b-τ Unification and Large Atmospheric Mixing: A Case for a Noncanonical Seesaw Mechanism

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We study the second and third generation masses in the context of the minimal renormalizable SO(10) theory. We show that if the seesaw takes the noncanonical (type II) form, large atmospheric neutrino mixing angle requires b-τ unification.

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Introduction.—A suspected quark-lepton symmetry is, as we know, badly broken by the difference in their mixing angles. Small $V_{\rm CKM}$ mixing should be contrasted with the maximal mixing for atmospheric neutrinos and probably large mixing for solar neutrinos. Why is this so? This has become one of the major issues in the so-called fermion mass and mixing problem.

In this Letter we address this question in the minimal renormalizable SO(10) theory, without any additional symmetries or interactions. We focus only on the second and third generations for three reasons: (i) in this case the neutrino mixing angle is maximal and experimentally established; (ii) it is much likely that in the case of the first family we cannot ignore higher dimensional operators; (iii) in this simple 2×2 case we can actually present analytic expressions.

Our main result is the following: We show that in the case of noncanonical seesaw, large neutrino mixing angle requires b-τ unification. The rest of the paper is a proof of this statement and a discussion of its implications.

The choice of SO(10) theory is highly natural. It unifies a family of fermions; it unifies their interactions (except for gravity); it has a seesaw mechanism [1] of small neutrino mass naturally built in; it has charge conjugation as a gauge symmetry; and, in its supersymmetric version, leads naturally to a theory of R parity [2,3].

The last result holds true in the renormalizable version of the theory with a 126_H dimensional Higgs supermultiplet used to give masses to the right-handed neutrinos.

Canonical (type I) versus noncanonical (type II) seesaw mechanism.—The minimal Higgs that breaks SU(2)×U(1) symmetry and gives mass to the fermions is under the Pati-Salam $SU(2)_L \times SU(2)_R \times SU(4)_C$ symmetry

$$10_H = (2, 2, 1) + (1, 1, 6), \tag{1}$$

and so $\langle 10_H \rangle = \langle (2, 2, 1) \rangle \neq 0$ implies the well-known quark-lepton symmetric relation for fermion masses

$$m_D = m_E, \tag{2}$$

which works well for the 3rd family, and fails badly for the first two. You can correct this by adding more Higgses, or appealing to higher dimensional operators (see, for example, [4–6]). However, a nice and important point was raised around 20 years ago [7]. Ten years ago Babu and Mohapatra utilized it to study neutrino masses and mixings [8]. With 10_H and 126_H the Yukawa sector of the Lagrangian is given by

$$\mathcal{L}_Y = 10_H \psi Y_{10} \psi + 126_H \psi Y_{126} \psi, \tag{3}$$

where ψ stands for the 16 dimensional spinors which incorporate a family of fermions, and Y_{10} and Y_{126} are the Yukawa coupling matrices in generation space.

From

$$126_H = (3, 1, 10) + (1, 3, \overline{10}) + (2, 2, 15) + (1, 1, 6) \tag{4}$$

one has

$$M_{\nu_R} = Y_{126} \langle (1, 3, \overline{10})_{126} \rangle, \tag{5}$$

where $\langle (1, 3, \overline{10})_{126} \rangle = M_R$, the scale of $SU(2)_R$ gauge symmetry breaking.

It can be shown that, after the $SU(2) \times U(1)$ breaking through $\langle 10_H \rangle = \langle (2, 2, 1) \rangle \approx M_W$, the (3, 1, 10) multiplet from 126_H gets a small vacuum expectation value (vev) [9,10]

$$\langle (3, 1, 10)_{126} \rangle \propto \frac{M_W^2}{M_{\rm parity}}, \tag{6}$$

where $M_{\rm parity}$ is the scale of the breakdown of parity. In general M_R and $M_{\rm parity}$ are not necessarily equal, but typically one breaks parity through the breaking of $SU(2)_R$ symmetry, in which case $M_R = M_{\rm parity}$. This is what we take hereafter.

In turn, neutrinos pick up small masses

$$M_{\nu_L} = Y_{126} \langle (3, 1, 10)_{126} \rangle + m_D^T M_{\nu_R}^{-1} m_D, \tag{7}$$

where m_D is the neutrino Dirac mass matrix. It is often assumed, for no reason whatsoever, that the second term dominates. This we call canonical (often called type I) seesaw. In what follows we explore the opposite case, which we call noncanonical (type II) seesaw. After all, it does not involve Dirac mass terms and so there is no reason *a priori* in this case to expect quark-lepton analogy of mixing angles. In this sense the noncanonical seesaw is physically more appealing. More than that, we will show that the large leptonic mixing fits perfectly with the small quark mixing, as long as $m_b = m_\tau$.

The crucial ingredient is the fact [8] that through a nonvanishing tadpole a (2, 2, 15) field in 126_H also picks up a vev:

$$\langle (2,2,15)_{126}\rangle \approx \left(\frac{M_R}{M_{\rm GUT}}\right)^2 \langle (2,2,1)_{10}\rangle. \tag{8}$$

In the supersymmetric version of the theory this requires a 210 dimensional Higgs at the grand unified theory (GUT) scale.

Noncanonical seesaw: b-τ unification and large atmospheric neutrino mixing.—Most of the study throughout the years has assumed the canonical seesaw, i.e., the second term dominates in (7). The original claim of [8] that the leptonic mixing matrix V_l had a small 2–3 element was questioned by using a nonminimal model [11] or the freedom to adjust the phases in the mixing matrices [12]. Last year we studied [13] the opposite case, the noncanonical seesaw and noticed that it fitted nicely with a large 2–3 mixing angle responsible for atmospheric neutrinos.

We give here a simple argument in favor of this. We show how maximal μ-τ mixing fits nicely with b-τ unification.

To see this, notice that fermion masses take the following form:

$$M_U = Y_{10} v_{10}^u + Y_{126} v_{126}^u, \tag{9}$$

$$M_D = Y_{10} v_{10}^d + Y_{126} v_{126}^d, \tag{10}$$

$$M_E = Y_{10} v_{10}^d - 3Y_{126} v_{126}^d, \tag{11}$$

$$M_N = Y_{126}\langle (3,1,10)_{126}\rangle, \tag{12}$$

where U, D, E, N stand for up quark, down quark, charged lepton, and neutrino, respectively, while $v_{10}^{u,d}$ and $v_{126}^{u,d}$ are the two vevs of (2, 2, 1) in 10_H and (2, 2, 15) in 126_H, and the last formula is the assumption of the noncanonical seesaw. The result is surprisingly simple. Notice that [14]

$$M_N \propto Y_{126} \propto M_D - M_E. \tag{13}$$

Now, let us study the 2nd and 3rd generations, and work in the basis of M_E diagonal. The puzzle then is why a small mixing in M_D corresponds to a large mixing in M_N. For simplicity take the mixing in M_D to vanish, $\theta_D = 0$, and ignore the second generation masses, i.e., take $m_s = m_\mu = 0$. Then

$$M_N \propto \begin{pmatrix} 0 & 0 \\ 0 & m_b - m_\tau \end{pmatrix}. \tag{14}$$

Obviously, unless $m_b = m_\tau$, neutrino mixing vanishes. Thus, large mixing in M_N (the physical leptonic mixing in the above basis) is deeply connected with the b-τ unification. Notice that we have done no model building whatsoever; we assumed only a renormalizable SO(10) theory and the noncanonical seesaw.

Before we discuss (14) more carefully by switching on m_μ, m_s and the mixings, let us comment on the implication of our result. First, notice that it does not depend on the number of 10_H's. Notice also that it is not easily generalized to three generations, i.e., it is not easy to give the same reasoning why the solar neutrino mixing should be large.

In short, our results should be taken as an argument in favor of the noncanonical seesaw: large atmospheric mixing angles and b-τ unification seem to prefer clearly this form of the seesaw mechanism.

Quantitative analysis.—Let us now be more quantitative and turn on m_s, m_μ, and θ_D. Notice that θ_D is not a 2–3 $V_{\rm CKM}$ mixing angle, but rather a difference between charged lepton and down quark mixing angles (recall that we choose M_E diagonal).

It is important to notice that not all the 32 doublets in (2, 2, 1) and (2, 2, 15) remain light; with the minimal fine tuning we end up with only two of them at M_Z. Let us denote their vevs by v_i $(i = u, d)$, where $M_W = g\sqrt{v_u^2 + v_d^2}/2$ and we adopt as usual $\tan\beta = v_u/v_d$. Then we can write

$$v_{10}^i = v_i \cos\alpha_i, \qquad v_{126}^i = v_i \sin\alpha_i, \qquad (i = u, d), \tag{15}$$

where α_i are unknown angles. Defining

$$x = \frac{\tan\alpha_u}{\tan\alpha_d}, \qquad y = \frac{\cos\alpha_d}{\cos\alpha_u} \tag{16}$$

(notice that either $x^2 \le y^2 \le 1$ or $x^2 \ge y^2 \ge 1$), it is a simple exercise to derive from (9)–(12)

$$Y_E = \frac{1}{1-x}[4yY_U - (3+x)Y_D], \tag{17}$$

$$Y_N = c(Y_E - Y_D), \tag{18}$$

where $M_U = v_u Y_U$, $M_D = v_d Y_D$, $M_E = v_d Y_E$, $M_N \propto Y_N$, and c is an unknown constant in this theory. Since Y's are symmetric, we can write for species X

$$Y_X = X Y_X^d X^T, \tag{19}$$

where Y_X^d are diagonal Yukawa matrices and X are in general unitary. In what follows we do not wish to play with the adjustment of phases and so take X to be orthogonal matrices for simplicity and transparency.

Let θ_l, θ_D, and θ_q denote the rotation angles in $E^T N$, $D^T E$, and $D^T U$, respectively (θ_l and θ_q are the leptonic and quark weak mixing angles, respectively). From (18) we get

$$\tan 2\theta_l = \frac{\sin 2\theta_D}{\frac{y_\tau - y_\mu}{y_b - y_s} - \cos 2\theta_D}. \quad (20)$$

Next, we wish to connect θ_D with θ_q in order to have the dependence of θ_l with θ_q. From (17) one has ($c_D = \cos\theta_D$, $c_q = \cos\theta_q$, etc.)

$$\begin{pmatrix} c_D^2 y_\tau + s_D^2 y_\mu - y_b & c_q^2 y_t + s_q^2 y_c \\ s_D^2 y_\tau + c_D^2 y_\mu - y_s & s_q^2 y_t + c_q^2 y_c \end{pmatrix} \begin{pmatrix} x \\ 4y \end{pmatrix} = \begin{pmatrix} c_D^2 y_\tau + s_D^2 y_\mu + 3y_b \\ s_D^2 y_\tau + c_D^2 y_\mu + 3y_s \end{pmatrix}. \quad (21)$$

After introducing

$$\epsilon_u = \frac{y_c}{y_t}, \qquad \epsilon_d = \frac{y_s}{y_b}, \qquad \epsilon_e = \frac{y_\mu}{y_\tau}, \qquad \epsilon = \frac{y_b - y_\tau}{y_b}, \quad (22)$$

and after some computational tedium we get from (17) and (21)

$$(1 - \epsilon_e)\tan\theta_D[(1 - \epsilon_u\epsilon_d)\tan^2\theta_q + (\epsilon_u - \epsilon_d)] = (1 - \epsilon_u)\tan\theta_q[(1 - \epsilon_e\epsilon_d)\tan^2\theta_D + (\epsilon_e - \epsilon_d)]. \quad (23)$$

In the limit $\epsilon_i = 0$ ($i = u, d, e$) there are two solutions: $\tan\theta_D = 0$ and $\tan\theta_D = \tan\theta_q$. The first solution can be shown to be unrealistic, whereas the second one gives the important relation between the physical mixing angles of quarks and leptons:

$$\tan 2\theta_l = \frac{\sin 2\theta_q}{2\sin^2\theta_q - \epsilon}. \quad (24)$$

Since $\theta_q = \theta_{bc}$ of $V_{\rm CKM}$, $\theta_q \approx 10^{-2}$, (24) shows manifestly that $\tan\theta_l \approx 1$ requires $\epsilon \approx 0$, i.e., $y_b \approx y_\tau$ as we argued repeatedly.

Let us now switch on the second generation masses, i.e., let us take $\epsilon_i \neq 0$. From (23) one can see that the physically acceptable solution is

$$\tan\theta_D = \mathcal{O}(\delta), \qquad \delta = \epsilon_i, \tan\theta_q \approx 10^{-2}. \quad (25)$$

From (20) is then obvious that b-τ unification $y_\tau = y_b + \mathcal{O}(\delta)$ is sufficient to make the mixing angle large, i.e., $\tan 2\theta_l = \mathcal{O}(1) \gg \delta$. This is our main result, rather nontrivial in our opinion. A small quark mixing angle automatically leads to a large leptonic mixing in the 2–3 case.

From high to low energy: running.—Our expressions are valid at the unification scale $M_{\rm GUT}$. Thus we must run the physical parameters from $M_{\rm GUT}$ to M_Z in order to be precise. However, in this case the running is not so important as it may seem. Namely, in this Letter we want to study the implications of the SO(10) symmetry (in its minimal renormalizable version) on fermion masses and mixings. What we said up to now is equally valid in ordinary and supersymmetric (with 210_H Higgs) SO(10) gauge theory. We wish to emphasize the generic feature of the model, that is the connection between the large $\theta_{\rm atm}$ and b-τ unification and do not worry so much about the precise numerical estimates. This requires specifying precisely the nature of the low energy effective theory. Still, it is instructive to see the impact of running. We thus discuss briefly the supersymmetric case and leave the complete discussion for a longer paper now in preparation.

The neutrino matrix elements M_{ij} run at the 1-loop level and neglecting threshold effects according to [15–17]

$$16\pi^2 \frac{d}{dt} M_{ij} = \left[y_\tau^2 (k_i + k_j) + 6y_t^2 - 6g_2^2 - \frac{6}{5} g_1^2 \right] M_{ij}, \quad (26)$$

where $t = \ln(Q/M_Z)$, g_1 is normalized in the SU(5) fashion, $i, j = 2, 3$ stand for the second and third generations, and $k_2 = 0$, $k_3 = 1$. The neutrino mixing angle at the electroweak scale is

$$\tan 2\theta_l|_{M_Z} = \frac{2M_{23}(0)}{M_{22}(0) - M_{33}(0)} = \frac{2M_{23}(t_{\rm GUT})B_\tau}{M_{22}(t_{\rm GUT}) - B_\tau^2 M_{33}(t_{\rm GUT})}, \quad (27)$$

where $t_{\rm GUT} = \ln(M_{\rm GUT}/M_Z)$ and

$$B_\tau = \exp\left[-\frac{1}{16\pi^2} \int_0^{t_{\rm GUT}} y_\tau^2(t) dt \right]. \quad (28)$$

The elements $M_{ij}(t_{\rm GUT})$ are exactly the ones discussed throughout the paper. We can thus recalculate (20) (valid at $M_{\rm GUT}$) at M_Z:

$$\tan 2\theta_l|_{M_Z} = \frac{B_\tau \sin 2\theta_D}{\frac{y_\tau - y_\mu}{y_b - y_s} - \frac{1+B_\tau^2}{2}\cos 2\theta_D - \frac{1-B_\tau^2}{2}(1 + 2\frac{y_\tau - y_b}{y_b - y_s})}. \quad (29)$$

All the parameters of the right-hand side are to be evaluated at the GUT scale. For this reason the same Eq. (23) is again used to express θ_D. Clearly, as before, large neutrino mixing angle comes out as soon as y_b and y_τ unify at the GUT scale. Of course, the precise value of the neutrino mixing angle depends on this running, however the qualitative behavior does not change.

A more detailed approach would require to use numerical techniques to account for (1) the running as function of $\tan\beta$; (2) the inclusion of threshold corrections [18]; and (3) first generation effects. However, threshold effects in SO(10) are bound to be important and high precision calculations may actually not be so useful, see for example [19].

What about the values of neutrino masses? We do not enter into this issue here since we have no new results beyond [13].

Summary and outlook.—The sharp contrast of quark and lepton mixings is often considered a deep puzzle. We argued here that it is actually quite natural in the minimal SO(10) renormalizable theory. All that is required is that the seesaw mechanism takes a noncanonical form free from Dirac masses. The approximate formula (24) expresses it clearly: a small quark mixing $\theta_q = \theta_{bc} \approx 10^{-2}$ gives naturally a large $\theta_l = \theta_{\rm atm}$ if $\epsilon \approx 0$, i.e., $y_b \approx y_\tau$. Actually, the essence of our work lies in formulas (13) and (14). Formula (14), valid in the approximation of vanishing second generation masses and vanishing quark and lepton mixings speaks eloquently: unless $m_b = m_\tau$ at the large scale, we will have a vanishing atmospheric neutrino mixing. In short, the noncanonical seesaw marries nicely b-τ unification with the maximal atmospheric neutrino mixing. This can be of great help in trying to pinpoint the nature of the seesaw mechanism: our study points in favor of the noncanonical version.

Strictly speaking, a numerical study showed that in the 3×3 case, by playing with CP phases, even the canonical seesaw can be made to work [12]. However, in our case, the 2–3 family study offers physical insight into the question, and after all the first family of fermions may suffer from the higher dimensional operators. The 10_H and 126_H, the minimal Higgses needed to give masses to all fermions, work beautifully: 10_H offers $m_b = m_\tau$, and 126_H offers $3m_s = -m_\mu$ at the GUT scale; and in this framework a small θ_{cb} (θ_{ts}) and a large θ_{atm} become naturally connected. Thus, the observational evidence that quarks and leptons have sharply different mixing angles fits nicely with the belief that they are one and the same object at a fundamental level.

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