New Limit on Signals of Lorentz Violation in Electrodynamics

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(Received 19 November 2002; published 12 February 2003)

We describe the results of an experiment to test for spacetime anisotropy terms that might exist from Lorentz violations. The apparatus consists of a pair of cylindrical superconducting cavity-stabilized oscillators operating in the TM_{010} mode with one axis east-west and the other vertical. Spatial anisotropy is detected by monitoring the beat frequency at the sidereal rate and its first harmonic. We see no anisotropy to a part in 10^{13}. This puts a comparable bound on four linear combinations of parameters in the general standard model extension, and a weaker bound of $<4 \times 10^{-9}$ on three others.

DOI: 10.1103/PhysRevLett.90.060403 PACS numbers: 03.30.+p, 11.30.Cp

Tests of spacetime anisotropy are generally divided into two main classes, one involving angle dependent effects and the other absolute velocity effects. Following Robertson [1], a treatment of potential Lorentz invariance violations involving idealized rods and clocks was developed by Mansouri and Sexl [2], who considered the possibility of an anisotropic propagation velocity of light relative to a preferred frame. In this model, if a laboratory is assumed to be moving with a velocity v at an angle θ relative to the axis of a preferred frame, the speed of light as a function of θ and v is given by

$$\frac{c(\theta, v)}{c} = 1 + \left(\frac{1}{2} - \beta + \delta\right)\left(\frac{v}{c}\right)^2 \sin^2\theta + (\beta - \alpha - 1)\left(\frac{v}{c}\right)^2, \qquad (1)$$

where α is the time dilation parameter, β is the Lorentz contraction parameter, and δ tests for transverse contraction. In special relativity, the last two terms on the right-hand side of the equation are zero. Classical Michelson-Morley experiments attempt to measure the amplitude of the θ-dependent term, while Kennedy-Thorndyke experiments set limits on the amplitude of the θ-independent term. While useful to help categorize experiments, this approach has a number of limitations. For example, it fails to include effects on the measurement system itself and it does not take into account the full range of anisotropies allowable in nature.

Recently, Kostelecký and Mewes [3] (KM) have pointed out that, in the standard model extension (SME) that describes general Lorentz violations [4], additional terms may exist which show signatures different from those expressed in Eq. (1). In particular, $\sin\theta$ and $\sin 2\theta$ terms may exist independent of v which could be detectable in the experiments, as well as terms first order in v/c. No systematic search for these terms appears to have been undertaken in the optical experiments conducted until now, although a number of tests have been performed with fermions and with astrophysical sources [5]. Also, experiments involving Earth's rotation typically use co-rotating frequency references which complicates the analysis. A simple configuration that can be analyzed easily consists of a pair of cylindrical microwave cavity resonators operating on radial modes with their axes aligned in the east-west direction and optimally at 45° to Earth's axis, as indicated in Fig. 1. This apparatus will in general have a different sensitivity to the coefficients of the Lorentz violating terms than an optical cavity experiment because of the radial nature of the wave motions involved. Each cavity will provide its own fractional offset signal $\delta\nu/\nu$ from its unperturbed frequency. The beat signal from such a pair, $\Delta\nu/\nu = \delta\nu_1/\nu - \delta\nu_2/\nu$, takes the general form

$$\frac{\Delta\nu}{\nu} = \mathcal{A}_s \sin\omega t + \mathcal{A}_c \cos\omega t + \mathcal{B}_s \sin 2\omega t + \mathcal{B}_c \cos 2\omega t + C, \qquad (2)$$

where the coefficients are linear combinations of potential Lorentz violating terms in the SME and $2\pi/\omega$ is Earth's sidereal period. The term C has an annual variation that can be neglected here. In zero and first order of v/c, the quantities $\mathcal{A}$ and $\mathcal{B}$ contain exclusively SME terms, while higher order terms would of course include the more traditional effects described by Eq. (1). The cavity which is oriented in the east-west direction is maximally sensitive to the second harmonic terms in

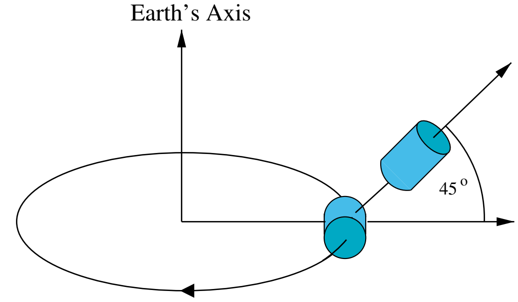

FIG. 1 (color online). Ideal arrangement of two microwave cavities relative to Earth's rotational axis which maximizes sensitivity to sidereal and twice-sidereal variations in the beat frequency. Actual tilt angle is the latitude of the laboratory.

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Eq. (2), while the cavity oriented 45° to Earth's axis is maximally sensitive to the first harmonic terms [6]. A search for the lower order terms would probe for new physics that might, for example, correspond to residual effects left over from the birth of the universe. Because of the extreme sensitivity of modern cavity resonators and clocks, it is possible to put useful bounds on such possibilities.

When limited to the photon sector, the Lagrangian describing the SME can be written in the form [7]

$$L_{\text{photon}} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu} + \tfrac{1}{2}(k_{AF})^{\kappa}\epsilon_{\kappa\lambda\mu\nu}A^{\lambda}F^{\mu\nu}. \tag{3}$$

Here, A_μ are the photon fields, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, and the coefficients k_F and k_{AF} control the magnitude of the Lorentz violations. Stringent limits exist on the size of the k_{AF} term, but the *CPT*-even k_F term is only partially constrained [3]. KM define matrices $\tilde{\kappa}_{e+}$, $\tilde{\kappa}_{e-}$, $\tilde{\kappa}_{o+}$, $\tilde{\kappa}_{o-}$, and $\tilde{\kappa}_{tr}$ with elements that are parity even and parity odd combinations of the coefficients k_F. These matrices arise naturally in the analogous situation of wave propagation in a homogeneous anisotropic medium. Astrophysical tests constrain $\tilde{\kappa}_{e+}$ and $\tilde{\kappa}_{o-}$ at the 10^{-32} level [8], while the other matrices are currently only weakly constrained. These include nine additional coefficients of k_F, of which eight are in principle accessible via the present experimental configuration. Of these, four contribute directly to a possible frequency shift and three at first order in v/c, leading to high sensitivity tests. Detailed expressions relating the coefficients of Lorentz violation to those in Eq. (2) have been given by KM up to first order in v/c.

In our experiment, we compare the frequencies of two cylindrical superconducting microwave cavities operating in the TM_{010} mode [9] which gives sensitivity to the velocity of light in radial directions. One cavity has its axis oriented to the local vertical at our latitude of 37.4°, while the other axis is oriented to the local horizontal in the east-west direction. The cavities are made of niobium and are operated at about 1.4 K in conventional helium Dewars. Microwave synthesizers are locked to the 8.6 GHz modes of the cavities using Pound frequency discrimination systems, and the difference frequency is mixed with an intermediate frequency oscillator to produce a beat signal in the 20–30 Hz range. Data was collected at irregular intervals over a 98-day period during a development program for a related experiment to be performed in space [10]. Nine records were obtained, each corresponding to a continuous segment of data at least 24 h long. Frequency sampling was at 1 s intervals, with averaging of 100 s segments before curve fitting was performed. An example of one of the records is shown in Fig. 2(a). Typically, the records were collected after some other form of testing on the apparatus was completed. This situation leads to arbitrary offsets of up to a few Hz between the records. Also, mechanical disturbances occasionally gave rise to a perceptible drift of the beat frequency amounting to a few mHz per day. We therefore fitted each record with the function

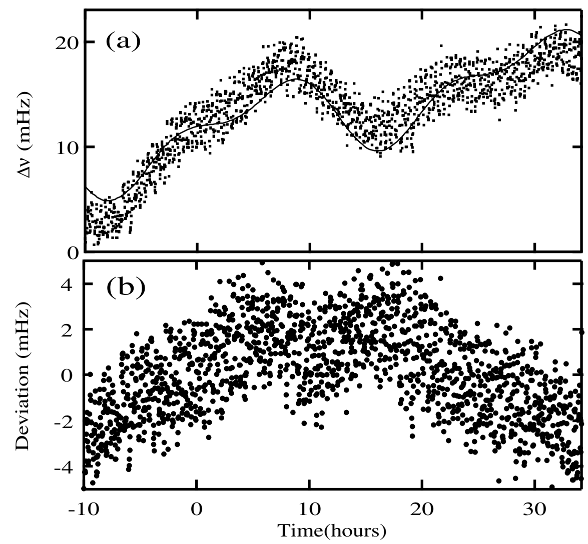

FIG. 2. (a) Example of a beat frequency record $\Delta\nu$ as a function of time. $\Delta\nu$ is measured from 26 Hz. Line shows best fit with Eq. (4). (b) Residuals of data in (a) after subtraction of best fit.

$$\frac{\Delta\nu}{\nu} = \nu_0 + \nu_1 t + \mathcal{A}_s \sin(\omega t) + \mathcal{A}_c \cos(\omega t) + \mathcal{B}_s \sin(2\omega t) + \mathcal{B}_c \cos(2\omega t), \tag{4}$$

where ν_0 and ν_1 were additional free parameters. The residuals from the fit to the record in Fig. 2(a) are shown in Fig. 2(b). The values obtained for the coefficients of the sinusoidal terms are listed in Table I along with the day of the record [11]. It can be seen that the amplitudes $\mathcal{A}_s$, $\mathcal{A}_c$, $\mathcal{B}_s$, and $\mathcal{B}_c$ are in the low 10^{-13}–10^{-14} range, with no obvious trend with time. *XY* plots of the sine and cosine coefficients are shown in Figs. 3(a) and 3(b). To test for an alignment of the observed signals with inertial space, we also made the *XY* plots after phase shifting the sine and cosine coefficients to sidereal time, obtaining the results in Figs. 3(c) and 3(d). For these plots, the time origin was also shifted to the 2002 vernal equinox. No significant reduction of the scatter is evident.

A study of the behavior of the apparatus at other times disclosed a high sensitivity of the signal from one of the cavities to tilt which appears to be the dominant limit to the experiment. A second effect was the stability of the temperature of the cavity. This was controlled to within $\pm 5 \times 10^{-6}$ K using a germanium resistance thermometer mounted on the cavity and a servoed heater. The dominant source of temperature fluctuation was the 1.4 K cooling system. A proportional-integral temperature controller was used, but thermal gradients within the cavity assembly could cause some undetected coupling with room temperature. A ratio transformer bridge was used

TABLE I. Coefficients of sinusoidal terms from best fits to the raw data [11] with Eq. (4). Uncertainties in the coefficients from the fit are given in parentheses.

Day	$\mathcal{A}_s \times 10^{13}$	$\mathcal{A}_c \times 10^{13}$	$\mathcal{B}_s \times 10^{13}$	$\mathcal{B}_c \times 10^{13}$
1	0.731 (0.04)	2.520 (0.04)	−0.216 (0.04)	−0.204 (0.04)
3	−0.081 (0.03)	1.189 (0.03)	−1.680 (0.03)	0.605 (0.03)
18	3.699 (0.12)	0.368 (0.12)	−1.817 (0.11)	0.691 (0.12)
26	2.286 (0.07)	0.108 (0.07)	−0.950 (0.07)	0.459 (0.07)
59	2.503 (0.08)	−0.697 (0.09)	−1.347 (0.08)	1.101 (0.08)
78	−0.329 (0.10)	1.776 (0.13)	0.535 (0.10)	−0.457 (0.11)
80	1.006 (0.06)	0.515 (0.06)	−0.156 (0.06)	−0.135 (0.06)
95	−0.809 (0.06)	0.107 (0.06)	−0.212 (0.06)	0.145 (0.06)
98	−0.306 (0.04)	−1.336 (0.04)	0.823 (0.04)	−0.558 (0.04)

for the germanium thermometer with a reference resistor at 1.4 K. This configuration typically gives stabilities of better than 1 μK. The temperature coefficient of the cavity frequency was -28 Hz/K, which would imply frequency offsets on the order of ± 0.15 mHz, but this could be amplified by thermal gradient effects. With servo powers of the order of 10^{-5} W, temperature differences of as much as 5×10^{-5} K would be expected in the cavity support structure. A number of correlation studies were performed but only a modest reduction of the amplitudes in Table I was obtained. Discussion of this aspect of the analysis is lengthy and will be presented elsewhere. We suspect that the signal amplitudes in Table I are dominated by mechanical effects in the low temperature apparatus.

From the plots in Fig. 3, it seems reasonable to conclude that there is no significant evidence for an inertially oriented frequency shift in our experiment. Treating the variation of the observed signal amplitudes as locally generated "noise," we can average the data in each direction and derive bounds on any signal. Using the coefficients from Figs. 3(c) and 3(d), we obtain $\overline{\mathcal{A}}_c = -8.5 \pm 10.4 \times 10^{-14}$, $\overline{\mathcal{A}}_s = 4.2 \pm 8.8 \times 10^{-14}$, $\overline{\mathcal{B}}_c = -2.0 \pm 4.3 \times 10^{-14}$, and $\overline{\mathcal{B}}_s = 5.8 \pm 5.9 \times 10^{-14}$, where the errors correspond to the statistical 2σ level. Because of the likely presence of unmodeled systematic effects and the small number of records, we consider the confidence level in these results to be closer to the 60% or 1σ level. The results imply that certain linear combinations of the k_F coefficients are constrained at the 10^{-13} level. For example, neglecting contributions of order v/c and higher, the $\mathcal{A}_s$ term can be written as $\mathcal{A}_s^0 = 1/4 \sin 2\chi (3\tilde{\kappa}_{e+} + \tilde{\kappa}_{e-})^{YZ}$, where χ is the colatitude of the experiment [12]. Setting $(\tilde{\kappa}_{e+})^{YZ} = 0$ on the basis of the extremely tight astrophysical bound $<10^{-32}$, it is easy to show that $\mathcal{A}_s^0 = (1/16)\sin 2\chi[(k_F)^{ZXYX} + (k_F)^{XZXY} - (k_F)^{ZXXY} - (k_F)^{XZYX} - 4(k_F)^{0Y0Z}]$, which reduces to $\sin 2\chi[(k_F)^{XYXZ} - (k_F)^{0Y0Z}]/4$. Similar relations can be derived for the other amplitudes in Eq. (2). Alternatively, the experiment can be viewed as setting the following bounds on elements of the $(\tilde{\kappa}_{e-})$ matrix: $(\tilde{\kappa}_{e-})^{YZ} < 1.7 \pm 3.6 \times 10^{-13}$; $(\tilde{\kappa}_{e-})^{XZ} < -3.5 \pm 4.3 \times 10^{-13}$; $(\tilde{\kappa}_{e-})^{XY} < 1.4 \pm 1.4 \times 10^{-13}$; $[(\tilde{\kappa}_{e-})^{XX} - (\tilde{\kappa}_{e-})^{YY}] < -1.0 \pm 2.1 \times 10^{-13}$.

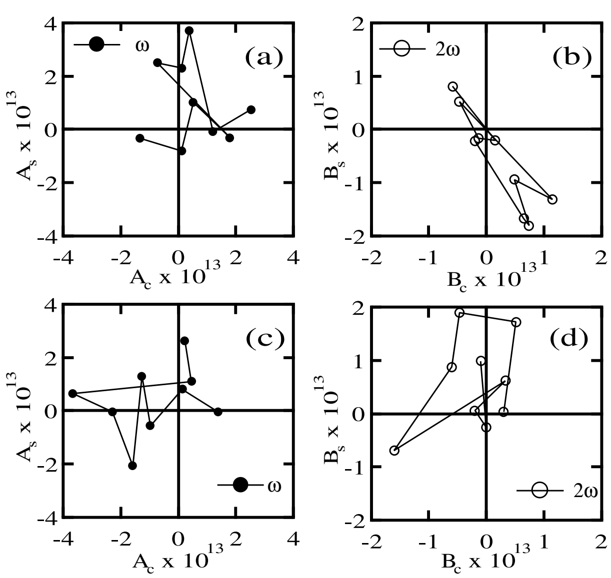

FIG. 3. XY plots of best fit coefficients with Eq. (4) for all records. (a) $\mathcal{A}_s$ vs $\mathcal{A}_c$, and (b) $\mathcal{B}_s$ vs $\mathcal{B}_c$ using solar time; (c) and (d): same as (a) and (b) but with sidereal phase shifts added to align results to inertial space. Lines link the data points in time sequence.

As described by KM, three more coefficients are introduced at the level v/c, where v is the velocity of Earth around the Sun. The additional term in $\mathcal{A}_s$ can be written as

$$\mathcal{A}_s^1 = \frac{v}{4c} \sin 2\chi \cos \Omega T [\sin \eta (\tilde{\kappa}_{o+})^{ZX} - \cos \eta (\tilde{\kappa}_{o+})^{YX}], \quad (5)$$

where $2\pi/\Omega$ is the orbital period of Earth, T is the time measured from the vernal equinox, and η is the angle between Earth's orbital and equatorial planes. Evaluating this expression for our situation, we obtain $\mathcal{A}_s^1 = 9.75 \times 10^{-6}[(\tilde{\kappa}_{o+})^{YX} - 0.432(\tilde{\kappa}_{o+})^{ZX}]$, where we have used the midvalue of T over our data collection period. Clearly, this expression is dependent on the duration of

TABLE II. Experimental constraints on coefficients in the SME assuming no cancellation effects.

Constrained quantity	Bound
$(\tilde{\kappa}_{e-})^{YZ}$	$1.7 \pm 3.6 \times 10^{-13}$
$(\tilde{\kappa}_{e-})^{XZ}$	$-3.5 \pm 4.3 \times 10^{-13}$
$(\tilde{\kappa}_{e-})^{XY}$	$1.4 \pm 1.4 \times 10^{-13}$
$(\tilde{\kappa}_{e-})^{XX} - (\tilde{\kappa}_{e-})^{YY}$	$-1.0 \pm 2.1 \times 10^{-13}$
$(\tilde{\kappa}_{o+})^{YX} - 0.432(\tilde{\kappa}_{o+})^{ZX}$	$4.0 \pm 8.4 \times 10^{-9}$
$(\tilde{\kappa}_{o+})^{XY} - 0.209(\tilde{\kappa}_{o+})^{YZ}$	$4.0 \pm 4.9 \times 10^{-9}$
$(\tilde{\kappa}_{o+})^{XZ} - 0.484(\tilde{\kappa}_{o+})^{YZ}$	$1.6 \pm 1.7 \times 10^{-9}$
$(\tilde{\kappa}_{o+})^{YZ} + 0.484(\tilde{\kappa}_{o+})^{XZ}$	$0.6 \pm 1.9 \times 10^{-9}$

the experiment. In conjunction with $\mathcal{A}_s^0$, this term is also bounded at the level of 4.2×10^{-14}, implying a constraint on the term inside the square brackets of $<4.0 \pm 8.4 \times 10^{-9}$, assuming no cancellation between the terms [13]. Similar expressions can be derived for the other coefficients in Eq. (4). For clarity, the entire set of constraints obtained from the measurements is given in Table II. A more direct bound on the components of $\mathcal{A}_s^1$ could be obtained by extending the data gathering period to a larger fraction of a year when it would become reasonable to include the annual modulation terms in the fit to the data. At the present level, the experiment sets bounds of $<4 \times 10^{-9}$ on four expressions of the type in square brackets in Eq. (5). We note that by mixing the experimental bounds a cleaner separation of the $\tilde{\kappa}_{o+}$ components can be obtained.

By restricting the Lagrangian in Eq. (3) to the photon sector, the model omits a range of potential effects from the material that makes up the apparatus. Within the full SME, these possibilities lead to considerable complexity, but KM argue that complete cancellation of the photon sector effects is improbable due to the complexity of the forces involved. In very recent work, Müller *et al.* [14] have considered these effects and find a small enhancement of the photon effects in the case of ionic crystalline materials.

In summary, we have set bounds at the 10^{-13} level on four combinations of parameters in the SME, and bounds at the 10^{-9} level on four others. These bounds now constrain seven of the nine unknown coefficients k_F in the model. We note that in a space-based version of this experiment [10], substantially greater sensitivity could be achieved, perhaps approaching the 10^{-17} level for the four primary bounds.

We wish to thank J. Turneaure and S. Buchman for support in the early phases of the project, the NASA Office of Biological and Physical Research for its support with Grants No. NAG3-1940 and No. NAG8-1439, and the Jet Propulsion Laboratory for Contract No. 1203716.

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[11] For these coefficients, time is set to zero at the first midnight during data collection. Day 1 corresponds to May 30, 2002.
[12] The XYZ coordinate system defined in Ref. [3] is Sun centered and has X and Y in Earth's equatorial plane. The positive X axis is in the direction of the vernal equinox, which we took to be at midnight on March 20, 2002.
[13] If a statistically significant signal was observed, some progress could be made untangling the effects of the various contributions.
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