

# Nonlinear Quasiparticle Tunneling between Fractional Quantum Hall Edges

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Remarkable nonlinearities in the differential tunneling conductance between fractional quantum Hall edge states at a constriction are observed in the weak-backscattering regime. In the  $\nu = 1/3$  state a peak develops as temperature is increased and its width is determined by the fractional charge. In the range  $2/3 \leq \nu \leq 1/3$  this width displays a symmetric behavior around  $\nu = 1/2$ . We discuss the consistency of these results with available theoretical predictions for interedge quasiparticle tunneling in the weak-backscattering regime.

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Under the application of intense perpendicular magnetic fields, the kinetic energy spectrum of the two dimensional electron gas (2DEG) breaks into a sequence of macroscopically degenerate Landau levels. At "magic" fractional values of the filling factor  $\nu$  the 2DEG condenses into collective phases that display the fractional quantum Hall (FQH) effect [1,2]. The emergence of these collective states was discussed by Laughlin in terms of new quasiparticles carrying fractional charge [3]. Additionally, the lowest-energy charged excitations of FQH liquids are confined at the edge of the sample in one-dimensional branches whose excitations propagate only in one direction. Wen [4] was the first to describe such one-dimensional modes in terms of a chiral Luttinger liquid. Wen's proposal stimulated a considerable amount of work aimed at understanding this non-Fermi liquid state [5].

The departure from Fermi-liquid behavior is predicted to show up in many properties of an interacting one-dimensional system [6]. Most of the experiments in quantum Hall systems concentrated on the power-law behavior of the electron tunneling from a metal to the FQH edge [7–9] and between two FQH edges [10–12] in the *strong backscattering* limit. The interedge tunneling process, in particular, can be experimentally induced at a quantum point contact (QPC) constriction. In the *strong backscattering* regime one observes the tunneling of *electrons* between two quantum Hall fluids separated by the QPC. For simple fractions (i.e.,  $\nu = 1/q$ , where  $q$  is an odd integer), this leads to a dc tunneling current at temperature  $T = 0$  given by  $I_T \propto V_T^{(2/\nu-1)}$ . Notably  $I_T$  vanishes when  $V_T$ , with  $V_T$  labeling the potential difference between the two edges, tends to zero [5].

In the opposite limit of *weak backscattering* the quantum Hall fluid is weakly perturbed by the QPC constriction. In this case the interedge tunneling current (again, at  $\nu = 1/q$ ) consists of Laughlin quasiparticles of charge  $e^* = \nu e$  that scatter between the edges through the quantum Hall fluid [see Fig. 1 inset (a)]. At  $T = 0$  the quasiparticle tunneling rate is predicted to grow at low voltages as  $I_T \propto V_T^{(2\nu-1)}$  in contrast to the electron-tunneling case discussed above. At finite temperature, below a critical value  $V_{T,max}$  of the order of  $k_B T/e^*$ , the

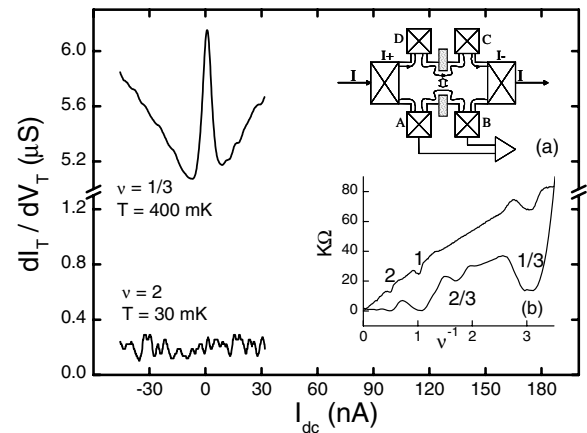


FIG. 1. Main panel: differential tunneling conductance ( $dI_T/dV_T$ ) as a function of driving current ( $I_{dc}$ ) at the constriction for filling factors  $\nu = 2$  at  $T = 30$  mK and  $\nu = 1/3$  at  $T = 400$  mK. Inset (a): sketch of the device and experimental setup. Inset (b): longitudinal ( $R_{AB}$ ) and transverse ( $R_{AC}$ ) resistances as a function of magnetic field at  $I_{dc} = 0$  and  $T = 30$  mK.

tunneling current reverts to the linear Ohmic behavior. In the differential tunneling characteristics ( $dI_T/dV_T$ ) this leads to a peak centered at  $V_T = 0$  with a width  $\Delta V_T \approx 2V_{T,\max}$  [13]. The complex phenomena associated to nonlinear quasiparticle tunneling, however, are still largely experimentally unexplored. The resonant regime was investigated by Maasilta *et al.* [14], but magnetotransport experiments in the *weak backscattering* regime mostly concentrated on shot noise measurements aimed at detecting the quasiparticle fractional charge [15].

In this Letter we report the observation of nonlinear interedge tunneling in the *weak backscattering* regime. We present  $dI_T/dV_T$  as a function of  $V_T$  in a wide range of  $T$  and  $\nu$ . In the FQH regime our data cannot be described by electron tunneling but display the features of Wen's theory of quasiparticle tunneling. We demonstrate that while the differential tunneling characteristic shows the tendency towards a diverging behavior as the temperature is lowered at  $\nu = 1/3$ , it develops the peak at  $V_T = 0$  as  $T$  is increased. Width and shape of this zero-bias peak are determined by the fractional charge of the quasiparticle. We also discuss the results obtained for filling factors between  $\nu = 2/3$  and  $\nu = 1/3$  that display an evolution not explained by current theories. We believe that these results combine to provide the first evidence of nonlinear quasiparticle tunneling in FQH systems.

The devices here studied were realized starting from a high-mobility GaAs/Al<sub>0.3</sub>Ga<sub>0.7</sub>As 2DEG with low-temperature mobility  $\mu \sim 1 \times 10^6$  cm<sup>2</sup>/Vs and density  $n \sim 5 \times 10^{10}$  cm<sup>-2</sup>. The 2DEG was located 140 nm from the surface. The measurements were performed in a dilution refrigerator. The QPC constriction was nanofabricated on Hall-bar mesas using e-beam lithography and Al metallization. The width and length of the QPC constriction were 300 nm and 600 nm, respectively.

Differential interedge tunneling conductance as a function of  $V_T$  was controlled by exploiting the QPC [see Fig. 1 inset (a)] in order to force edge states to flow close to each other in the constriction. The split gate was biased at  $V_g = -0.4$  V which is just below the 2DEG depletion leading to 2D-1D threshold at zero magnetic field. At these bias conditions, therefore, the conduction takes place through the point contact only. A current  $I$  with both dc and ac components was supplied to the Hall bar. This current causes a Hall voltage drop  $V_H = \rho_{xy}I$  (with dc and ac components and  $\rho_{xy} = h/\nu e^2$ ) between the counterpropagating edge channels within the constriction [16]. In the weak-backscattering regime  $V_H$  coincides with the tunneling bias, i.e.,  $V_T = V_H$ . Four-wire differential-resistance measurements were carried out using an ac lock-in technique [see Fig. 1 inset (a)] with  $0 \leq I_{dc} \leq 45$  nA and  $I_{ac} = 250$  pA. The measured quantity is the resistivity drop at the constriction (the differential longitudinal resistance  $dV/dI$ ). In the weak-backscattering regime  $dV/dI$  is directly related to the differential tunneling conductance by  $dV/dI = \rho_{xy}^2 dI_T/dV_T$ . The pres-

ence of residual backscattering outside the constriction leads to a background signal. In the present devices this background is significant at  $\nu = 1/3$ . Even away from the constriction the longitudinal resistivity is around 4 k $\Omega$ . However, it does not display a sizable variation as a function of  $V_T$  (data not shown) at least for low biases. This and additional control experiments carried out at lower values of  $V_g$  allow us to unambiguously attribute the observed structures in  $dV/dI$  to quasiparticle tunneling at the QPC.

Inset (b) in Fig. 1 shows the longitudinal and transverse resistances at  $I_{dc} = 0$  as a function of magnetic field at  $V_g = -0.4$  V. The longitudinal resistance, in particular, displays Shubnikov-de Haas oscillations in the presence of the constriction. At  $I_{dc} \neq 0$  marked nonlinearities are observed in the FQH regimes. In the main panel of Fig. 1 two representative differential conductance curves at filling factors 2 and 1/3 are shown. As expected, the behavior in the integer regime is linear even at the lowest temperatures explored. In order to emphasize the differences observed in the fractional regime, the marked nonlinear behavior measured in the case  $\nu = 1/3$  is shown at the comparatively high temperature of  $T = 400$  mK.

Figure 2(a) shows the temperature evolution at  $\nu = 1/3$  for temperatures up to 900 mK. At this FQH state, the lowest- $T$  curve (at 30 mK) shows a minimum at zero bias. The zero-bias peak, however, is recovered at higher temperatures. It develops above 400 mK and tends to disappear as  $T$  is increased above 900 mK. We can understand the data in Fig. 2 in the framework of the *weak-backscattering* theory for interedge tunneling originally proposed by Wen [13]. The weak-backscattering limit can be quantified comparing the tunneling current ( $I_T$ ) to the

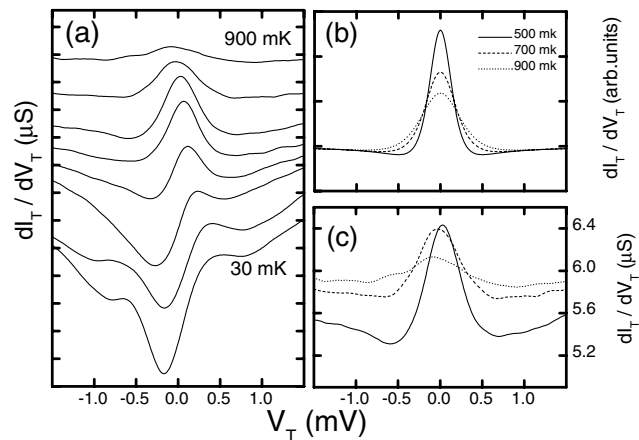


FIG. 2. (a) Differential tunneling conductance ( $dI_T/dV_T$ ) for filling factor  $\nu = 1/3$  at different temperatures (30, 100, 200, 300, 400, 500, 700, 900 mK from bottom to top). (b) Calculated  $dI_T/dV_T$  [derivative of Eq. (1)] in the weak-backscattering regime at  $T = 500, 700,$  and  $900$  mK. (c) Selected differential tunneling conductance curves at the same temperatures as in (b).

total current ( $I$ ) flowing through the device. We define the scattering to be *weak* when  $I_T \ll I$  (or  $dV/dI \ll h/\nu e^2 \sim 75$  k $\Omega$ ). In this regime we expect

$$I_T(V_T) = \frac{2\pi|t|^2}{\Gamma(2g)} \left| \frac{2\pi T}{T_0} \right|^{2g-1} \left| \Gamma\left(g + \frac{ix}{2\pi}\right) \right|^2 \sinh\left(\frac{x}{2}\right), \quad (1)$$

where  $g = e^*/\nu e^2$ ,  $x = e^*V_T/k_B T$ ,  $\Gamma$  is the Euler gamma function, and  $k_B T_0$  is an energy scale of the order of  $\hbar v/l$ , where  $v$  is the speed of edge waves,  $l$  is the magnetic length, and  $t$  is the interedge tunneling amplitude. Note that in Wen's original theory the chiral Luttinger liquid was assumed to exist only at the filling factors of the FQH effect. In (1), however,  $\nu$  is allowed to vary continuously. Such an extension of the original formulation can be justified on the basis of a hydrodynamic model, which allows us to derive the chiral Luttinger liquid at generic values of  $\nu$  without relying on the occurrence of the FQH effect [17]. The value of the effective charge  $e^*$ , on the other hand, is known with reasonable certainty only at  $\nu = 1/q$ , in which case  $e^* = \nu e$  and  $g = \nu$ . A complete theoretical or experimental determination of the dependence of  $e^*$  on  $\nu$  remains an open challenge.

The behavior of  $I_T$  depends crucially on the relative size of  $e^*V_T$  and  $k_B T$ . At low temperatures (1) predicts the nonlinear behavior  $I_T \propto V_T^{2g-1}$ , which, for  $g < 1/2$ , leads to a growing current with decreasing bias. At higher temperatures (1) predicts an Ohmic behavior  $I_T \propto V_T$ . The crossover between the two regimes occurs at  $V_T = V_{T,\max}$ , where  $I_T$  reaches a maximum. Figure 2(b) shows the calculated differential tunneling conductance [the derivative of (1)] at  $\nu = 1/3$  ( $g = 1/3$ ) at different temperatures. The peak at zero bias arises from the Ohmic region of the  $I_T$ - $V_T$  relation. As the temperature lowers this peak is expected to grow in intensity without saturation and to shrink in size. Indeed the width of this peak (defined here as the distance between the two zeros of the differential conductance) is  $2V_{T,\max} \sim 4.79k_B T/e^*$  [13]. For  $V_T > V_{T,\max}$  the differential conductance becomes negative and eventually tends to zero as  $V_T^{2g-1}$ .

Figure 2(c) reports the experimental results obtained for temperatures higher than 400 mK. The observed overall trend is in good agreement with the calculated behavior shown in Fig. 2(b) and discussed above. The agreement is particularly satisfactory for what concerns the width of the conductance peak determined by the effective charge of the quasiparticle. At higher temperatures the peak broadens and its amplitude decreases following qualitatively the theory. At temperatures higher than 1 K the tunneling is completely linear in this voltage range. The data also show evidence of a *negative* contribution to the tunneling conductance at larger values of the bias voltage although the large background raises the average value of the conductance above zero.

At temperatures lower than 400 mK [see Fig. 2(a)], however, the tunneling conductance exhibits a minimum

at zero bias that was observed in different devices with electron densities up to  $n \sim 9 \times 10^{10}$  cm $^{-2}$ . The disappearance of the zero-bias peak can be related to its progressive shrinking with temperature: for a given current modulation intensity a threshold temperature can be estimated below which the peak cannot be detected. In the weak-backscattering limit  $V_{T,\text{ac}} \sim \rho_{xy} I_{\text{ac}} \sim 0.02$  mV  $\sim$  100 mK. This value is not in agreement with our experimental finding and suggests that the system by lowering the temperature may evolve into the strong backscattering regime [18]. In addition, curves display a sample-dependent asymmetric behavior probably originating from the slight asymmetry of split-gate grounding at nonzero biases. This effect can lead to a large modification of the tunneling conductance in the low- $T$  regime at  $\nu = 1/3$  when tunneling close to zero voltage becomes very sensitive to the effective width of the constriction.

Next we examine the dependence of tunneling current on the filling factor shown in Fig. 3 for different temperatures (here higher values of the tunneling conductance are in yellow, lower values in black). The measured  $I_T$ - $V_T$  relation was linear at integral filling factor (data not shown) and for temperatures above 900 mK. Otherwise, above 400 mK, the zero-bias peak is visible in the whole range  $1/3 \leq \nu \leq 2/3$  and presents an interesting evolution: the peak is strongest and narrowest at  $\nu = 1/3$  and  $\nu = 2/3$  and rapidly broadens and loses its strength as  $\nu = 1/2$  is approached symmetrically from either side. This is shown in more detail in Fig. 4 where the differential tunneling conductance is plotted at three representative values of the magnetic field. The inset of Fig. 4 reports the width of the zero-bias peak as a function of

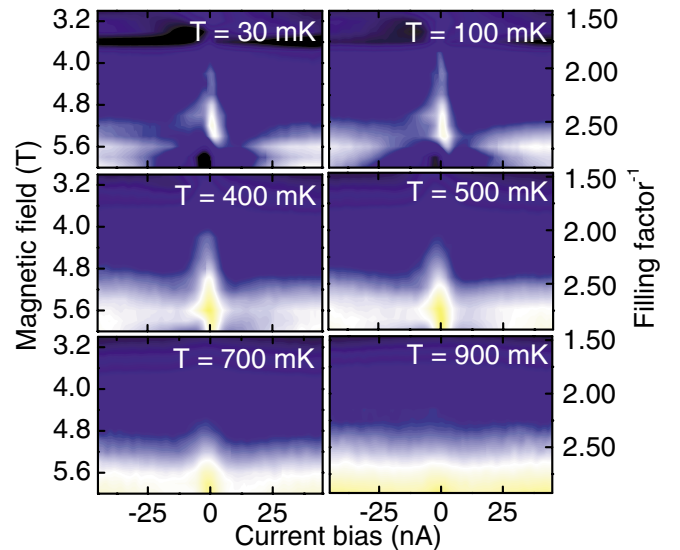


FIG. 3 (color). Color plots of the differential tunneling conductance ( $dI_T/dV_T$ ) as a function of the driving current  $I_{\text{dc}}$  and magnetic field [ $V_T = \rho_{xy}(B)I_{\text{dc}}$ ] at different temperatures.  $\nu = 1/3$  occurs at  $B \approx 6$  T. Bright yellow regions correspond to a high value of the tunneling conductance.

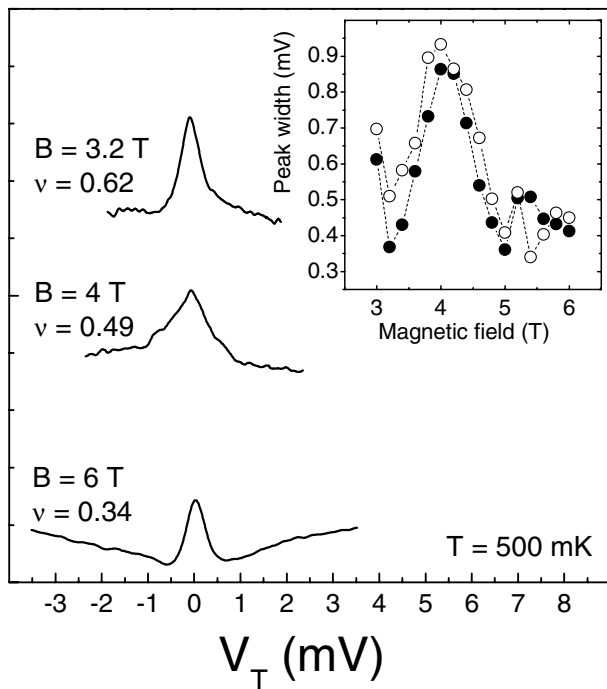


FIG. 4. Representative differential tunneling conductance ( $dI_T/dV_T$ ) at three values of magnetic field and  $T = 500$  mK. The inset reports the evolution of the peak width (as derived from a Lorentzian best-fit procedure) as a function of magnetic field for  $T = 400$  mK (filled circles) and  $T = 500$  mK (open circles). [See Fig. 1 inset (b) to relate magnetic field to the filling factor.]

magnetic field at  $T = 400$  mK (filled circles) and  $T = 500$  mK (open circles). A similar behavior was found also at  $T = 700$  mK (data not shown).

A convincing theory of this is not available yet. Setting  $g = \nu$  in (1) yields a tunneling conductance curve with a peak at zero bias; however, the width of the peak increases monotonically with  $\nu$ , while its amplitude decreases. It should be pointed out that, at  $\nu = 2/3$  [and more generally at filling factors of the Jain sequence  $np/(np + 1)$ ], the result of (1) with  $g = \nu$  is in agreement with the weak-backscattering theory of composite edge states developed by Kane and Fisher [5]. Finally, we point out that also the width of the zero-bias peak observed at  $\nu > 1/3$  and low temperature is not compatible with (1). One is tempted to associate the nonmonotonic behavior of the peak with the different structure of the zero-temperature fixed points controlling the charge mode for the Jain sequences with positive and negative  $p$ . However, additional theoretical analysis is needed [19].

In conclusion, we reported nonlinear behavior in the interedge tunneling in the fractional quantum Hall regime in the weak-backscattering limit. The observation of a zero-bias peak in the differential tunneling conductance

has been interpreted as evidence for quasiparticle tunneling between fractional edge states. In selected ranges of temperatures and filling factors our data are consistent with Wen's theory of quasiparticle tunneling. Results as a function of filling factors reveal intriguing features not predicted by current theories.

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