Above Threshold Ionization in Tightly Focused, Strongly Relativistic Laser Fields

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The dynamics of electrons ionized from high charge states by lasers with intensity $> 10^{20}\ \mathrm{W/cm^2}$ have been studied. At these intensities $\mathbf{v} \times \mathbf{B}$ forces drive the electrons subsequent to ionization in a trajectory nearly parallel to the laser propagation direction. This gives rise to large energy gains as the electron rides in phase with the laser field over a long distance. Monte Carlo simulations illustrate that, unlike in case of ionization in sub- and near-relativistic intensity fields ($< 10^{19}\ \mathrm{W/cm^2}$), the electron dynamics in the ultrarelativistic case are strongly influenced by the longitudinal electric fields found near the focus of a tightly focused laser.

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The physics of electron energy increase subsequent to ionization by a strong optical field, known as above threshold ionization, has been studied for many years. Even in the very earliest above threshold ionization (ATI) experiments ponderomotive forces on the electron subsequent to ionization were important in shaping the electron energy spectrum and spatial distributions upon ejection from the laser focus [1]. For example, ponderomotive scattering led to pulse duration effects on observed peak shifts in nonperturbative ATI studies. At intensity greater than about $10^{18}\ \mathrm{W/cm^2}$ relativistic effects start to play a role in the ATI dynamics of ionized electrons [2–11]. For example, Moore *et al.* observed tunnel-ionized electrons in a weakly relativistic intensity regime, around $10^{18}\ \mathrm{W/cm^2}$, and found that ponderomotive forces led to a folding of the ejected ATI electrons toward the laser propagation direction [12]. This forward folding of electrons arose from the increasing importance of $\mathbf{v} \times \mathbf{B}$ forces on the electron subsequent to its tunnel ionization. This effect increased with increasing charge state since high charge states are ionized at higher field strengths. The $\mathbf{v} \times \mathbf{B}$ also plays an important role in suppressing recollision dynamics and double ionization at relativistic intensity [13,14].

At the ultrahigh intensities now accessible with the latest generation of petawatt-class lasers, $\mathbf{v} \times \mathbf{B}$ forces play a dramatic role on the dynamics of tunnel-ionized electrons. The dynamics of electron ponderomotive scattering of tunnel-ionized electrons by strongly relativistic laser pulses has been recently examined by Hu *et al.* [15] in numerical simulations. Their simulations were conducted assuming electric and magnetic fields purely transverse to the laser propagation axis, and it was found that the ejected ATI electrons had a well defined relationship between their final energy and ejection angle. At sub- and near-relativistic intensity ATI electrons can be well described by models which assume that the laser electric and magnetic fields are purely transverse to the laser propagation axis. It is known, however, that the actual electric and magnetic fields near a focus have components that are parallel to the laser propagation axis [16]. The magnitude of these longitudinal fields is smaller than the transverse fields by a factor of $\sim 1/kw_0$ (where k is the laser wave number and w_0 is the focal radius). Since these fields are typically at least an order of magnitude smaller than the transverse fields, even for tightly focused beams, they are usually unimportant. However, Quesnel and Mora have examined free electron scattering from a focused, relativistic intensity laser beam including the effects of the longitudinal fields and found that they do, indeed, play an important role in determining the energy and spatial distributions of the scattered electrons [16]. In this Letter we examine ATI from tunnel ionization in the ultrarelativistic regime using tightly focused, petawatt-class laser pulses. We find that, while the electron energies produced can be quite high, approaching 1 GeV at an intensity of $5 \times 10^{21}\ \mathrm{W/cm^2}$, they are strongly influenced by the presence of longitudinal fields near the focus. These longitudinal electric fields serve to decelerate the electrons, pushing them out of phase with the field. This decreases significantly the energy the electrons can acquire in the field and leads to large differences in the ATI electron angular distribution.

In quasiclassical ATI at subrelativistic intensities, the tunnel-ionized electron picks up a drift velocity that is related to the phase in the laser oscillation in which it is ionized [17]. In subrelativistic (linearly polarized) fields, the electrons are ejected predominately in the plane of the laser polarization. As the intensity is increased toward $10^{18}\ \mathrm{W/cm^2}$ (where the normalized vector potential of the laser field, a_0, approaches one), $\mathbf{v} \times \mathbf{B}$ forces start to affect the trajectories of the electrons and the electrons begin to be ejected in a direction forward of the polarization axis.

In a strongly relativistic field, these dynamics will be different. Ionization of a very highly charged ion effectively "injects" a free electron into the field at phase near the field peak. When the laser field $a_0 \gg 1$, the $\mathbf{v} \times \mathbf{B}$ force curves the trajectory of the electron along the laser propagation direction in a small fraction of the field cycle.

The electron with $v_z \sim c$ can then "surf" along with the wave, acquiring energy from the electric field. In an infinite plane wave, the electron has velocity slightly smaller than c, and will fall out of phase with the field. Consequently the electron will be decelerated in the following E-field half-cycle. The finite extent of a focused laser beam, however, can allow some electrons to exit the region of high field before this phase reversal occurs, and the ejected electron will acquire quite substantial energy from the laser. This phenomenon was described in detail in Ref. [15]. Hu *et al.* found that electrons ionized from V^{22+} by 8×10^{21} W/cm^2 pulses would be ejected with energy up to 2 GeV. In a pure plane wave, this effect can be simply described by the equation [12]

$$\tan^2\theta = \frac{2}{\gamma - 1}, \tag{1}$$

where θ is the electron ejection angle with respect to the laser propagation axis and the electron energy is $E_e = \gamma m_e c^2$. It should be noted that this free-wave acceleration of ionized electrons is more dramatic than in the interaction of a pulse with initially free electrons. Ponderomotive expulsion of free electrons from the focus prohibits the very large acceleration that can be achieved when the electron is born in the field at high intensity and the optimum phase for acceleration [16].

The application of Eq. (1) to a focused laser is not straightforward because the interaction is complicated by the presence of longitudinal fields at the focus. The most dramatic consequence of these fields is to decelerate the electron in the z direction and aid in the dephasing of the electron as it travels along with the laser pulse. The consequence of this effect can be determined by examining the dephasing time of the electron as it surfs along with the field. Upon ionization in the field by tunneling, the electron changes its energy as

$$\frac{dE}{dt} = \frac{d}{dt}(\gamma mc^2) = -e\mathbf{E} \cdot \mathbf{v}. \tag{2}$$

The extent to which an electron can acquire energy from the field as it is expelled from a laser focus is related to the dephasing time $\delta\tau$, which is the time during which the sign of $\mathbf{E} \cdot \mathbf{v}$ stays the same (negative) so that the electron accelerates. In an infinite plane wave the dephasing time $\delta\tau$ will be set by the small difference between the z component of the electron velocity and the light phase velocity, c. The dephasing time for an electron born near the peak of the field is given by the time required for the light phase at the electron (whose position is z, and was born at $z = 0$) to shift by $\pi/2$, i.e., assuming a constant v_z, $\pi/2 = \omega t - kz = k(ct - z) = kt(c - v_z)$. For large γ, $k(ct - z) \approx kct/2\gamma^2$ so that

$$\delta\tau_{\text{plane}} \approx \pi\gamma^2/kc. \tag{3}$$

In a focused laser pulse, however, the dephasing time, hence the electron energy gain, is much smaller. As shown by Ref. [16], the fields near a Gaussian focus, to first order in the small parameter $\varepsilon = 1/kw_0$, are

$$E_x = E_o \frac{w_0}{w} \exp\left(-\frac{r^2}{w^2}\right)\sin\phi_G, \qquad B_y = \frac{E_x}{c}, \tag{4a}$$

$$E_z = 2E_o \varepsilon \frac{xw_0}{w^2} \exp\left(-\frac{r^2}{w^2}\right)\cos\phi_G^{(1)}, \qquad B_z = 2\frac{E_o}{c}\varepsilon \frac{yw_0}{w^2} \exp\left(-\frac{r^2}{w^2}\right)\cos\phi_G^{(1)}, \tag{4b}$$

$$E_y = B_x = 0, \tag{4c}$$

where $\phi_G = \omega t - kz + \tan^{-1}(z/z_R) - zr^2/z_R w^2 - \phi_0$, $\phi_G^{(1)} = \phi_G + \tan^{-1}(z/z_R)$, $w = w_0\sqrt{1 + z^2/z_R^2}$, $z_R = kw_0^2/2$ is the Rayleigh length, and $\phi_0 = \pi/2$, if $t = 0$, $z = 0$ correspond to the peak field strength. For illustrative purposes, we consider the case of an electron born by ionization at $t = 0$ and $x, y, z = 0$, i.e., the peak of the field, where the ionization probability is highest. In the case of strongly relativistic fields, we can, to good approximation, say that $z \approx ct$ and $x \approx v_x t \approx \theta z \approx \theta ct$. From Eq. (2), it is clear that this electron will pick up energy from the field until a time, $\delta\tau$, at which $\mathbf{E} \cdot \mathbf{v} = 0$. Using Eqs. (4a) and (4b) we have that

$$\mathbf{E} \cdot \mathbf{v} \propto v_x \sin\phi_G + v_z \cdot 2\varepsilon \frac{x}{w}\cos\phi_G^{(1)} \approx v_x\left(\sin\phi_G + 2\varepsilon\frac{ct}{w}\cos\phi_G^{(1)}\right). \tag{5}$$

If we ignore the effects of longitudinal fields, considering only the field in the paraxial approximation, the dephasing distance for $\gamma \gg 1$ is much greater than the Rayleigh range. Expanding $\tan^{-1}z/z_R \approx \pi/2 - z_R/z$ it follows that $\sin\phi_G = 0$, for electrons traveling nearly parallel to the laser when (for $\theta \ll 1/\gamma$)

$$\delta\tau_{\text{paraxial}} \approx \frac{w_0}{c}\left(\frac{1}{\gamma^2} - \theta^2\right)^{-1/2} \approx \frac{\gamma w_0}{c}. \tag{6}$$

The dephasing time in the focused field, given by Eq. (6), scales linearly with electron energy and is much smaller than the dephasing time in a plane wave, which lengthens as the square of the electron energy. This can be attributed to the fact that the field undergoes the $\pi/2$ Guoy phase shift as the electron propagates out of the focus, leading to complete reversal of the field in the frame of the electron much sooner.

It can be seen from Eq. (5), however, that the effect of the longitudinal E_z field is to decelerate the electron even before it reaches the time given by Eq. (6); (the sign of $\cos\phi_G^{(1)}$ is opposite that of $\sin\phi_G$ during the half cycle after tunnel ionization if the ionization occurs near the peak of the oscillation). This leads to a dephasing time shorter than that given by Eq. (6). It can be seen that when $\gamma \gg 1$ the term in parentheses in Eq. (5) is zero at $z = z_R$. Or, Taylor expanding to first order with z near z_R, we find that the dephasing time in a properly treated Gaussian focus is

$$\delta\tau_{\text{Gaussian}} \approx \frac{z_R}{c}\left(1 - \frac{kz_R}{2\gamma^2}\right). \tag{7}$$

At large γ, the dephasing time of Eq. (7) is smaller than the case of ignoring the longitudinal fields by a factor of $\sim kw_0/2\gamma$. This effect is more important at higher fields (larger γ) and with tighter focused beams (larger ε).

It is possible to estimate the energy pickup by an electron ejected in the forward direction using Eq. (2). Since the electric field falls from its maximum to zero during the flight of an electron ionized near the field peak to the point in the focus where the field phase changes, we can roughly estimate the maximum γ by integrating Eq. (5). To do this we assume an average field strength over the optical cycle of $E_{\text{max}}/2$ and factor this out of the integral, resulting in the estimate for the maximum energy gain,

$$\gamma_{\text{max}} \approx \frac{e}{m_e c^2}\frac{E_{\text{max}}}{2}\theta z_R \sinh^{-1}\left(\frac{c\delta\tau}{z_R}\right), \tag{8}$$

or, using Eq. (7) at large γ, the energy gain is $\gamma_{\text{max}} \approx eE_{\text{max}}\theta z_R/2m_e c^2$. This simple estimate suggests that an electron ionized from Ar^{+17} at peak intensity of 5×10^{21} W/cm^2, with a laser focused to a w_0 of 5 μm at an ejection angle of 3°, will acquire a maximum γ of about 950 (i.e., 500 MeV). Using Eqs. (6) and (7) yields the ratio of the maximum energy without longitudinal fields to that when they are properly included as $\approx \ln(4\gamma/kw_0)$ (i.e., about 5 times higher in our example).

To examine these dynamics in more detail we have performed numerical simulations of ATI in a strongly focused, relativistic intensity femtosecond laser pulse. Our simulation solves the ionization rate equations for atoms at randomly selected points in the focus (similar to the simulations of Ref. [15]). We use Ammosov-Delone-Krainov rates [18] for ionization below the barrier suppression ionization (BSI) threshold [19], using Monte Carlo techniques to determine the ionization time of a given ionization state. If the ion survives to the BSI threshold, it is automatically ionized at that point. Ionized electrons are born with zero energy, and their trajectories are then calculated using the relativistic equations of motion.

At mildly relativistic fields, the electron trajectories are not significantly affected by the longitudinal fields, and the electrons are ejected with an angle well described by Eq. (1). Calculated electron energies as a function of ejection angle from the ionization of Ne at intensity 1×10^{18} W/cm^2 are shown in Fig. 1. A laser wavelength of 800 nm, a 30 fs pulse duration (full width at half

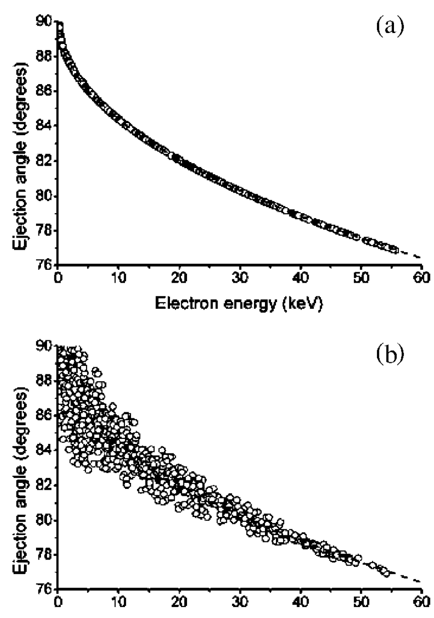

FIG. 1. Simulation results showing electron ejection angle as a function of electron energy for Ne ionized with a 800 nm, 30 fs laser pulse at an intensity of 1×10^{18} W/cm^2. The dashed line shows the prediction of Eq. (1). (a) Results when longitudinal fields are excluded from the calculation. (b) Results when longitudinal fields are included.

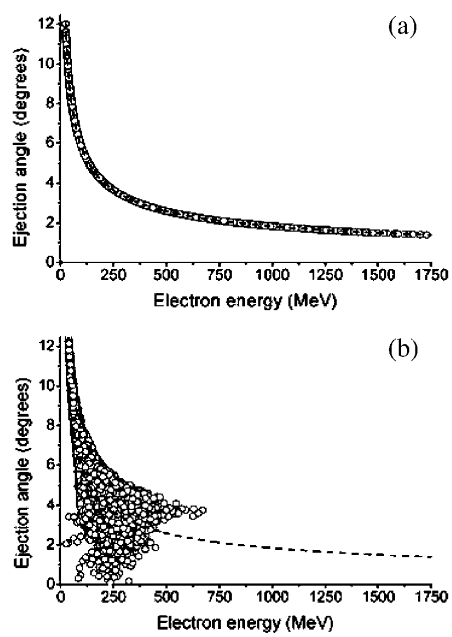

FIG. 2. Simulation results showing electron ejection angle as a function of electron energy for Ar^{+17} ionized with a 800 nm, 30 fs laser pulse at an intensity of 5×10^{21} W/cm^2. The dashed line shows the prediction of Eq. (1). (a) Results when longitudinal fields are excluded from the calculation. (b) Results when longitudinal fields are included.

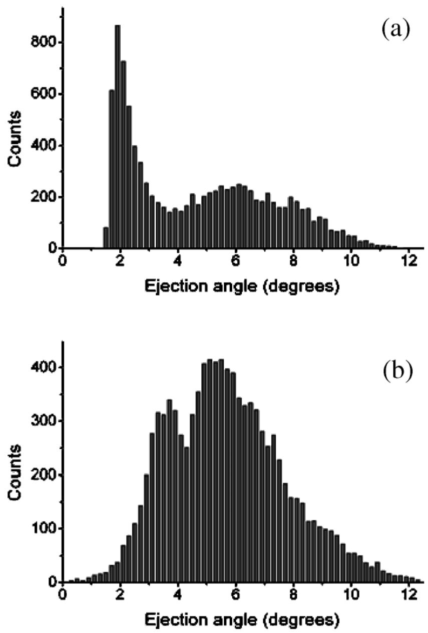

FIG. 3. Integrated angular distributions of electrons from Ar^{+17} ionized under the conditions of Fig. 2. (a) Results when longitudinal fields are excluded from the calculation. (b) Results when longitudinal fields are included.

maximum), and a focal spot radius of 5 μm ($1/e^2$) were used in the simulation. Results for both cases of a plane wave [Fig. 1(a)] and the times when longitudinal fields are properly included [Fig. 1(b)] are shown in this figure. Even though in Fig. 1(b) some broadening occurs, on average the electrons in both cases follow the behavior of Eq. (1), which agrees with the experimental results of [12]. In this weakly relativistic case, the electrons are ejected mainly in the polarization plane, and the longitudinal fields do not significantly affect the energies.

On the other hand, ionization in strongly relativistic fields is clearly affected by dephasing resulting from deceleration by the longitudinal fields. We calculated ionization of Ar^{+17} ions by an 800 nm, 30 fs laser focused to a spot size radius of 5 μm and an intensity of 5×10^{21} W/cm^2. The longitudinal electric field was deliberately excluded from the simulation presented in Fig. 2(a). As expected, the electrons follow the behavior predicted by Eq. (1). This calculation suggests that electrons with gamma up to 3500 are possible ($E = 1.75$ GeV) and is similar to the result of Hu *et al.* [15]. The integrated angular distribution of electrons, shown in Fig. 3(a) for this case, exhibits a well defined hole on the laser axis, with a minimum ejection angle given by $\theta \approx \sqrt{2/\gamma_{\max}}$ where $\gamma_{\max}$ is the maximum gamma achieved by the accelerated electrons.

A calculation of the identical situation when the longitudinal fields are properly included is shown in Fig. 2(b). Here the maximum energy observed in the electrons is significantly lower than in Fig. 2(a) with a maximum γ of 1400 ($E = 0.7$ GeV). This is consistent with the simple estimate of Eq. (8). The ejection angle of the electrons deviates significantly from that expected in the paraxial approximation, as predicted by Eq. (1) [shown as a dashed line in Fig. 2(b)]. This demonstrates the much greater impact of the longitudinal components in the very high intensity regime. Furthermore, the calculated integrated angular distribution, shown in Fig. 3(b), is filled in on axis, with no minimum angle, differing substantially from the calculation of Fig. 3(a).

In conclusion, we have examined the above threshold ionization behavior of electrons in tightly focused, strongly relativistic laser beams. We have shown that the forward directed electrons produced by tunnel ionization at intensity well above 10^{18} W/cm^2 are strongly accelerated but that their ultimate energy gain from the laser is clamped by a dephasing of the electron with the field. In strongly focused light fields, this dephasing is increased by the deceleration of the electrons by the longitudinal component of the E field.

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[1] P. Agostini *et al.*, Phys. Rev. A **36**, 4111 (1987).
[2] V. P. Krainov, J. Opt. Soc. Am. B **14**, 425 (1997).
[3] V. P. Krainov and B. Shokri, Laser Phys. **5**, 793 (1995).
[4] V. P. Krainov and S. P. Roshupkin, J. Opt. Soc. Am. B **9**, 1231 (1992).
[5] C. H. Keitel, J. Phys. B **29**, L873 (1996).
[6] J. Prager and C. H. Keitel, J. Phys. B **35**, L167 (2002).
[7] M. Protopapas, C. H. Keitel, and P. L. Knight, Rep. Prog. Phys. **60**, 389 (1997).
[8] U.W. Rathe *et al.*, J. Phys. B **30**, L531 (1997).
[9] R. Taieb, V. Veniard, and A. Maquet, Phys. Rev. Lett. **81**, 2882 (1998).
[10] R. Taieb, V. Veniard, and A. Maquet, Phys. Rev. Lett. **87**, 053002 (2001).
[11] P. Monot *et al.*, Phys. Rev. Lett. **70**, 1232 (1993).
[12] C. I. Moore, J. P. Knauer, and D. D. Meyerhofer, Phys. Rev. Lett. **74**, 2439 (1995).
[13] E. Lenz, M. Dorr, and W. Sandner, Laser Phys. **11**, 216 (2001).
[14] E. A. Chowdhury, C. P. J. Barty, and B. C. Walker, Phys. Rev. A **63**, 042712 (2001).
[15] S. X. Hu and A. F. Starace, Phys. Rev. Lett. **88**, 245003 (2002).
[16] B. Quesnel and P. Mora, Phys. Rev. E **58**, 3719 (1998).
[17] N. H. Burnett and P. B. Corkum, J. Opt. Soc. Am. B **6**, 1195 (1989).
[18] M.V. Ammosov, N. B. Delone, and V. P. Krainov, Sov. Phys. JETP **64**, 1191 (1986).
[19] S. Augst *et al.*, J. Opt. Soc. Am. B **8**, 858 (1991).