Reactive Glass and Vegetation Patterns

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The formation of vegetation patterns in the arid and the semiarid climatic zones is studied. Threshold for the biomass of the perennial flora is shown to be a relevant factor, leading to a frozen disordered pattern in the arid zone. In this "glassy" state, vegetation appears as singular plant spots separated by irregular distances, and an indirect repulsive interaction among shrubs is induced by the competition for water. At higher precipitation rates, the diminishing of hydrological losses in the presence of flora becomes important and yields spatial attraction and clustering of biomass. Turing patterns with characteristic length scale emerge from the disordered structure due to this positive-feedback instability.

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Vegetation patterns in the arid and the semiarid climatic zones [1,2] are an interesting example of spontaneous symmetry breaking in complex systems. Competition of shrubs for a limited supply of water is the relevant process that dictates the spatial organization. The struggle for water induces an indirect interaction among shrubs, as the flora is extinct if its water supply is insufficient.

Competition for common resources has been considered for many years as one of the basic processes in population dynamics [3,4]. It may be shown that, if two species compete for a common resource, the one that is able to survive at a lower resource level prevails and displaces the other species population. Stable coexistence of N species is possible if there exist N different resources and each of the species is a superior competitor for one of the supplies, that is, it has one *biological niche*.

The situation becomes more complicated if the resource admits spatial dynamics. Recent theoretical and experimental work reveals the dynamics of competing populations in water, where light, the limiting resource, is consumed gradually by the upper layers of aquatic phytoplankton [5]. This model may be extended to include spatial dynamics of the fauna, but it does not support time independent patterning.

Vegetation patterns are an example of one species (shrubs) and one resource (water) system, where field studies revealed a wide variety of stable, or almost stable, spontaneous segregation modes. Understanding the underlying mechanism for generation of such patterns and their observed resilience is considered as an important step toward a comprehension of the desertification process, where environmental effects such as climate changing and grazing destroy the natural balance toward stable aridity.

Technically, the water-biomass system has been considered to be a spatially extended nonlinear system, that, in some parameter range, may yield stripes, spots, labyrinth, and other ordered arrangements attributed to a positive-feedback mechanism, i.e., to the inhibition of water runoff and evaporation by the flora [6]. However, the typical perennial vegetation patterns in the semiarid zone are disordered, as one can easily see in Fig. 1. The generic spatial organization of perennial flora varied along the precipitation gradient: from scattered "green spots" in the arid zone through clusters of shrubs in the semiarid zone, to an almost full coverage of the soil by biomass in the humid/subhumid climate. Analysis of the transverse correlations in the three panels of Fig. 1 shows that the correlation length in the semiarid zone is larger (by a factor of 2–3) than in the other regions, and seems to indicate weak long-range oscillations in the Mediterranean site, perhaps a precursor of Turing instability.

In this Letter we present a general and simple model of the water-shrubs reaction that is able to yield all these features. Our model takes into account the intrinsic "noise," i.e., the amplification of initial fluctuations due to the minimal size needed for the survival of perennial flora. The resulting pattern is disordered but robust, thus it may be considered as the reactive equivalent of a static glass [7].

To present the model, let us begin from its zero dimensional (''flower pot'') deterministic and continuous dynamics. With water supplied to the system at some rate R and continuum vegetation growth, the time evolution of the water-shrub system is described by the following, nondimensionalized rate equations:

$$\frac{\partial B}{\partial t} = wB - \mu B, \qquad \frac{\partial w}{\partial t} = R - w - \lambda w B, \qquad (1)$$

where w stands for the available water density, B is the density of shrubs biomass, the term wB represents growth of shrubs as they consume water, while $-\lambda w B$ is the corresponding consumption of water by shrubs. μ is the "death rate" of the vegetation in the absence of water and

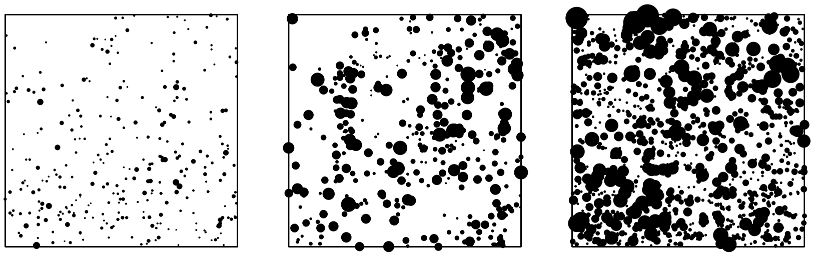

FIG. 1. Results of direct field measurements at three different locations along the precipitation gradient. The distribution of perennial shrubs (annual flora not included) is presented for an area of 100 square meters at each site. Each black spot presents a shrub and the size of a spot is proportional to the area size of the canopy. Shrubs distribution on hillslopes has been taken at three sites representing mildly arid, semiarid, and subhumid climate conditions in Israel. The mildly arid site (left panel, with a mean annual rainfall of 260 mm, hillslope gradient 11°) is located at Mishor Adumim, 10 km east of Jerusalem. The semiarid site (middle, with mean annual rainfall 330 mm and gradient of 16°), is located at Ma'ale Adumim, 8 km east of Jerusalem. The Mediterranean site (right, mean annual rainfall 620 mm, hillslope gradient 13°), is located at Giv'at Ye'arim, 11 km west of Jerusalem. All three sites have hard calcareous bedrock and southeast exposure (azimuth 140°–150°).

the term $-w$ represents water losses by percolation and evaporation.

The set of differential Eqs. (1) admits two non-negative fixed points. The trivial one at $B_0 = 0$, $w_0 = R$, becomes unstable to small perturbations as $R = \mu$, while above this value a "coexistence" fixed point at

$$w_1 = \mu, \quad B_1 = \frac{R - \mu}{\lambda \mu}, \tag{2}$$

becomes a stable node [if $R^2 < 4(R - \mu)\mu^2$] or a stable focus [if $R^2 > 4(R - \mu)\mu^2$]. Adding lateral water flow to the above model leads to a reaction-diffusion equations of the form

$$\begin{aligned} \frac{\partial B}{\partial t} &= wB - \mu B, \\ \frac{\partial w}{\partial t} &= D\nabla^2 w - \mathbf{v} \cdot \nabla w + R - w - \lambda w B, \end{aligned} \tag{3}$$

where $\mathbf{v}$ pointed down the hillslope. Simple linear analysis implies that in the absence of cross-diffusion effects (such as those considered recently by [8]), no Turing-like instability exists in that system; the steady state is a uniform covering of all the plane by the amount of flora that corresponds to the stable fixed point and fluctuations of wavenumber k decay like $e^{-k^2 t}$.

In the desert area considered here there are two seasons, dry summer and humid winter. Equations (2) and (3) present the winter, with "smeared" rain events. While annual flora wilt in the summer, perennial shrubs have to survive, so they must reach some *threshold size* before the dry season. If the winter is not long enough to allow for a full development of the plant to its stable fixed point, the survival of a shrub depends on its size at the end of the rainy season, which, in turn, depends on small fluctuations in its initial size and the consumption of water by its neighbors.

In the next winter, the existence of a shrub causes a depletion of the available soil moisture in its immediate neighborhood (roughly speaking, in an area of typical linear size $\sqrt{1/\mu}$), and the chance for another shrub that pops out in the depletion region to reach the threshold is lowered. The whole area is then segregated into a mosaic of water accepting and water contributing patches. This is a non-Turing mechanism that has nothing with the effect of shrubs on the overland flow. While Turing instability is characterized by some typical wavelength that sets the linear size of vegetation and bare soil patches to be equal, our model allows for clusters of arbitrary size, as indicated in Fig. 1.

The optimal segregation of the hillslope, that gives maximal biomass per unit area, is an ordered array of shrubs, each located in the lower end (or, on a flat plane, in the middle) of its contributing area. A regular or distorted "lattice" of flora is then formed, similar to the structure of atoms in a two-dimensional crystal. However, this optimal scenario is rarely accomplished in nature, due to the stochastic character of the growth process itself. For simplicity, let us assume that the seed bank in the soil ensures the development of a perennial shrub if some water exists at the site. As the first shrub pops out in an empty region, the soil moisture in its surroundings (primarily downhill) is depleted, and the next shrub will not grow in this "shadowed" area. Nevertheless, as the next shrub also occurs at random, its position will be uncorrelated with the first, except that it cannot pop out at the shadowed region of the first one, and so on. The process continues until all the slope is shaded ("jammed"). This stochastic growth process yields a random covering of the slope by shrubs, with typical distance between nearest neighbors, but no long-range structure. The random arrangement is, however, extremely robust; although the slope is covered inefficiently by the shrubs, there is not enough source area for the next shrub. Essential changes, such as the death of a plant and a formation of another one, are discontinuous, though they are very rare, unless some intervention (grazing, climate changes) comes from the outside.

In Fig. 2, typical results of a numerical simulation of a threshold-noise model [integration of Eqs. (3) with a threshold at the end of each rainy season, see caption] with no slope are presented. In the simulation, the system freezes rapidly (in 1–2 rainy seasons) to a robust configuration (slower dynamics will increase the efficiency of biomass growth), and then persists, with negligible fluctuations, up to 500 winters. The average amount of flora is much smaller than B_1, and the average amount of water is larger than w_1, i.e., there is an inefficient use of the water due to the stochastic arrangement of the shrubs. Annual flora, that flourish during the spring, use this access water. To guide the eye, soil moisture contours are also plotted (dotted lines, with the water level inside smaller than 0.46) and reveal the depletion zone around each shrub [9]. No empty site in the region maintains enough water to allow for new shrubs to develop, and the whole region is "shaded" by the existing flora. Figure 3 presents the results of a simulation with the same growth and diffusion parameters, but with nonzero downhill slope, and the effect on the moisture depletion zones is evident.

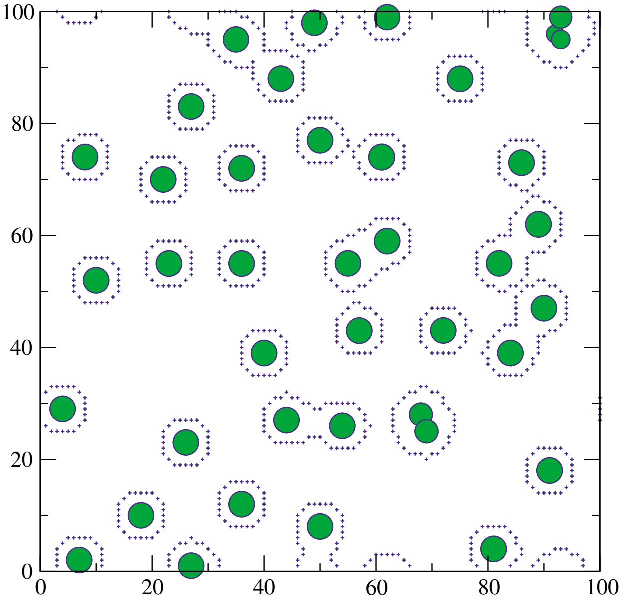

FIG. 2 (color online). Numerical results of forward Euler integration of the reaction-diffusion equations (3) on 100 × 100 sites grid with periodic boundary conditions. A gray spot is plotted around the location of each shrub, with the size of the spot proportional to its biomass. The dotted lines present soil moisture contours around the shrubs. The simulation parameters are $v = 0$ (no slope) and $D = 10$, $\mu = 0.2$, $R = 0.5$, $\lambda = 1.2$ (this implies $B_1 = 1.25$ and $w_1 = 0.2$). Initial conditions are no water and a seed of biomass taken from a square distribution between [0,0.01]. The effect of the summer is modeled at the end of each "winter" (21 time cycles) by setting all water to zero while the flora at a site is dropped to the seed level if the biomass is smaller than a threshold, with $B_{\text{th}} = 1.2$. The average (per site) values of water (about 0.32) and flora (0.57) reflect the inefficient use of water attributed to the glasslike structure.

Various aspects of this competition scenario are similar to the adsorption of large particles at an interface [10]. In the model of random sequential adsorption, "hard" particles are added sequentially to a D dimensional volume at random positions with the condition that no trial particle can overlap previously inserted ones. The addition process is then repeated until the system reaches its "jamming limit," at which the density saturates, and the configuration of adsorbed particles froze at some disordered pattern. A shrub above threshold, with its excluded volume of depleted moisture is similar to an adsorbed "disc." The shrubs-water model, although nonlocal (excess water transferred downhill) and reversible (new shrub may remove an existing plant by depleting its water resources), yields a similar jamming disordered and inefficient covering of the slope.

The above considerations about glassy structure elucidate the existence and the robustness of the vegetation patterns in the arid zone, but fail to explain the aggregation of shrubs (larger correlation length) in the semiarid zone, or the Turing patterns [6,8]. To explain patchiness one should consider the positive-feedback mechanism, i.e., the inhibition of water dynamics induced by the shrubs themselves. In the absence of shrubs (and other meso topography factors), water flows downhill, with some typical lateral displacement per unit length. In the presence of shrubs the flow in their vicinity

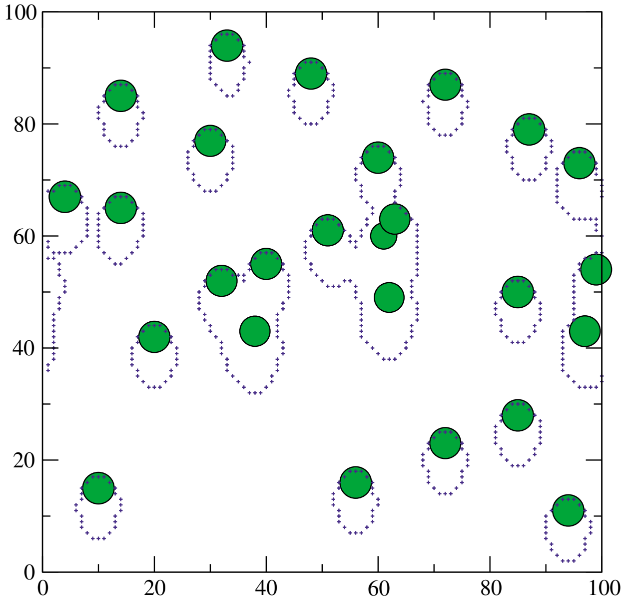

FIG. 3 (color online). Same as Fig. 3 but with a downhill drift of water where the asymmetry parameter is $v = 0.5$. Note the change in the shape of the water contours. Average biomass and water levels are almost the same as in Fig. 2.

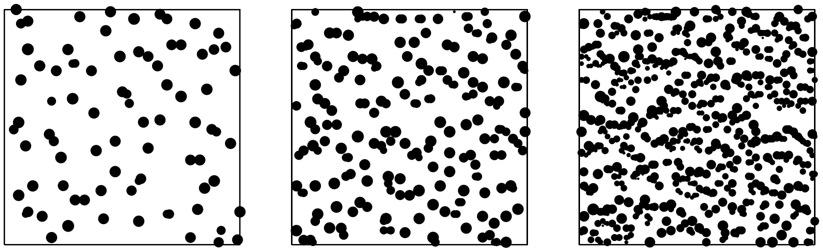

FIG. 4. Effect of the positive feedback mechanism. While at low precipitation ($R = 0.45$, left panel) the repulsive interaction wins and no clustering occurs, at higher humidity ($R = 0.47$, middle panel) clustering is more pronounced, and at $R = 0.5$ (right) vegetation stripes may be recognized. All other parameters are the same as in Fig. 3.

is slower than that in bare soil, as a result of higher infiltration rates. In addition, the microclimate under shrubs is characterized by less direct radiation and smaller evaporation rate [6]. This means that close to the shrub there are favorable soil water conditions and more flora may grow. The "repulsive" interaction among shrubs due to the struggle for water is then balanced by an "attraction." Accordingly, the size of a typical cluster changes along the precipitation gradient, from a single shrub at the arid limit to large clusters in the semiarid regime. The response of the system to external parameters (climate change) and grazing seems to depend on its phase. In particular, it seems that hysteresis loops (desertification transition) such as those described in [8] are a characteristic of the clustered phase. A detailed discussion of these issues, along with more quantitative characteristics of a glassy structure like correlation/response functions and aging will be presented elsewhere.

In Fig. 4, the results of the same simulation program with positive feedback are presented. Diffusion and evaporation remain the same as in (3), and the only new ingredient added to the simulation is a suppression of the asymmetry in the downhill water flow, with the asymmetry term $\mathbf{v}$ multiplied by $\exp[-5B(\mathbf{r})/B_{\mathrm{th}}]$. This is a cross-convection effect (the convective downhill flow is diminished by the shrubs) and it yields naturally dynamical instability, spatial segregation and pattern formation. As the downhill flow becomes smaller, water tends to accumulate at shrub's neighborhood. The transition from the arid (left) zone, with no clustering and glassy structure, to more clustered patterns and even some linear order in the semiarid (right) is evident.

In conclusion, the generic spatial patterns due to the struggle for water are disordered frozen patterns, and the threshold-noise process dictates the vegetation spatial organization in the arid zone. The instability that yields this glassy structure is not Turing-like, thus the inhomogeneity is not characterized by typical length scale. Shrub clusters and Turing patterns emerge as further instability of this glassy structure if the precipitation is large enough, where the positive feedback dominates. It then leads first to clustering of shrubs and then to a global order in the form of "tiger bush" patterns.

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[1] D. J. Tongway and J. A. Ludwig, Aust. J. Ecol. **15**, 23 (1990).

[2] C. Valentin, J. M. Herbes, and J. Poesen, Catena **37**, 1 (1999).

[3] G. F. Gause, *The Struggle for Existence* (Williams and Wilkins, Baltimore, 1934).

[4] D. Tilman, *Resource Competition and Community Structure* (Princeton University Press, Princeton, 1982); J. D. Murray, *Mathematical Biology* (Springer-Verlag, New York, 1993).

[5] J. Huisman and F. J. Weissing, Ecology **75**, 507 (1994); F. J. Weissing and J. Huisman, J. Theor. Biol. **168**, 323 (1994); J. Huisman, R. R. Jonker, C. Zonneveld, and F. J. Weissing, Ecology **80**, 211 (1999).

[6] J. B. Wilson and A. D. Q. Agnew, Adv. Ecol. Res. **23**, 263 (1992); R. Lefever and O. Lejeune, Bull. Math. Biol. **59**, 263 (1997).

[7] *Oscillating* spatial patterns have been observed in a system of coupled chaotic maps due to synchronization failure; see K. Kaneko, Physica (Amsterdam) **34D**, 1 (1989); S.V. Petrovskii and H. Malchow, Theor. Pop. Biol. **59**, 157 (2001).

[8] J. von Hardenberg, E. Meron, S. Shachak, and Y. Zarmi, Phys. Rev. Lett. **87**, 198101 (2001).

[9] Actual field experiments usually find the soil moisture under a shrub to be greater than the moisture a few meters away from it. The reason for that has to do with the positive-feedback mechanisms that reduce water losses in this region. However, up to some limit, this water resource serves the shrub during the summer, and is not available for the use of another planet. One may regard this fraction of soil moisture as part of the biomass, while the contours in Figs. 2–4 represent available water density.

[10] See, e.g., J. Talbot, G. Tarjus, P. R. Van-Tassel, and P. Viot, Colloids Surf. A, **165**, 287 (2000), and references therein.