This phase  $\delta$  may be set equal to zero by convention. Thus we have  $A^{+0} = (e^{i\gamma} - \delta_{EW})a^{+0}$  and  $A^{-0} = \bar{A}^{+0} = (e^{-i\gamma} - \delta_{EW})a^{+0}$ . Here [13]

$$\delta_{EW} = \frac{-b^{+0}}{a^{+0}} \approx -\frac{3}{2} \left| \frac{\lambda_t}{\lambda_u} \right| \frac{C_9 + C_{10}}{C_1 + C_2} = 0.66 \pm 0.15, \quad (11)$$

where  $\lambda_q = V_{qb}^* V_{qs}$ . The angle  $2\tilde{\gamma}$  is then given by

$$\tan\tilde{\gamma} = \frac{\sin\gamma}{\cos\gamma - \delta_{EW}}. \quad (12)$$

Since the angle  $\tilde{\gamma}$  is determined, it follows that angle  $\gamma$  is now calculable from Eq. (12).

It turns out that we can determine  $\gamma$  without having to use the theoretically computed value of  $\delta_{EW}$ , given by Eq. (11). As we will show below,  $\gamma$  can be determined cleanly by relying only on the Neubert-Rosner observation that the amplitude  $A^{+0}$  has a single common strong phase. We emphasize that the observation of a common strong phase  $\delta$  is based on very firm grounds within the frame work of the SM. It relies essentially, only on isospin and the operator structures contributing within the SM. An experimentally verifiable consequence of this hypothesis would be the vanishing of direct  $CP$ -violating asymmetry for the mode  $A^{+0} \equiv A[K^0(\pi^+\pi^0)_e]$ . If however, a sizable direct  $CP$ -violating asymmetry is observed it could imply either isospin violation or NP. NP could result in enhancement of operators negligible within the SM or additional new operators. Isospin violation can be tested by the comparison of the Dalitz plot for  $B^+ \rightarrow K^0(\pi^+\pi^0)_e$  and  $B^0 \rightarrow K^+(\pi^-\pi^0)_e$  [see Eq. (4)]. In the absence of such isospin violating signals, sizable direct  $CP$ -violating asymmetry in the  $A^{+0}$  amplitude would signal NP.

Using the amplitudes  $A^{+-}$ ,  $\bar{A}^{+-}$ ,  $A^{00}$ , and  $\bar{A}^{00}$  one can construct a maximum of seven independent observables (the amplitudes  $A^{+0}$ ,  $A^{-0}$  are not independent as they can be obtained using isospin relations). The two triangles can be completely defined in terms of seven observables: the three sides of each of the triangles and a relative angle between the two triangles. The amplitudes under consideration involve the following 11 variables:  $a^{+-}$ ,  $b^{+-}$ ,  $a^{00}$ ,  $b^{00}$ ,  $a^{+0}$ ,  $b^{+0}$ ,  $\delta_a^{+-}$ ,  $\delta_b^{+-}$ ,  $\delta_a^{00}$ ,  $\delta_b^{00}$ , and  $\gamma$ . These variables are connected by two isospin relations [see Eq. (2) and the corresponding relation for the conjugate process], which results in four constraints, reducing the number of independent variables to seven, as we will illustrate below. Hence, all variables including  $\gamma$  can be determined purely in terms of observables.

In order to determine  $\gamma$ , we express all the amplitudes and strong phases, in terms of observables and  $\gamma$ . The variables  $a^{+-}$  and  $b^{+-}$  may be solved as a function of  $\gamma$  and other observables as follows:

$$|a^{+-}|^2 = \frac{B^{+-}}{2\sin^2\gamma} [1 - y^{+-} \cos(2\theta^{+-})], \quad (13)$$

$$|b^{+-}|^2 = \frac{B^{+-}}{2\sin^2\gamma} [1 - y^{+-} \cos(2\theta^{+-} - 2\gamma)]. \quad (14)$$

Similar solutions may be obtained for  $a^{+0}$  ( $a^{00}$ ) and  $b^{+0}$  ( $b^{00}$ ) with  $B^{+-}$  replaced by  $B^{+0}$  ( $B^{00}$ ) and  $2\theta^{+-}$  replaced by  $2\tilde{\gamma}$  ( $2\theta^{00}$ ), respectively. The branching ratio,  $B^{+-} = (|\bar{A}^{+-}|^2 + |A^{+-}|^2)/2$ , with similar relations for  $B^{00}$  and  $B^{+0}$ . The angle  $2\theta^{00}$  between  $A^{00}$  and  $\bar{A}^{00}$  need not be measured but can be determined from geometry of the two triangles and is given by

$$\cos(2\theta^{00} - 2\tilde{\gamma}) = \frac{B^{00} - B^{+-} + |A^{+-}| |\bar{A}^{+-}| \cos(2\theta^{+-} - 2\tilde{\gamma})}{|A^{00}| |\bar{A}^{00}|}.$$

We define  $\delta^{+-} = \delta_b^{+-} - \delta_a^{+-}$  and  $\delta^{00} = \delta_b^{00} - \delta_a^{00}$ , with  $\delta^{+-}$  expressed in terms of  $\gamma$  and observables as

$$\tan\delta^{+-} = \frac{a_{\text{dir}}^{+-} \tan\gamma}{1 - y^{+-} [\cos 2\theta^{+-} - \sin 2\theta^{+-} \tan\gamma]}, \quad (15)$$

with an analogous expression for  $\tan\delta^{00}$ . Our task now is to express the strong phases  $\delta_a^{+-}$  and  $\delta_a^{00}$  in terms of  $\gamma$  and observables, just as we have done for the other variables. One finally intends to solve for  $\gamma$ , only in terms of observables.

The isospin triangle relation given by Eq. (2) and the similar relation for the conjugate process may be expressed as

$$(a^{+-} e^{i\delta_a^{+-}} + a^{00} e^{i\delta_a^{00}}) e^{\pm i\gamma} + (b^{+-} e^{i\delta_b^{+-}} + b^{00} e^{i\delta_b^{00}}) = (a^{+0} e^{\pm i\gamma} + b^{+0}). \quad (16)$$

The ‘‘four’’ equations contained in Eq. (16) may be used to solve for  $\cos\delta_a^{+-}$  and  $\cos\delta_a^{00}$ :

$$\cos\delta_a^{+-} = \frac{|a^{+0}|^2 + |a^{+-}|^2 - |a^{00}|^2}{2|a^{+0}||a^{+-}|}, \quad (17)$$

$$\cos\delta_a^{00} = \frac{|a^{+0}|^2 + |a^{00}|^2 - |a^{+-}|^2}{2|a^{+0}||a^{00}|}, \quad (18)$$

as well as, obtain the relation,

$$|b^{+-}|^2 + |b^{00}|^2 + 2b^{+-}b^{00}\cos(\delta_b^{+-} - \delta_b^{00}) = |b^{+0}|^2. \quad (19)$$

Now  $\delta_b^{+-} = \delta^{+-} + \delta_a^{+-}$  and  $\delta_b^{00} = \delta^{00} + \delta_a^{00}$ . Hence, Eq. (19) is expressed completely in terms of observables and  $\gamma$ .  $\gamma$  can thus be determined cleanly, in terms of observables. Having measured  $\gamma$  one can use Eq. (12) to estimate the value of  $\delta_{EW}$  in terms of observables. We can thus verify our understanding of electroweak penguin contributions.

One may ask if it is possible to determine  $\gamma$  using  $B \rightarrow K\pi\pi$  without resorting to the Neubert-Rosner hypothesis. If one includes in the analysis  $B \rightarrow K_S(\pi^+\pi^-)_o$  with subscript  $o$  representing the two pions being in an isospin odd state, one adds four new variables corresponding to the amplitudes and strong phases of the two parts with different weak phases. However, one can at best obtain four new independent observables. Three of which arise from time-dependent measurement for this mode, and one results from the interference between states with pions in isospin even and isospin odd. We hence conclude that it is not possible to determine  $\gamma$  without at least one theoretical observation, even if one uses all the information possible from  $B \rightarrow K\pi\pi$  decays.

Current experimental data [7,8] indicate that a statistically significant contribution in the  $K_S\pi^+\pi^-$  mode is from the  $K^{*+}\pi^-$ . It can be easily seen by a simple isospin analysis that  $K^{*+}\pi^-$  final state *cannot* result in  $K^0(\pi^+\pi^-)_o$ , but must contribute to  $K^0(\pi^+\pi^-)_e$  final state. If one takes the preliminary data of Ref. [8] seriously, then based on an integrated luminosity of  $43.1 \text{ fb}^{-1}$ , there are  $19.1_{-5.9}^{+6.8} K^{*+}\pi^-$  events in a total of  $60.3 \pm 11.0 K^0\pi^+\pi^-$  events. With the increased luminosity of  $100 \text{ fb}^{-1}$  achieved recently, one may expect about  $40 K^{*+}\pi^-$  events corresponding to  $K(\pi^+\pi^-)_e$ . This should already allow for a time-dependent measurement to be done even with the present sample of data. Additional  $K(\pi^+\pi^-)_e$  events will occur at other regions of the Dalitz plot. Now, if one side of the isospin triangle,  $A^{+-}$ , is large enough to be observable, then clearly one other side must also be large (i.e., half of  $A^{+-}$  or larger, as it has to form a part of the triangle). Thus, at least two sides of the triangles in Fig. 1 may be readily measurable.

While  $B^+ \rightarrow K_S\pi^+\pi^0$  has not yet been observed, the mode  $B^0 \rightarrow K^+\pi^-\pi^0$  has been seen. The two amplitudes are related by Eq. (4). Again, if the  $K^{*0}\pi^0$  contribution to the  $K^+\pi^-\pi^0$  is significant, it must result in  $K^+(\pi^-\pi^0)_e$ . In future, data from both  $B^+ \rightarrow K_S\pi^+\pi^0$  and  $B^0 \rightarrow K^+\pi^-\pi^0$  modes could be combined to improve statistics.

To conclude, the weak phase  $\gamma$  can be measured using a time-dependent asymmetry measurement in the three body decay,  $B \rightarrow K\pi\pi$ . A detailed study of the Dalitz plot can be used to extract the  $\pi\pi$  even isospin states. These states obey certain isospin relations which allow us to not only obtain  $\gamma$ , but also determine the size of the electroweak penguin contribution. In contrast to methods of determination of  $\gamma$  using the two body decay modes  $B \rightarrow K\pi$ , this technique does not require any theoretical assumptions such as SU(3) or neglect of any contributions to the decay amplitudes. By studying different regions of the Dalitz plot it may be possible to reduce the ambiguity in the value of  $\gamma$ .

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