Stochastic Resonance in Geomagnetic Polarity Reversals

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Among noise-induced cooperative phenomena a peculiar relevance is played by stochastic resonance. In this paper we offer evidence that geomagnetic polarity reversals may be due to a stochastic resonance process. In detail, analyzing the distribution function $P(\tau)$ of polarity residence times (chrons), we found the evidence of a stochastic synchronization process, i.e., a series of peaks in the $P(\tau)$ at $T_n \simeq (2n+1)T_\Omega/2$ with $n = 0, 1, ..., j$ and $T_\Omega \sim 0.1$ Myr. This result is discussed in connection with both the typical time scale of Earth's orbit eccentricity variation and the recent results on the typical time scale of climatic long-term variation.

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A special class of noise-induced cooperative phenomena is represented by stochastic resonance (SR) [1]. SR was originally proposed as a possible explanation of the nearly periodic recurrence of Earth's ice ages in a series of papers [2], which triggered a rather limited reaction in some theoretical studies and experimental papers [3]. The experimental paper by McNamara *et al.* [4] marked the renaissance of SR, and, in the last decade, stochastic resonance has been observed to occur in a wide variety of physical and biological systems. SR is now a well established phenomenon.

SR requires three basic ingredients: (i) an energetic activation barrier, (ii) a weak coherent input, and (iii) a source of noise that is inherent in the system or that adds to the coherent input. An intuitive picture of the stochastic resonance is provided by the one-dimensional motion of an overdamped point mass in a double-well potential profile in the presence of two stimuli: a random noise and a weak periodic force. The periodic force is weak in the sense that transitions between the two wells cannot occur if this force is applied alone. The application of an optimal amount (in terms of strength) of noise may enhance the synchronous transitions between the two potential minima. This resembles a resonance phenomenon, which, however, should be not confused with standard dynamical resonance [5]. In other words, SR is a peculiar nonlinear cooperative process in which an optimal amount of noise is able to enhance the synchronous switches in a bistable system driven by a weak coherent periodic signal. Formally speaking, the optimal statistical synchronization takes place when $2T_k \simeq T_\Omega$, where T_k is the average waiting time between two noise-induced interwell transitions and T_Ω is the period of the driving force.

SR can be conveniently described in terms of the distribution function of the residence times (RTDF), i.e., the time intervals between two successive switches with opposite phases. Since the modulation perturbs the temporal switching symmetry, the RTDF shows a sequence of exponentially decaying peaks centered close to

$$T_n \simeq (2n+1)\frac{T_\Omega}{2} \tag{1}$$

with $n = 0, 1, \ldots$ and where T_Ω is the period of the weak modulating periodic force. Gammaitoni and co-workers [6] showed that the area under the RTDF peak at $T/2$ goes through a maximum when the optimal synchronization condition is satisfied. This characterization of SR was named as *bona fide* SR [6].

The major phenomenological discovery of the 20th century about the geomagnetic field is the occurrence of polarity reversals of the dipolar component of the magnetic field during the Earth's geological history [7]. In the past, several hypotheses were put forward on the possible mechanisms responsible for polarity reversals. All these hypotheses can be grouped into two main classes. In the first case reversals are considered as the result of magnetic hydrodynamic instabilities triggered by finite-amplitude perturbations (of internal and/or external origin) of an otherwise stable dynamo. Conversely, in the second case polarity reversals are considered to be due to irregular oscillations of a nonlinear dynamo.

One of the most fascinating and oldest theories involves a possible link among the geomagnetic reversals, paleoclimatic changes, and the variations of the Earth's orbital parameters (the Milankovitch orbital frequencies) [8], suggesting that the orbital forcing or paleoclimatic changes may energize the geodynamo. Although some preliminary evidence of this link was found [9], there is no general consensus on it [10]. Nevertheless, very recently, by studying a set of paleomagnetic records covering the past 2.25 Myr, Yamazaki and Oda [11] have again found the presence of a 100 Kyr periodicity in the geomagnetic intensity and inclination which suggests that the geomagnetic field is, in some way, modulated by orbital eccentricity variations. Their results reopen the question of the possible existence of a link between geomagnetic polarity reversals and periodic changes of the Earth's orbital parameters.

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In this Letter, we address the problem of geomagnetic polarity reversals in the framework of SR. In detail, analyzing the statistics of the polarity time intervals (chrons) in the last 166 Myr, we show that the distribution function $P(\tau)$ of chrons shows a sequence of exponentially decaying peaks in good agreement with the prediction of SR as described by Eq. (1) with a characteristic time scale T_Ω of the order of the Earth's orbital eccentricity variation.

The geomagnetic field is originated by dynamo action in the Earth's fluid outer core [12]. The equation that describes the time evolution of the geomagnetic field is the magnetic induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \qquad (2)$$

where η is the magnetic diffusivity and $\mathbf{v}$ is the fluid velocity. In the *mean field electrodynamics* approximation [13] the evolution of the large scale field $\langle \mathbf{B} \rangle$ is given by

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times [\langle \mathbf{v} \rangle \times \langle \mathbf{B} \rangle + \alpha \langle \mathbf{B} \rangle + (\beta - \eta) \nabla \times \langle \mathbf{B} \rangle], \qquad (3)$$

where $\langle \mathbf{v} \rangle$ is the mean flow velocity, and the parameters α and β are related to the mean properties of the turbulent convection:

$$\alpha \simeq -\tfrac{1}{3} \langle \mathbf{u} \cdot \nabla \times \mathbf{u} \rangle \tau_c, \qquad \beta \simeq \tfrac{1}{3} \langle |\mathbf{u}|^2 \rangle \tau_c, \qquad (4)$$

where $\mathbf{u}$ is the turbulent convection velocity field superposed to the mean flow velocity $\langle \mathbf{v} \rangle$ (i.e., $\mathbf{v} = \langle \mathbf{v} \rangle + \mathbf{u}$), and τ_c is the eddy turnover time. The validity of the mean field approach [Eq. (3)] requires that the diffusion term might be irrelevant, i.e., $\eta \ll \beta$. Because Eqs. (2) and (3) are invariant under parity transformation of the geomagnetic field ($\mathbf{B} \rightarrow -\mathbf{B}$), two stable solutions are possible: a normal polarity field, as we have today, and a reversal polarity field. The symmetry of the equations is such that the two polarities should have identical statistical properties apart from the sign of the field. In other words the geodynamo is equivalent to an inherent bistable system.

Recently, it has been shown that polarity reversal may be approached in terms of the motion of a thermally activated strongly damped particle in a bistable potential well with minima representing normal and reversed polarity [14]. In detail, Hoyng and co-workers [14] developed a mean field model for the evolution of the axisymmetric dipolar component of the geomagnetic field, where random fluctuations in the α effect over very long times may cause reversals. It is possible to demonstrate that this model leads to a Fokker-Planck equation for the amplitude of the fundamental dipole mode which is analogous to the one describing the motion of an overdamped point mass, randomly forced, in a bistable potential. Here reversals are fast random events that can be studied in the framework of the Kramers reaction rate theory [15]. Given the stochastic nature of the aforementioned model and the inherent short memory of the fluctuations, the distribution function of polarity residence times should follow a Poissonian distribution function $P(\tau) \propto \exp(-\tau/\langle \tau \rangle)$, where $\langle \tau \rangle$ is the average residence time.

Figure 1 displays the normalized probability density function $P(\tau)$ of the polarity time intervals τ (i.e., the RTDF). The time scale used for evaluating $P(\tau)$ results from the merging of two scales compiled by Cande and Kent [16] and by Ogg [17] and contains more than 300 polarity intervals dating back to 166 Myr. Time-scale resolution is 0.001 Myr. Furthermore, we note that no regularities are evident looking at the time series of polarity residence times. Because of the poor statistics and the uncertainties in the data set, to evaluate the distribution of the polarity intervals we used a moving box technique, defining the probability density in each box as follows:

$$P(\tau_i) = \frac{n_i}{N \Delta \tau}, \qquad (5)$$

where n_i is the number of events in the range $[\tau_i - \Delta\tau/2, \tau_i + \Delta\tau/2]$ with $\tau_i = (0.02 + 0.01 i)$ Myr, $i \in N$, and $\Delta\tau = 0.04$ Myr. The choice of the window $\Delta\tau$ has been optimized in order to attain a relevant statistics in each moving box and a good stability of the results. Furthermore, this moving box technique reduces the dependence of the results from the choice of the bin set. Although all the data set has been considered to evaluate the probability density function $P(\tau)$, we limit our discussion to polarity reversals lasting less than 1 Myr, since the statistics are very poor above this value. The number of polarity intervals that satisfies this condition is about

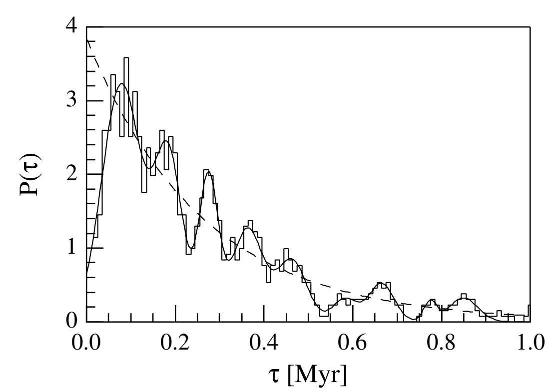

FIG. 1. The probability density function $P(\tau)$ of the geomagnetic polarity time intervals as evaluated on the basis of expression (5) in the text. The solid line refers to a nonlinear best fit using a superposition of nine Gaussian functions. The dashed line is the expected probability density function in the case of a Poissonian process with a mean polarity residence time $\langle \tau \rangle \sim 0.3$ Myr of the same order of the actual current mean residence time [14].

95% of the total amount. Thus, we do not discuss the superchron, i.e., the period extending from ≈ 118 Myr to ≈ 83 Myr, during which reversals apparently do not occur.

Conversely to what is expected in the case of a purely stochastic process, the $P(\tau)$ is not just a simple Poissonian distribution function but shows a series of decreasing and nearly equally spaced peaks. Although we cannot exclude other physical processes occurring in bistable systems subject to subthreshold periodic driving signals in noise, we believe that the quasiregular multimodal character of the RTDF might be due to a stochastic synchronization process as in SR. As a matter of fact, we believe that SR might be a reasonable process when the observational results of Yamazaki and Oda [11] are combined with the numerical simulations of Hoyng and co-workers [14].

To emphasize the multipeak nature of the RTDF, and to extract the features of each single peak, we have decomposed the RTDF by a superposition of nine Gaussian functions. In Fig. 2 we report the position T_n of each peak as a function of the odd numbers. A linear relationship is found between the peak position and the odd numbers. This result is in good agreement with the prediction of SR as described in Eq. (1), even if a complete proof of this hypothesis requires one to address the *bona fide* SR condition on peak strength, i.e., to check the result of the noise level change. In this respect, we note that this cannot be accomplished by means of observations.

Fitting the trend of T_n by Eq. (1) it is possible to define a characteristic time scale $T_\Omega \sim 0.1$ Myr. This characteristic time scale is in good agreement with previous findings [11] of a $\sim$100 Kyr periodicity in the geomagnetic intensity and inclination, suggesting a possible link between geomagnetic polarity reversals and the variation of the Earth's orbital eccentricity.

In Fig. 3 we report the behavior of the peak height A_n as a function of its position T_n as obtained by the Gaussian fit of the pdf of polarity residence times. The peak heights decay exponentially, $A_n(T_n) \sim \exp(-r_k T_n)$, defining a new characteristic time scale $T_k = r_k^{-1} = [310 \pm 20]$ kyr. This characteristic time scale T_k fits well in the current mean residence time $\langle\tau\rangle \sim 0.3$ Myr, and could be related to the spontaneous transition rate as stated, for example, by Hoyng *et al.* [14]. We note that the mean geomagnetic polarity residence time $\langle\tau\rangle$ decreased over the last 160 Myr from $\approx 10^7$ yr in the Cretaceous age to $\approx (2\text{–}3) \times 10^5$ yr during the past 10 Myr. This decrease of $\langle\tau\rangle$ could be due to a very small change in the fluctuation level of the α effect related to a gradual evolution of the Earth's inner core with time [14] and/or in the heath flux pattern at the core-mantle boundary [18].

We will now briefly discuss the origin of these two competing characteristic scales $T_\Omega \sim 0.1$ Myr and $T_k \sim 0.3$ Myr. As already mentioned, the characteristic scale T_Ω, associated with the modulation of the polarity residence time distribution, agrees with the typical scale of the Earth's orbital eccentricity variations, leading to the possible occurrence of a stochastic synchronization phenomenon as it occurs in SR. Besides the evidence of this stochastic sychronization, it is not trivial to understand how the orbital eccentricity variations might affect the geodynamo. One of the most accepted hypotheses states that changes of the core-mantle boundary conditions might strongly affect the geodynamo configuration. Among these conditions we surely have to include also the differential rotation between the outer fluid core and the mantle itself. In this framework, changes of the Earth's angular velocity $\mathbf{\Omega}$ should affect the geodynamo. Periodic variations of orbital eccentrity could introduce an almost periodic modulation of $\mathbf{\Omega}$ on the same time scale. For such a modulation various mechanisms can be involved: changes in the gravitational coupling between

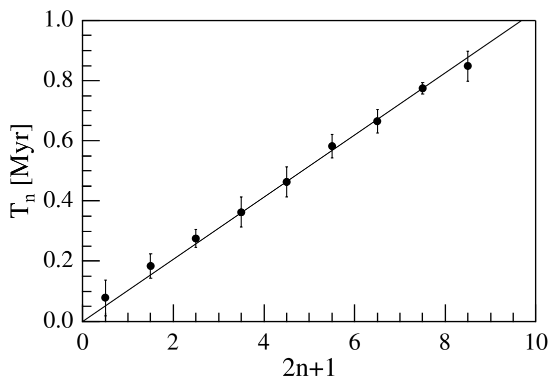

FIG. 2. The peak position T_n plotted versus the odd numbers $2n + 1$. Error bars are the variance of each Gaussian function associated with the peak T_n. The solid line refers to a weighted linear fit using Eq. (1)—see text. A characteristic time scale $T_\Omega = [103 \pm 2]$ kyr may be recovered by the fitting procedure.

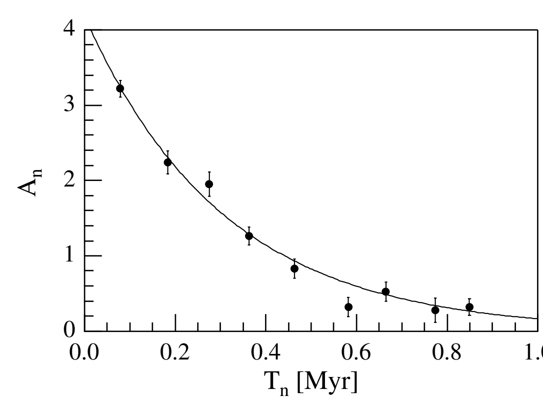

FIG. 3. The behavior of the peak height A_n as a function of the correspondent peak position T_n. The solid line refers to a weighted best fit using an exponential function. The resulting characteristic rate is $r_k = [3.2 \pm 0.2]$ Myr^{-1}.

the Earth's orbit and its spin, tidal effects, etc. At the present stage, we cannot exclude that climatic changes may affect the Earth's angular velocity. We note that the original papers on SR by Benzi and co-workers [2] were introduced to explain the presence of a 100 Kyr periodicity in the Earth's ice ages. Although this very small periodic modulation would not be energetically sufficient to directly trigger polarity reversals, the situation must be reconsidered in the framework of SR: in this case the slight periodic modulation nonlinearly couples to random fluctuations $\delta\alpha$ in the α effect [14], thus inducing synchronous transitions between the two polarities. In the framework of Eq. (3), being $\langle \mathbf{v} \rangle = \Omega \times \mathbf{r}$ ($\mathbf{\Omega}$ is the Earth's angular velocity and $\mathbf{r}$ is the Earth's radius), a slight periodic modulation of $\mathbf{\Omega}$, $\delta\mathbf{\Omega} \propto \mathbf{\Omega_0} \cos(2\pi t/T_\Omega + \phi)$, would change Eq. (3) as follows:

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times \{[\langle \mathbf{v} \rangle + \delta \mathbf{v} \cos(2\pi t/T_\Omega + \phi)] \times \langle \mathbf{B} \rangle + \alpha \langle \mathbf{B} \rangle + (\beta - \eta) \nabla \times \langle \mathbf{B} \rangle\}, \quad (6)$$

where $\delta\mathbf{v} = \delta\mathbf{\Omega} \times \mathbf{r}$. In spite of its daring character, we believe this to be the simplest hypothesis to fit in a coherent way our experimental signature of SR phenomenon in geomagnetic polarity reversals. Clearly, a further theoretical work is needed for a full discussion of this hypothesis. Thus, we postpone to future work the validation of this picture.

Summarizing our results in this paper, we have offered some experimental evidence for the occurrence of a SR phenomenon in geomagnetic polarity reversals. Our results suggest that the geomagnetic polarity reversal phenomenon might be due to the coincidence of two characteristic time scales T_Ω and T_k. Anyway, the possible relevance of SR in the geomagnetic reversals remains to be investigated, particularly from a theoretical point of view.

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