Comment on "Universal Intermediate Phases of Dilute Electronic and Molecular Glasses"

Phillips [1] has discussed the universal behavior of dilute and molecular glasses just above the critical density n_c for the metal-insulator transition and the origin of the scaling exponent of the conductivity σ [$\sigma(n, T \to 0) \propto (n/n_c - 1)^{\alpha}$] with $\alpha = 1/2$ for weakly compensated doped semiconductors (Si:P, Si:As, Si:B, and Ge:Ga) and suggests $\alpha = 1/2$ provides support for one-dimensional filamentary conduction (fc) paths. This Comment demonstrates how $\alpha = 1/2$ originates from the Boltzmann conductivity when the scattering is dominated by ionized impurity scattering (II*S*) [2]; how the density of states (DOS) $N(E)$ affects α; and considers other experimental evidence for fc and why cross-linking of the filaments is unimportant for $n < 2n_c$.

The Boltzmann conductivity $\sigma_B = n_i e^2 \tau(E_F)/m^*$ yields $\alpha = 1/2$ when the scattering is dominated by II*S*. This approach uses charge neutrality ($n_i = N_i$, n_i the density of itinerant carriers for $E > E_c$, N_i the density of ionized impurities), the two-component model [$n_i + n_{\rm loc} = n$, $n_{\rm loc}$ is the density of localized electrons (holes) with $E < E_c$] where E_c is the mobility edge, and the Friedel sum rule. The usual Boltzmann result $\sigma_B \propto (e^2/h){k_F}^2 \ell$ ($\ell \equiv$ mean-free path) for $k_F \propto [2m^*(E_F - E_c)]^{1/2} \propto (n/n_c - 1)^{1/2}$ yields $\sigma_B \propto (n/n_c - 1)^{3/2}$ for $\tau(E_F) \sim$ const, which is close to the $d = 3$ percolation (pc) prediction. This changes when II*S* is the source of $\tau(E_F)$. Use of $1/\tau(E_F) = N_i v_F \langle\sigma\rangle$ [the cross section $\langle\sigma\rangle = (4\pi/k_F^2) \times \sum_{\ell=0}(\ell + 1)\sin^2(\delta_\ell - \delta_{\ell+1})$ and the phase shifts δ_ℓ are evaluated at E_F] yields $\sigma_B \propto (e^2/h)k_F/\lambda_{i,h}$ where $\lambda_{i,h}(E_F) = \sum_{\ell=0}(\ell + 1)\sin^2(\delta_\ell - \delta_{\ell+1})$ is slowly varying with n leaving $\sigma_B \propto k_F \propto (n/n_c - 1)^{1/2}$ in agreement with experiment. This result giving $\alpha = 1/2$ is independent of the form of $N(E)$ for $E > E_c$, which is at odds with Phillip's discussion.

Some have argued k_F cannot go to zero as $n \to n_{c+}$ ignoring the fact that k_F depends only on n_i and not on the total dopant density n. $n_i(T = 0) = \int_{E_c}^{E_F} N(E)dE$ [$N(E) = a_d(E - E_c)^{d/2-1}$] giving

$$n_i(T = 0) = (2a_d/d)(E_F - E_c)^{d/2} \propto (n/n_c - 1)^{d/2}. \quad (1)$$

Equation (1) is consistent only with $d = 1$ fc for $\alpha = 1/2$. For $\sigma_B = n_i e\mu$ and $\alpha = 1/2$ any $d > 1$ would lead to a nonsensical diverging mobility [3] $\mu(n)$ as $n \to n_{c+}$. It is also apparent $\tau(E_F)$ is nearly constant with n for $N_i \propto (n/n_c - 1)^{1/2}$ which results from the charge neutrality condition $n_i = N_i$. If $\alpha = 3/2$ then for a constant $\tau(E_F)$ σ_B yields $n_i \propto (n/n_c - 1)^{3/2}$ for $d = 3$, but $n_i = N_i$ is no longer satisfied. Phillips mentions tunneling where the conductance $G(V)$ is measured versus the bias V. $G(V)$ is proportional to a DOS $N(E)$ but it differs from the $N(E)$ above because tunneling measures $N(E - E_F)$. Massey and Lee [4] measured tunneling into Si:B finding $G(V) \propto V^{1/2}$ for $n \geq 1.03n_c$ leading to $N(E - E_F) \propto (E - E_F)^{1/2}$, however this DOS is not relevant for $n_i(T = 0)$ and is not relevant to the exponent $\alpha = 1/2$ for $\sigma(n, T \to 0)$. The specific heat measures yet another DOS which varies smoothly thru n_c.

What evidence exists for a nearly constant $\tau(E_F)$ for $n_c < n < 2n_c$? The diffusivity $D(n) = {v_F}^2\tau(E_F)/d = (\hbar/dm^*)k_F\ell$ has been shown [2] to scale as $(n/n_c - 1)^t$ with $t \sim 1$ from the n dependence of the ee interaction correction [5] $\delta\sigma_D(n, T) = m(n)T^{1/2}$ [$m(n) \propto D(n)^{-1/2}$] in the diffusive regime $k_F\ell < 1$ and from the Einstein relation ($kT \ll E_F - E_c$) $eD = 2/3(E_F - E_c)\mu(n)$. This result suggests $\mu(n)$ does not scale and is slowly varying. A constant m^* then implies $\tau(E_F) \sim$ const for $n_c < n < 2n_c$. Moreover, the Einstein relation $\sigma = e^2(\partial n/\partial\mu)D$ leads to $\partial n/\partial\mu = N(E) \propto (E/E_c - 1)^{-1/2}$, which is consistent with $d = 1$ fc. New data [6] suggest the transverse dielectric response ε_t scales as $(n/n_c - 1)^{0.5}$ for $1.05n_c < n < 1.2n_c$. The Drude model for $\omega\tau \ll 1$ ($\varepsilon_t = \varepsilon_h - 4\pi\sigma_{DC}\tau$) suggests $\tau(n)$ is slowly varying. In zero magnetic field the evidence suggests the drift mobility is nearly constant just above n_c. Phillips mentions the branching of the $d = 1$ filaments as Si:P reverts to $d = 3$ behavior for larger n. In the diffusive regime $k_F\ell < 1$ $\ell(n)$ scales to zero as $(n/n_c - 1)^{1/2}$ and $\ell(n) < d(n) < \lambda_{\rm dB}(n)$ where $d(n)$ is the mean spacing of the donors. $\sigma(n, T \to 0) \propto \lambda_{\rm dB}^{-1}$ ($\lambda_{\rm dB} \equiv$ deBroglie wavelength). The inequality argues against coherent conduction, but suggests why fc is possible up to nearly $2n_c$.

Cohen and Jortner [7] suggested, based on pc theory, conducting metallic channels near n_c. The data showing $\alpha = 1/2$, much different than the $d = 3$ prediction of 1.6, led us away from the pc theory. Today there is substantial evidence for Si:P and Si:As supporting $d = 1$ fc.

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